

Surname ..... First name .....

**University of Rome “Tor Vergata”**  
**BSC in Business & Economics - A.Y. 2014-2015**  
**Course Mathematics**  
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**April 27, 2015**

**(Simulation of the) Written examination**

**Rules:** you cannot use any textbook, lecture notes, neither any electronic device (very welcome a traditional watch ...). Write your solutions on the enclosed sheets. You will receive additional sheets for the preliminary drafts. The solutions of the exercises must be detailed. The examination is divided in two Parts. You have **90 minutes** to finish each part. At the end of Part I there is a ten minutes break. An **identity card** or a passport is needed to participate.

**Part I**

**Exercise 1.** *20 points*

Study the following function  $f(x) = \sqrt{\frac{x-1}{x^2}}$  : domain, sign, limits, asymptotes, maximum and minimum points and graph.

**Exercise 2.** *5 points*

Give the definition of converging sequence and write an example of a sequence converging to  $-2$ .

**Exercise 3.** *5 points*

State and prove the Lagrange Theorem.

**Part II**

**Exercise 4.** *10 points*

Determine whether the following improper integral exists and, if so, evaluate it.

$$\int_{-3}^4 \left( \frac{1}{\sqrt{x+3}} - \frac{1}{\sqrt{4-x}} \right) dx$$

**Exercise 5.** *10 points*

For the matrix

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & k \\ 0 & -k & 1 \end{pmatrix}$$

find the values of  $k \in \mathbb{R}$  (if any) such:

i) the eigenvalues of  $A$  are real;

ii) the matrix  $A$  is orthogonal.

**Exercise 6.** *(10 points)*

Consider the function

$$f(x, y) = 2 \log(x^2 + y^2 + 2) - xy$$

Find: i) the domain; ii) the stationary points; iii) the character of the stationary points (local max, min, saddle).