

Surname First name

University of Rome “Tor Vergata”
BSC in Business & Economics - A.Y. 2014-2015
Course Mathematics
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(Simulation of the) Written examination

Rules: you cannot use any textbook, lecture notes, neither any electronic device (very welcome a traditional watch ...). Write your solutions on the enclosed sheets. You will receive additional sheets for the preliminary drafts. The solutions of the exercises must be detailed. The examination is divided in two Parts. You have **90 minutes** to finish each part. At the end of Part I there is a ten minutes break. An **identity card** or a passport is needed to participate.

Part I

Exercise 1. *20 points*

Study the following function $f(x) = xe^{\left(\frac{1}{x}+x\right)}$, domain, sign, limits, asymptotes, maximum and minimum points and graph.

Exercise 2. *5 points*

Find the infimum, the supremum, the minimum, the maximum, the accumulation points, the boundary points and the interior points of the set

$$A = \left\{ \frac{1}{n} \right\}_{n \geq 1} \cup \left\{ \frac{n-1}{n} \right\}_{n \geq 1}$$

Exercise 3. *5 points*

Establish for which $x \in \mathbb{R}$ the geometric series $\sum_{i=0}^{\infty} (x-1)^i$ converges.

Part II

Exercise 4. *10 points*

Determine whether the following improper integral exists and, if so, evaluate it.

$$\int_1^{+\infty} \frac{\log x}{x^3} dx$$

Exercise 5. *10 points*

Using the Rouché-Capelli theorem find the solutions of the following linear system.

$$\begin{cases} (1-k)x + (k-3)y = 1 \\ 3x - y - z = 1 \\ (4-k)y - 8z = 0 \end{cases}$$

Exercise 6. *(10 points)*

Consider the function

$$f(x, y) = \log \left(\frac{(x^2 + 2)(y^2 + 2)}{\sqrt{xy}} \right).$$

Find: i) the domain; ii) the stationary points; iii) the character of the stationary points (local max, min, saddle).