

Course in Macroeconomics and Global Economics
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Practice 8 Solutions

Exercise 1

1. $\frac{Y_t}{N} = \left(\frac{K_t}{N}\right)^\alpha$
2. $\frac{K_{t+1}-K_t}{N} = s \left(\frac{K_t}{N}\right)^\alpha - \delta \frac{K_t}{N}$
3. In steady state the change in capital per worker is zero. The expression in point 2 becomes:
$$0 = s \left(\frac{K^*}{N}\right)^\alpha - \delta \frac{K^*}{N}$$
$$s \left(\frac{K^*}{N}\right)^\alpha = \delta \frac{K^*}{N}$$
$$\frac{s}{\delta} = \frac{K^*}{N} \left(\frac{N}{K^*}\right)^\alpha$$
$$\frac{s}{\delta} = \left(\frac{K^*}{N}\right)^{1-\alpha}$$
$$\frac{K^*}{N} = \frac{s}{\delta}^{\frac{1}{1-\alpha}} = \left(\frac{0.32}{0.08}\right)^{\frac{3}{2}} = 4^{\frac{3}{2}} = 8;$$
$$\frac{Y^*}{N} = \left(\frac{K^*}{N}\right)^\alpha = 8^{\frac{1}{3}} = 2;$$
$$\frac{C^*}{N} = \frac{Y^*}{N} - \delta \frac{K^*}{N} = 2 - 0.08(8) = 2 - 0.64 = 1.36.$$
4. $\frac{K^*}{N} = \frac{s}{\delta}^{\frac{1}{1-\alpha}} = \left(\frac{0.16}{0.08}\right)^{\frac{3}{2}} = 2^{\frac{3}{2}} = 2.82$
$$\frac{Y^*}{N} = 2.82^{\frac{1}{3}} = 1.41.$$
5. The graph is the same as that reported at pag. 312 of the textbook, with the only difference that the investment per worker curve shifts downward, instead of upward, as in this exercise $s_0 > s_1$.

Exercise 2

1. $s = 0.16$; $\delta = 0.1$; $g_N = 0.02$; $g_A = 0.04$.

a) $s(\sqrt{\frac{K}{AN}})^* = (\delta + g_A + g_N)(\frac{K}{AN})^*$

$$\frac{s}{\delta + g_A + g_N} = (\frac{K}{AN})^* (\sqrt{\frac{AN}{K}})^*$$

$$\left(\sqrt{\frac{K}{AN}}\right)^* = \frac{s}{\delta + g_A + g_N}$$

$$\left(\frac{K}{AN}\right)^* = \left(\frac{s}{\delta + g_A + g_N}\right)^2 = \left(\frac{0.16}{0.1 + 0.04 + 0.02}\right)^2 = \left(\frac{0.16}{0.16}\right)^2 = 1;$$

b) $\left(\frac{Y}{AN}\right)^* = \left[\left(\frac{K}{AN}\right)^*\right]^{\frac{1}{2}} = 1$

c) $g\left(\frac{Y}{AN}\right)^* = g_{Y^*} - g_A - g_N = 0$ by definition

d) $g\left(\frac{Y}{N}\right)^* = g_{Y^*} - g_N = g_A = 4\%$

e) $g(Y)^* = g_{Y^*} = g_A + g_N = 6\%$

2. a) $\left(\frac{K}{AN}\right)^* = \left(\frac{s}{\delta + g_A + g_N}\right)^2 = \left(\frac{0.16}{0.1 + 0.08 + 0.02}\right)^2 = \left(\frac{0.16}{0.20}\right)^2 = (0.8)^2 = 0.64$

b) $\left(\frac{Y}{AN}\right)^* = \left[\left(\frac{K}{AN}\right)^*\right]^{\frac{1}{2}} = 0.8$

c) $g\left(\frac{Y}{AN}\right)^* = g_{Y^*} - g_A - g_N = 0$ by definition

d) $g\left(\frac{Y}{N}\right)^* = g_{Y^*} - g_N = g_A = 8\%$

e) $g(Y)^* = g_{Y^*} = g_A + g_N = 10\%$

After an increase in g_A we find a decrease in the steady state value of $\left(\frac{K}{AN}\right)^*$ and, consequently, of $\left(\frac{Y}{AN}\right)^*$. The reason is that *effective labor* now grows faster and it is not counterbalanced by an increase in the saving rate, s . Instead, $g\left(\frac{Y}{N}\right)^*$ and g_{Y^*} are now higher because of the higher rate of technological progress.

3. a) $\left(\frac{K}{AN}\right)^* = \left(\frac{s}{\delta + g_A + g_N}\right)^2 = \left(\frac{0.16}{0.1 + 0.04 + 0.06}\right)^2 = \left(\frac{0.16}{0.20}\right)^2 = (0.8)^2 = 0.64$

b) $\left(\frac{Y}{AN}\right)^* = \left[\left(\frac{K}{AN}\right)^*\right]^{\frac{1}{2}} = 0.8$

c) $g\left(\frac{Y}{AN}\right)^* = g_{Y^*} - g_A - g_N = 0$ by definition

d) $g\left(\frac{Y}{N}\right)^* = g_{Y^*} - g_N = g_A = 4\%$

e) $g(Y)^* = g_{Y^*} = g_A + g_N = 10\%$

Consumption per *effective worker* in point 1):

$$\left(\frac{C}{AN}\right)^* = \left(\frac{Y}{AN}\right)^* - (\delta + g_A + g_N)\left(\frac{K}{AN}\right)^* = 1 - 0.16(1) = 0.84.$$

Consumption per *effective worker* in point 3):

$$\left(\frac{C}{AN}\right)^* = \left(\frac{Y}{AN}\right)^* - (\delta + g_A + g_N)\left(\frac{K}{AN}\right)^* = 0.8 - 0.20(0.64) = 0.8 - 0.128 = 0.672.$$

Consumption per *effective worker* is higher in point 1) than in point 3). Again, this result comes from the fact that the increase in the growth rate of *effective labor* is not counterbalanced by an increase in the saving rate, s , and $\left(\frac{K}{AN}\right)^*$ is lower in 3) than in 1).