



Social Networks

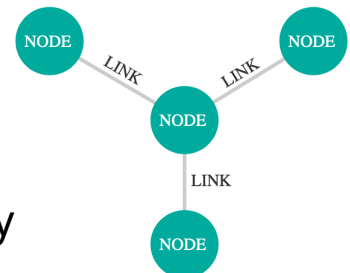
Algorithms, Data and Security
A.Y. 2025/26

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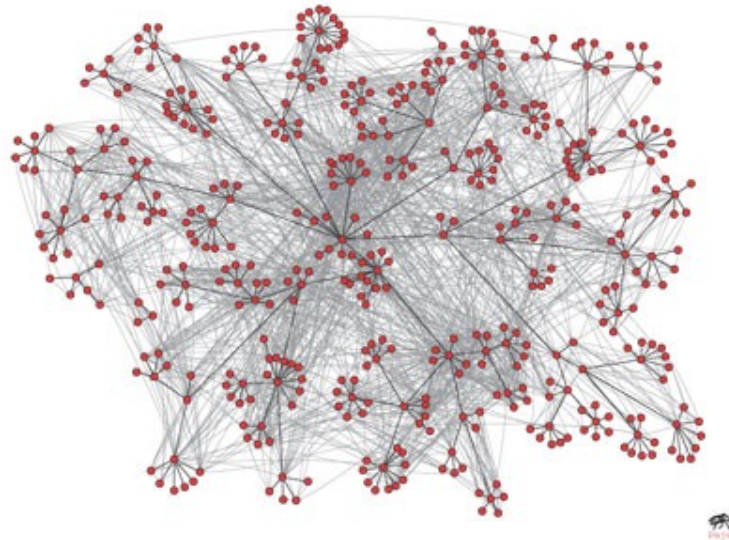
Networks

- Networks are representations of **interaction** structures among units
 - These units are **nodes**
 - Some pairs of nodes are connected by **links**
 - In social and economic networks, nodes represent individuals or organizations
- The study of networks encompasses different types of interactions, e.g.,
 - Transportation
 - Communication
 - **Social**
 - Spread of epidemics



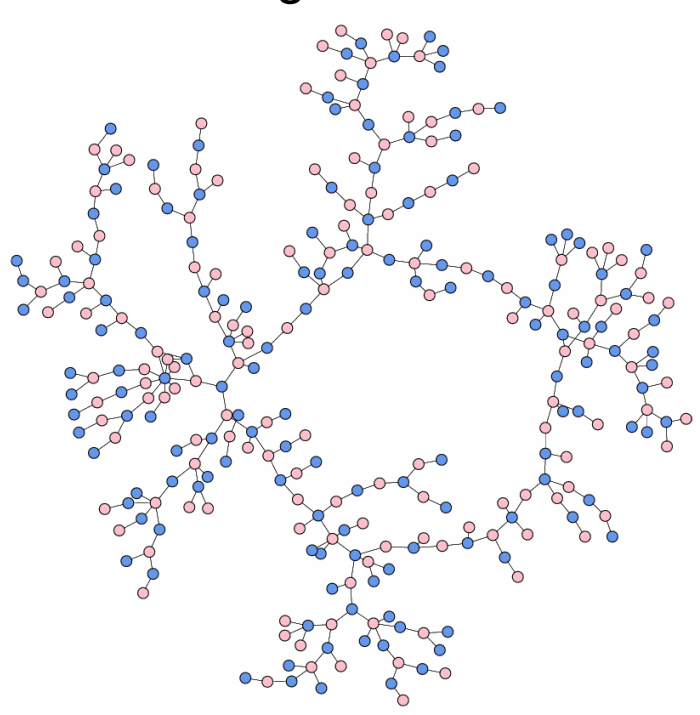
Examples of social networks

- Corporate e-mail communication
 - Links represent e-mail exchanges between individuals



Examples of social networks

- High school dating



Examples of social networks

- Trails of Flickr users in Manhattan

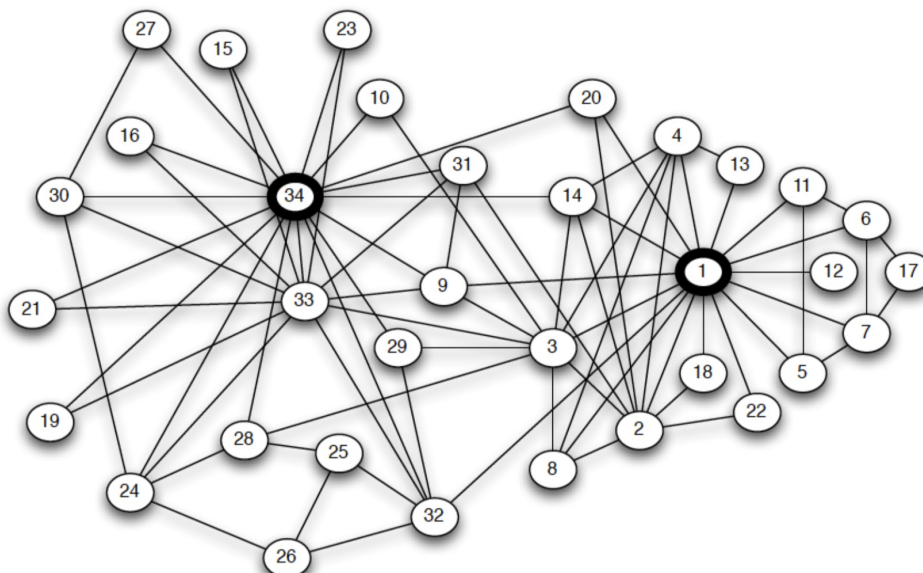


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Examples of social networks

- Friendships network within a 34-person karate club

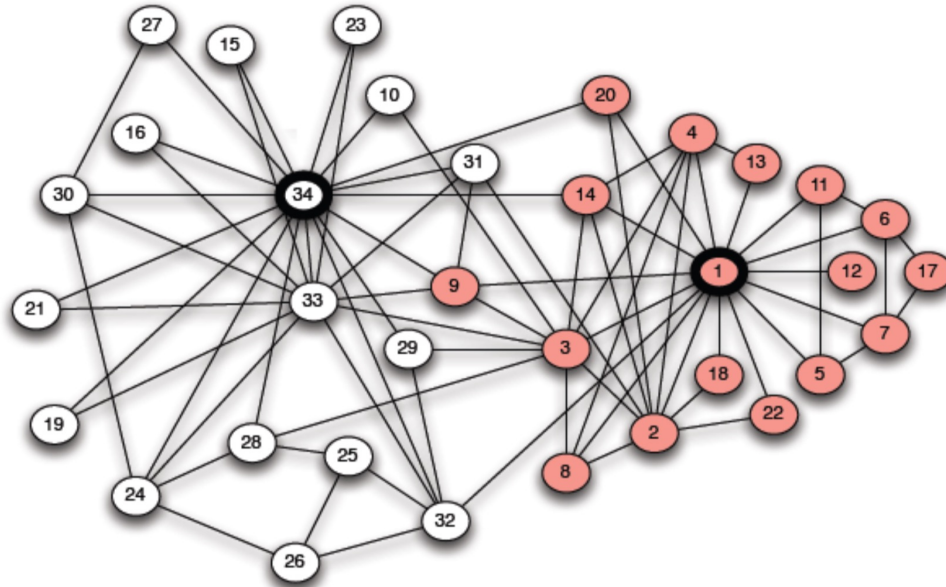


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Examples of social networks

- The friendship network within a 34-person karate club provides clues to the fault lines that eventually led to the split of the club



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Networks: structure and behavior

- Networks are complex and highly connected systems
- Two main perspectives to analyze them
 1. **Structural connectedness**
 - Focus: who is linked to whom (**graph theory**)
 - E.g., in a social network like Instagram: network structure, centrality measures, communities, etc.
 2. **Behavioral connectedness**
 - Focus: how individuals' actions have implicit consequences for the entire system (**game theory**)
 - E.g., users choose what content to share or engage with

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Network dynamics: population effects

- In large populations, ideas, innovations, and behaviors emerge and evolve over time
 - Can be grouped as **social practices** (opinions, products, behaviors) that individuals may adopt or not
- The spread of social practices depends largely on **social influence**
- Why do people follow others?
 - Due to the human tendency to **conform**
- But even rational individuals (no desire to conform) may copy others' behavior
 - Why? Because others' actions convey information: **information cascades**

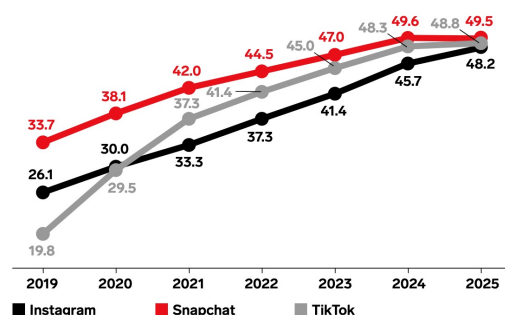
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Network dynamics: population effects

- Adoption of technologies or behaviors can spread because of:
 - Informational effects → others' choices signal useful information
 - Direct benefits → more value when many people adopt (**network effects**)

US Gen Z Instagram, Snapchat, and TikTok Users, 2019-2025
millions



Note: individuals born between 1997-2012 who access their account via any device at least once per month
Source: eMarketer, May 2021

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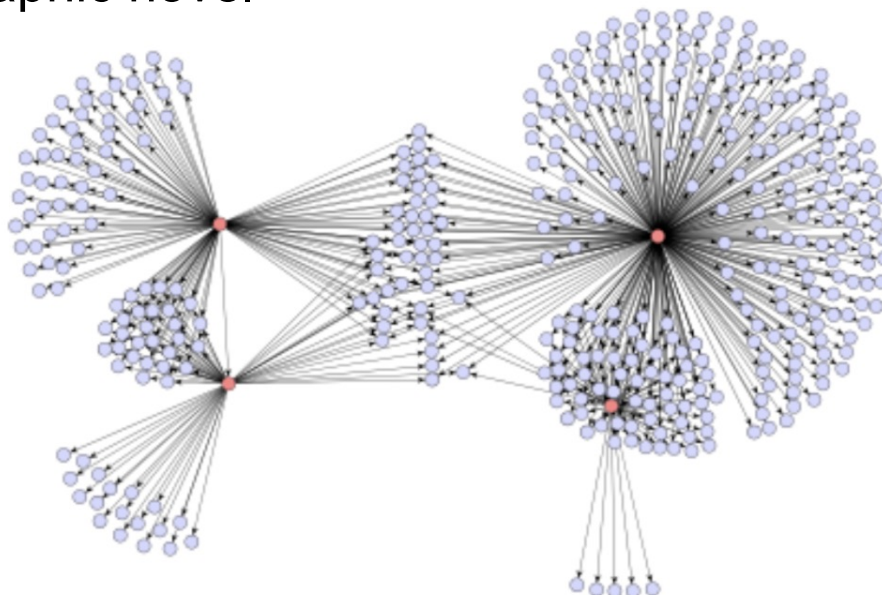
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Network dynamics: structural effects

- Taking network structure into account helps explain how influence spreads
- Mechanisms (information and direct benefits) act at:
 - Global level (entire population)
 - Local level (friends, colleagues, neighbors)
- Individuals are influenced by their network **neighbors**, enabling **cascades of adoption**

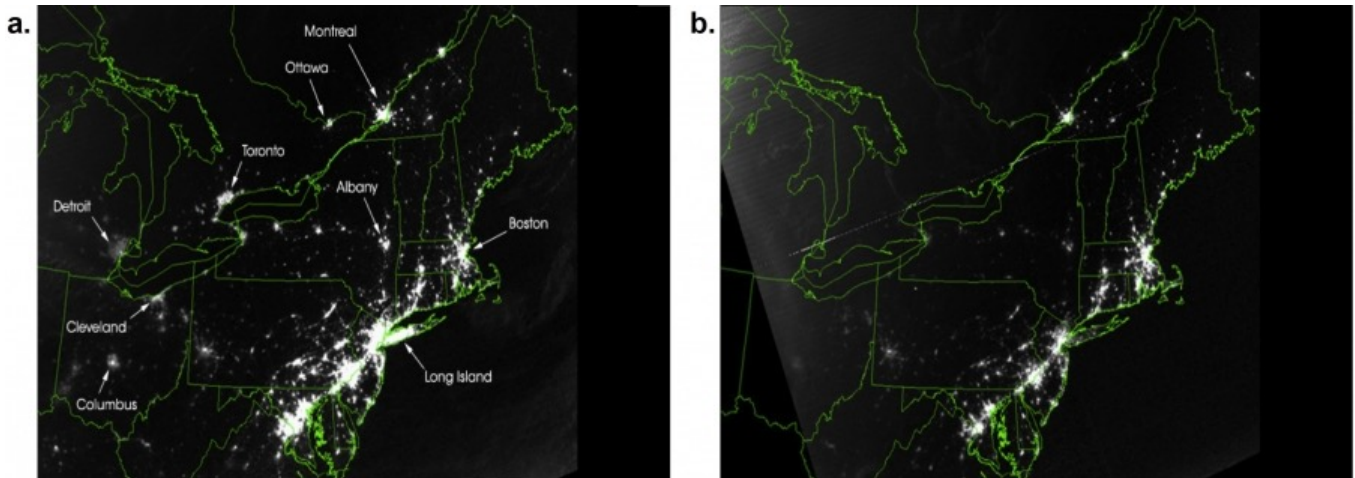
Network dynamics: structural effects

- Example of social contagion: recommendations by e-mail spreading a graphic novel



Network dynamics: structural effects

- Example of cascading failure: Northeast blackout of 2003



a) Satellite image on Northeast US on August 13th, 2003, at 9:29pm (EDT), 20 hours before the blackout

b) The same, but 5 hours after the blackout

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Vulnerability due to interconnectivity

- **Cascading failures** in networks
- Networks can act like transport systems
- A local failure redistributes load to nearby nodes
- If load is small → system absorbs it
- If load is large → neighbors overload and pass it on
- This can trigger a cascade, whose scale depends on:
 - Initial node position
 - Node capacity

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Networks: interdisciplinarity

- Theory of network structure and behavior addresses challenges across multiple fields:
- Economics
 - Strategic interaction among small groups
 - Aggregate behavior in large, homogeneous populations
- Sociology
 - Key insights into social network structure
 - Traditionally limited to small-scale data (10–100 individuals)
- Computer science
 - Rise of the Web and social media introduces large-scale systems
 - Constraints are both technical and human
 - Systems shaped by feedback from users (communication, self-expression, knowledge creation)

Network science

- Academic field which studies **complex networks**
 - Including telecommunication, computer, biological, cognitive, semantic, and social networks
- Networks are modeled as:
 - **Nodes** (vertices) → elements or actors
 - **Edges** (links) → connections between them
- Theories and methods from multiple disciplines
 - Mathematics: graph theory
 - Physics: statistical mechanics
 - Computer science: data mining, information visualization
 - Statistics: inferential modeling

Social networks

- We focus on **social networks** as a type of complex network
- Components:
 - Actors: individuals (e.g., students, employees) or organizations
 - Represented as nodes (vertices)
 - Relations: connections between actors
 - Represented as links (edges)
 - Relations can capture
 - Friendship
 - Advice
 - Communication
 - Hindrance

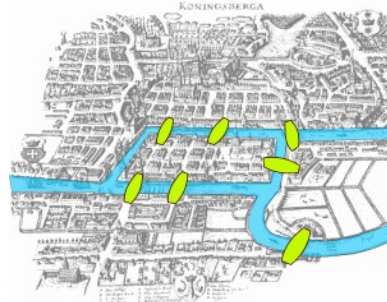
A brief history of networks (graphs)

- The field begins with Leonhard Euler (1735)
- His work on the Seven Bridges of Königsberg laid the foundations of graph theory
- *Problem*: in the city of Königsberg, 7 bridges connected parts of the town across the Pregel river
- *Question*: is it possible to cross all 7 bridges exactly once?
- *Insight*: model land areas as nodes and bridges as links

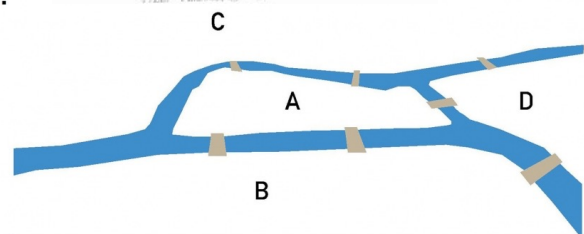
Seven bridges of Königsberg

- Is it possible to cross all 7 bridges exactly once?
- Euler represented the city as a graph:
 - Nodes → 4 land areas (A, B, C, D)
 - Links → 7 bridges connecting them
- Result:
 - No continuous path exists that crosses each bridge exactly once

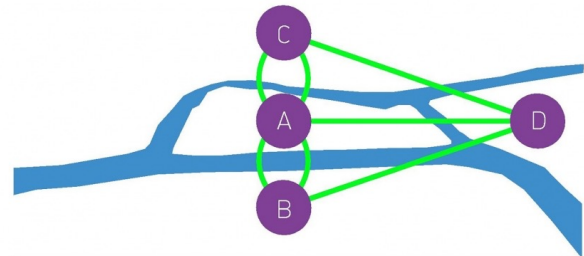
a.



b.



c.

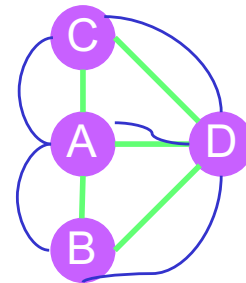
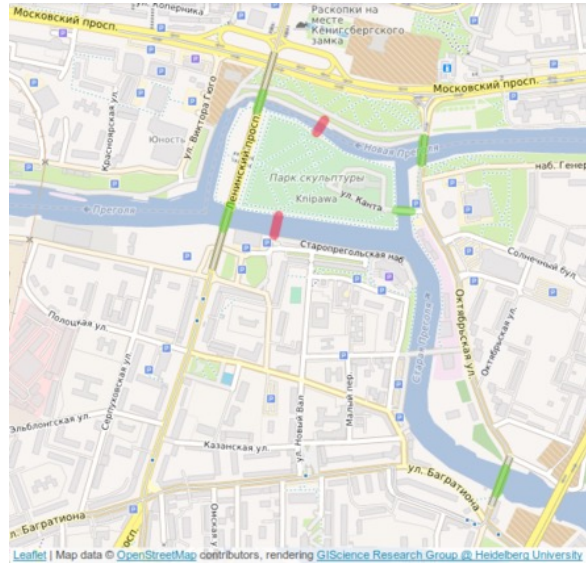


Seven bridges of Königsberg

- Euler showed that the possibility of traversing each link exactly once depends on node degrees
 - Node degree: number of links touching a node
- Necessary condition for Eulerian path: the graph is connected and exactly 0 or 2 nodes have odd degree
 - Königsberg graph: 4 nodes (A, B, C, and D) have all odd degree → no Eulerian path
- Euler's theorem: a graph is Eulerian if and only if:
 - All nodes have even degree, or
 - Exactly 2 nodes have odd degree

Seven Bridges of Königsberg: today

- 2 bridges were destroyed in World War II, 2 bridges later demolished and replaced, 3 bridges remain
- Now 2 nodes (B and C) have degree 2, and the other two have degree 3
- Eulerian path is now possible, must start at one odd-degree node (A or D) and end at the other (D or A)
 - E.g., A -> B -> D -> C -> A -> D



Networks or graphs?

- In scientific literature, network and graph are often used interchangeably:

Network science

Network

Node

Link

Graph theory

Graph

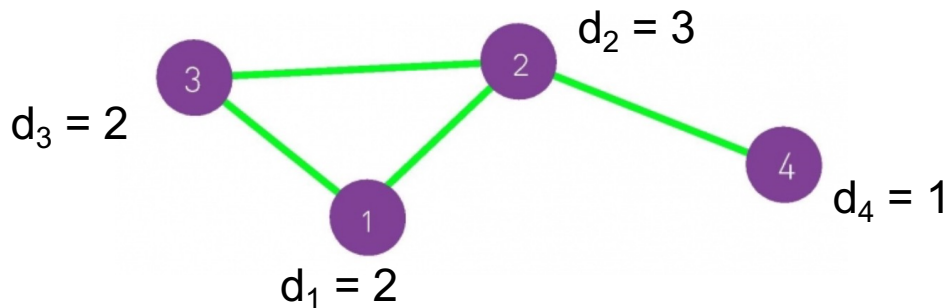
Vertex

Edge

- Network emphasizes real-world systems (e.g., social, biological, technological)
- Graph emphasizes the mathematical abstraction

Graph theory: degree

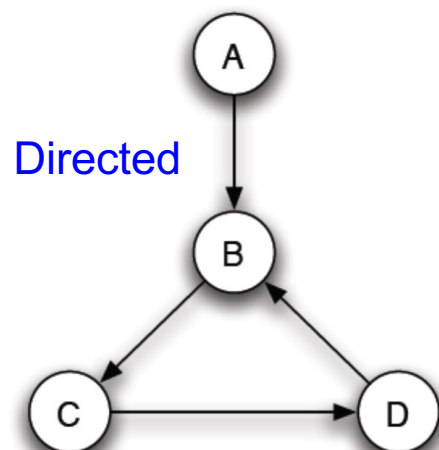
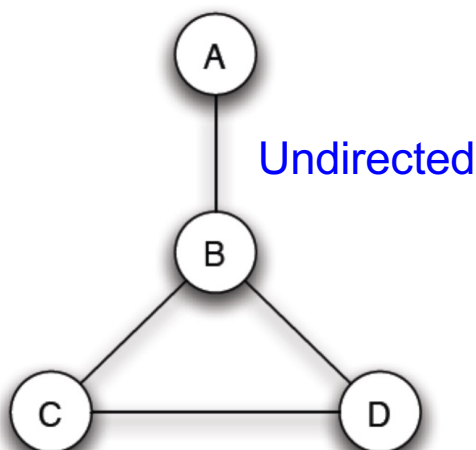
- **Node degree**: number of links a node has to other nodes



- **Average degree**: average of all node degrees in the graph
 - Example: $(2+3+2+1)/4 = 2$

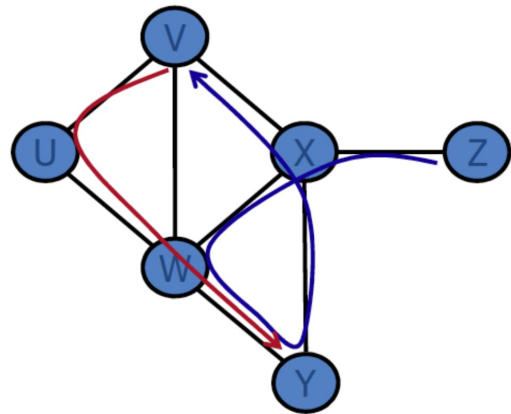
Graph theory: undirected or directed

- **Undirected** or **directed** graph
 - Links can be directed or undirected
 - Directed graph: all links have a direction (from one node to another)
 - Undirected graph: all links have no direction



Graph theory: path

- **Path**: a sequence of nodes where each consecutive pair is connected by a link
 - Red path: $V \rightarrow U \rightarrow W \rightarrow Y$
- **Length** of the path: number of links it contains
 - Red path has length 3
 - Blue path has length 5



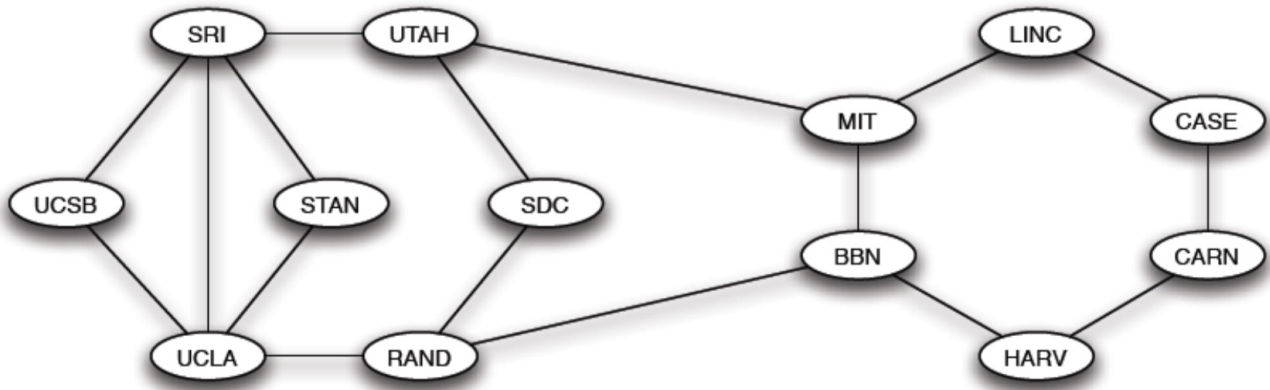
Graph theory: shortest path

- **Shortest path**: a path with the fewest links (or minimal cost) between nodes
- Several algorithms exist to find shortest paths
 - E.g., **Dijkstra's algorithm**: finds shortest paths from a source node to all other nodes
 - Applications:
 - Used by Google Maps to compute shortest routes
 - Routes can depend on current traffic conditions



Graph theory: cycle

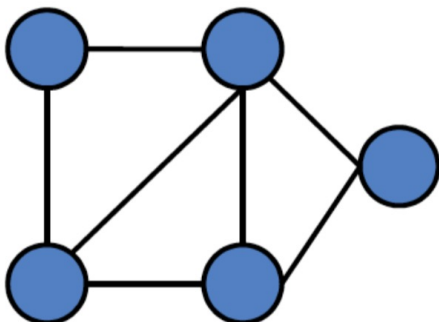
- **Cycle or loop:** a path where the first and last node are the same, and all other nodes are distinct
 - E.g., SRI -> STAN -> UCLA -> SRI



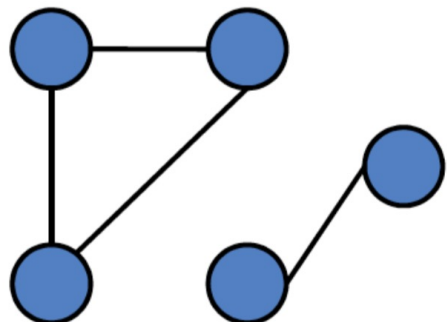
Graph theory: connectivity

- An **undirected** graph is **connected** if, there is a path between every pair of nodes

Connected



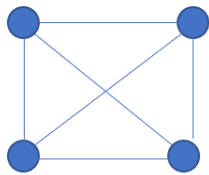
Disconnected



Graph theory: connectivity

- In a **fully connected undirected** graph with N nodes, every pair of nodes is connected by a link and the number of links is equal to

$$\frac{N \times (N - 1)}{2}$$



Example: $N=4$

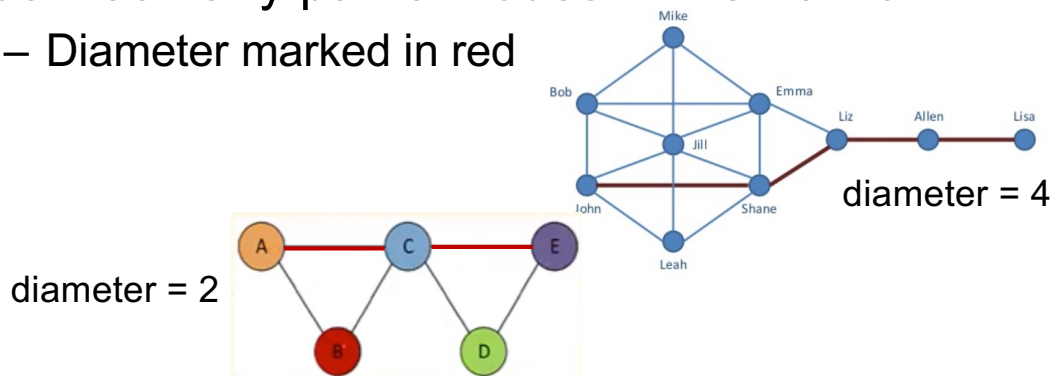
Number of links is $\frac{4 \times 3}{2} = 6$

Social network analysis

- **Social network analysis (SNA)** studies how individuals connect and how information flows in a network
- Helps identify: influencers/key players, communities, disconnected individuals
- Based on models of:
 - Relationships and interactions
 - Emerging patterns from these relationships and interactions
- Importance of SNA across many fields
 - Sociology, business, engineering, economics, politics, security, computer science, mathematics, physics, biology

SNA: network measures

- To characterize a network's structure, we consider some measures
- Network size**
 - Diameter
 - Average shortest path
- Diameter**: length of the longest shortest between any pair of nodes in the network
 - Diameter marked in red

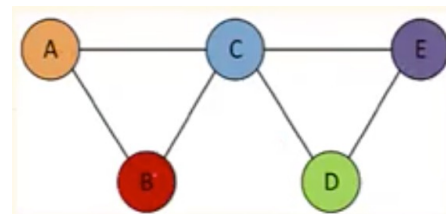


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SNA: network measures

- Average shortest path length**: average of the shortest path lengths (SPL) between all pairs of nodes
 - Less extreme than diameter: a more typical distance between nodes in the network
 - Example:



Average SPL is:

$$\frac{1 + 1 + 2 + 2 + 1 + 2 + 2 + 1 + 1 + 1}{10} = 1.4$$

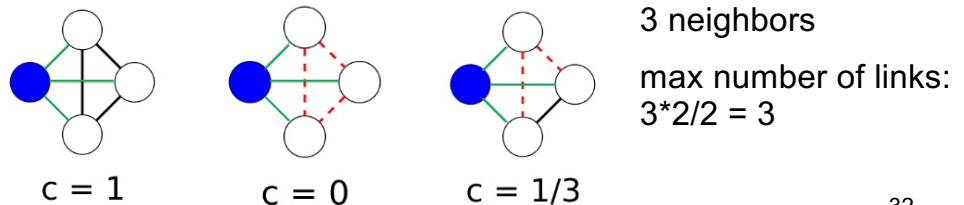
- SPL A-B: 1
- SPL A-C: 1
- SPL A-D: 2
- SPL A-E: 2
- SPL B-C: 1
- SPL B-D: 2
- SPL B-E: 2
- SPL C-D: 1
- SPL C-E: 1
- SPL D-E: 1

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SNA: network measures

- **Clustering coefficient**: measures the tendency of nodes to form clusters; can be local (node) or global (network)
 - **Clustering coefficient of a node**: number of links between neighbors of that node divided by the maximum number of possible links between them (if node v has m_v neighbors, $\max = m_v(m_v-1)/2$)
 - **Average clustering coefficient**: averaged over all nodes in the network
 - E.g., clustering coefficient of blue node



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How to build a network

- Network science aims to build models that reproduce properties of real networks
- How to build a **model for network formation**
- We consider 3 network models
 - **Erdos-Renyi** model: random networks
 - **Small-world** model (or Watts-Strogatz): small-world networks
 - **Scale-free** model (or Barabási-Albert): scale-free networks
- To evaluate models, we consider
 - **Average clustering coefficient**
 - **Average shortest path length**

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Network models: Erdos-Renyi

- **Random network**: nodes are randomly connected to each other (i.e., choices independent of current network structure)
- How to build the random network
 1. Choose parameters N and p
 - N : number of nodes in the network
 - p : probability that two nodes are connected
 2. Generate N nodes (no links yet)
 3. Add links randomly
 - For each pair of nodes (i, j) :
 - Flip a coin with probability p
 - If success $\rightarrow$ add a link between i and j
 - If not $\rightarrow$ no edge

<https://www.networkpages.nl/CustomMedia/Animations/RandomGraph/ERRG/AddoneEdgepATime.html>

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Network models: Erdos-Renyi

- Erdos-Renyi properties
 - Node degree follows **binomial distribution**

$P(deg(v) = k)$: probability that node v has degree k

$$P(deg(v) = k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

- Average degree is $p(N-1) \approx pN$
- Average clustering coefficient is p , a low value
- Average shortest path length is

$$\frac{\ln(N)}{\ln((N-1)p)} \approx \frac{\ln(N)}{\ln(Np)}$$

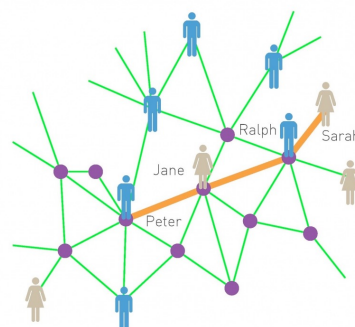
- i.e., very short paths even in large networks
- Homogeneous network: all nodes are similar

Network models: Erdos-Renyi

- Erdos-Renyi properties
 - Low clustering and homogeneity do not match many real-world networks
- Real-world network properties:
 - High clustering
 - Presence of hubs (high-degree nodes), i.e., non-homogeneous network
 - Small-world property: average distance between nodes grows logarithmically with N

Network models: Watts-Strogatz

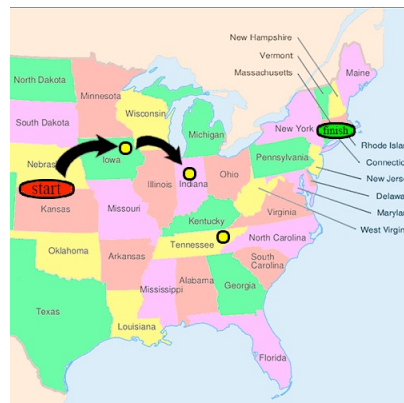
- Small-world network
- Small-world phenomenon (aka six degrees of separation): any two individuals anywhere are connected by a path of at most six acquaintances
 - Distance between nodes is unexpectedly small, even in large networks



Small-world: experimental confirmation

- Small-world experiments by Stanley Milgram (1967)
- Goal: measure **average path length** in US social networks of people
 - Letters sent from randomly chosen residents of Wichita (Kansas) and Omaha (Nebraska) to 2 target persons in Boston
 - Recipients instructed to forward letters to someone who could best reach the target
- Findings:
 - Human society behaves like a small-world network
 - **Short path lengths** connect people across large populations
- Note:
 - Experiment is associated with “six degrees of separation”, though Milgram did not use this term himself

Small-world: experimental confirmation



Example path in Milgram's small world experiment

https://en.wikipedia.org/wiki/Small-world_experiment



Small-world: experiment repeated

- Global, Internet-based social search experiment
 - Setup
 - 18 target persons in 13 different countries
 - 60,000+ participants: asked to forward an e-mail message to a social acquaintance “closer” to the target
 - Results
 - 24,163 message chains created
 - 384 reached their targets
 - average path length 4.0
 - Insights:
 - Confirms small-world property on a global scale
 - Success depends strongly on individual incentives and cooperation

Dodds et al., An Experimental Study of Search in Global Social Networks, *Science*, 2003 <https://www.science.org/doi/pdf/10.1126/science.1081058>

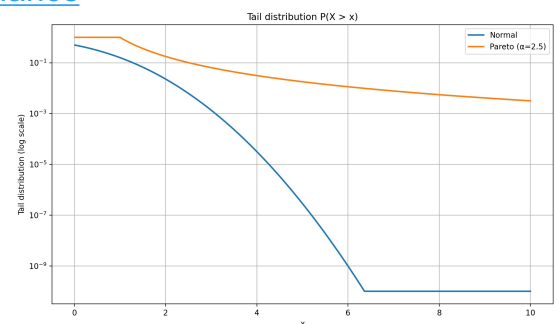
Network models: Watts-Strogatz

- Properties:
 - Small average shortest path length: most nodes reachable in few hops
 - High clustering coefficient: nodes tend to form tight clusters, independent of network size
- Most nodes are not direct neighbors, but can be reached via short paths
 - Example: social networks
- Small-world effect explains why information spreads quickly in social networks, epidemics, or Internet
- Limitation: does not generate **hubs** (nodes with very high degree)

Network models: Barabási-Albert

- **Scale-free** network
 - Node degree follows a **power law** distribution

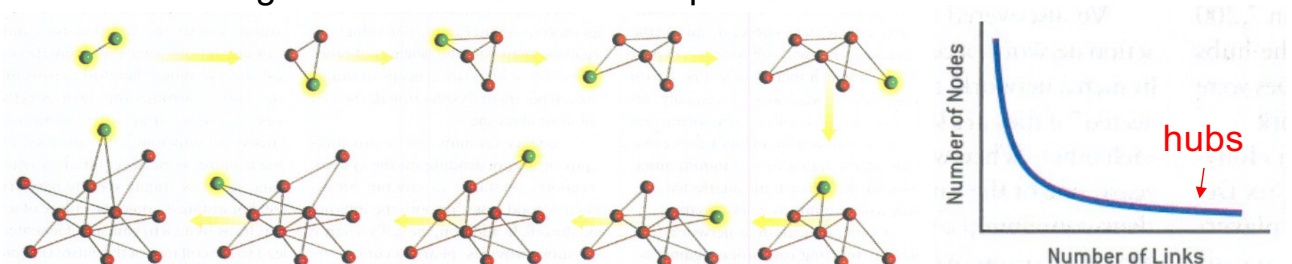
$$P(\deg(v) = k) \sim k^{-\alpha} \quad \text{where } 2 < \alpha < 3$$
 - A few nodes (**hubs**) have very high degree
 - Most nodes have low degree
- **Power law**: event frequency varies as a power of some attribute
 - Examples: wealth distribution, population size of cities
 - **Scale invariance**: same pattern observed at different scales
https://en.wikipedia.org/wiki/Scale_invariance
 - **Heavy-tailed**: the tail contains significant probability → small number of influential nodes
- In a scale-free network, hubs dominate connectivity



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Network models: Barabási-Albert

- New nodes connect to existing nodes based on **preferential attachment**
 - The more connected a node is, the more likely it is to receive new links (i.e., rich-gets-richer)
 - Think about connections in social networks
 - Consequences
 - Formation of hubs
 - Most nodes have few links
 - Degree distribution follows a power law

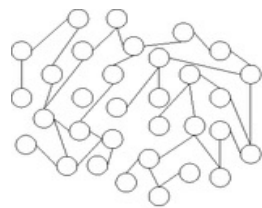


Source: <https://www.jstor.org/stable/26060284>

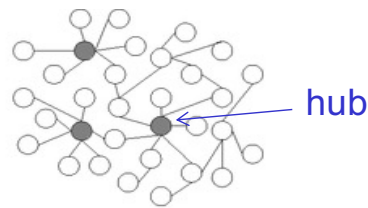
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Network models: Barabási-Albert

- **Scale-free property**: network diameter $\sim \ln(\ln N)$
 - Distances grow extremely slowly with network size
- Scale-free networks are **robust to random failures**
 - Removing random nodes $\rightarrow$ little impact
- But **vulnerable to targeted attacks**
 - Removing hubs $\rightarrow$ network quickly fragments
- Many networks are conjectured to be scale-free: Internet, Web, social networks, power grids



(a) Random network



(b) Scale-free network

Network models: comparison

	Random	Small-World	Scale-Free
Visualisation of the Topology			
Degree Distribution			
Robustness Characteristics	Responds similarly to both random and targeted attacks.		Resilient against random failures but very sensitive to targeted attacks.

Source: <https://appliednetsci.springeropen.com/articles/10.1007/s41109-017-0053-0>

SNA measures: centrality

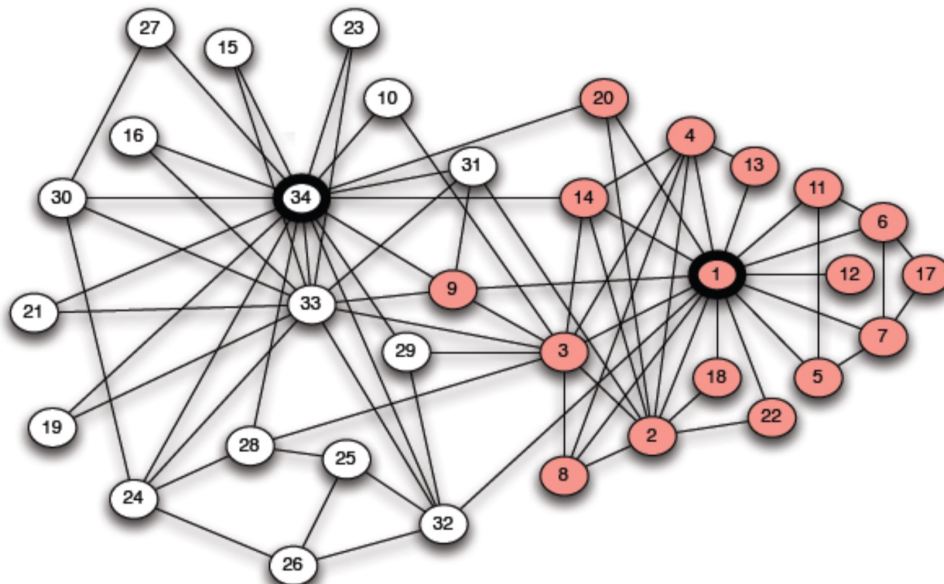
- We analyze **centrality** measures

Which are the most important nodes in the network?

- Useful to identify:
 - Influential individuals in social networks
 - Critical infrastructure nodes (Internet, transport, urban systems)
 - Super-spreaders in disease transmission

SNA measures: centrality

- The friendship network within a 34-person karate club provides clues to the fault lines that eventually led to the split of the club

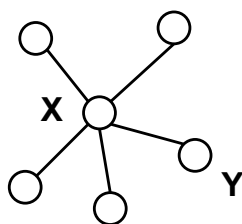


SNA measures: centrality

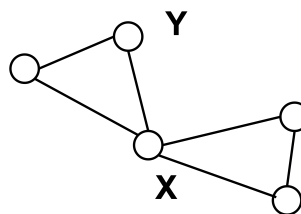
- Which nodes are the most central?
- Definition of centrality depends on context and purpose
- Local measure:
 - Degree centrality
- Global measures:
 - Betweenness centrality
 - Closeness centrality
 - PageRank centrality
- For each measure: the higher the score, the more central the node

Centrality: who is important?

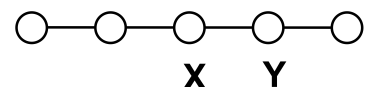
- Centrality measures importance based on network position
- Different measures highlight different kinds of importance
- Example: in each network X has higher centrality than Y according to a particular measure



degree

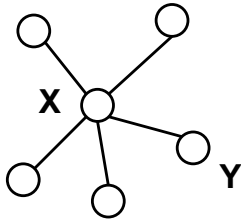


betweenness



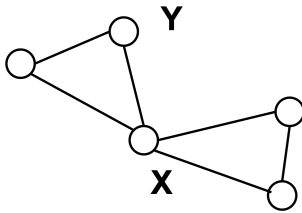
closeness

Centrality: who is important?



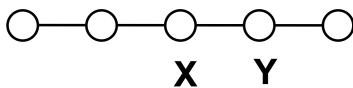
degree

- Degree: X has more direct connections than Y



betweenness

- Betweenness: X lies on more shortest paths connecting others



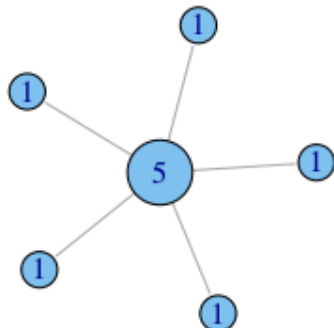
closeness

- Closeness: X can reach all other nodes faster than Y

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Degree centrality

- A node is important if it has many neighbors
- Count **how many neighbors a node has**



Degree centrality of a node: the **degree** of that node

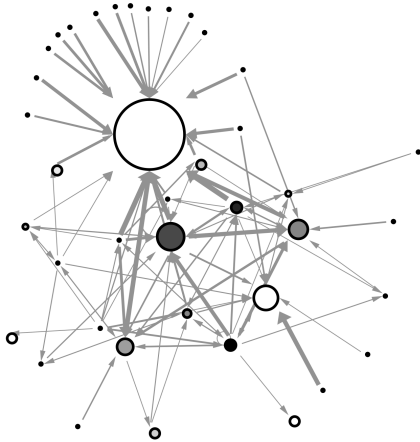
- Node degree centrality $C_D(i)$ can be extended to **whole graph**: C_D

$$C_D = \frac{\sum_{i=1}^N (C_D(n^*) - C_D(i))}{(N-1)(N-2)}$$

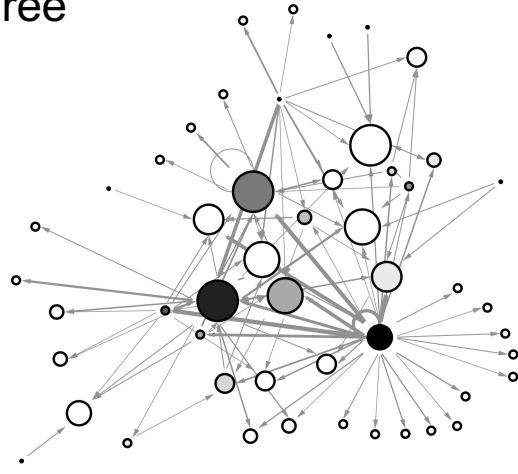
$C_D(n^*)$: degree of the most connected node

Degree centrality: examples

- Examples of financial trading networks
 - Nodes are sized by degree



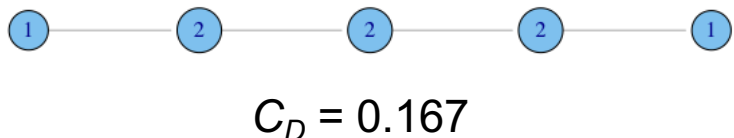
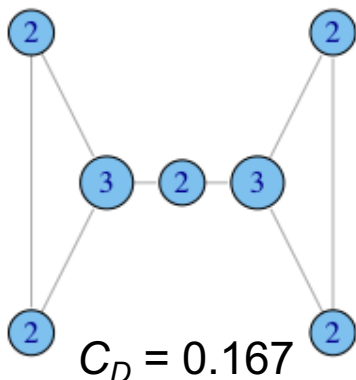
High centralization:
one node trading with
many others



Low centralization:
trades are more
evenly distributed

When degree is not everything

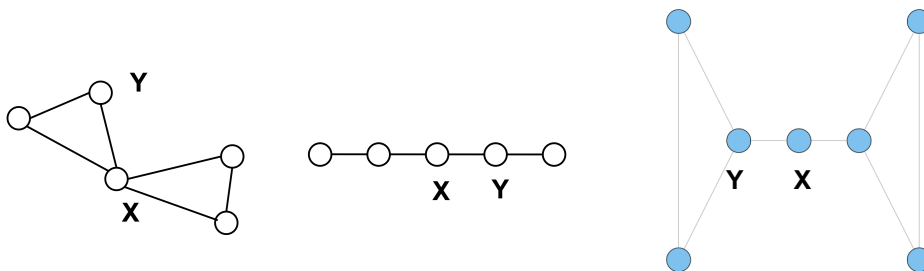
- In what ways does degree fail to capture global importance in the networks below?



- Ability to broker between groups
- Likelihood that information reaches you quickly

Betweenness: another centrality measure

- Intuition: how many pairs of individuals would have to go through you to reach each another node in the minimum number of hops?
- Betweenness centrality measures the **number of times a node acts as a bridge along the shortest path between each pair of nodes**
- Who has higher betweenness, X or Y?

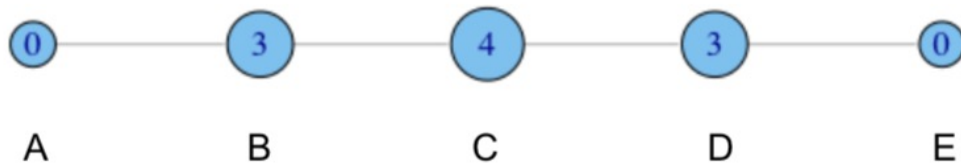


Betweenness: definition

- $C_B(i)$, betweenness of node i :
 - V : set of nodes
 - $\sigma_{jk}(i)$: number of shortest paths from j to k that pass through i
 - σ_{jk} : total number of shortest paths from j to k

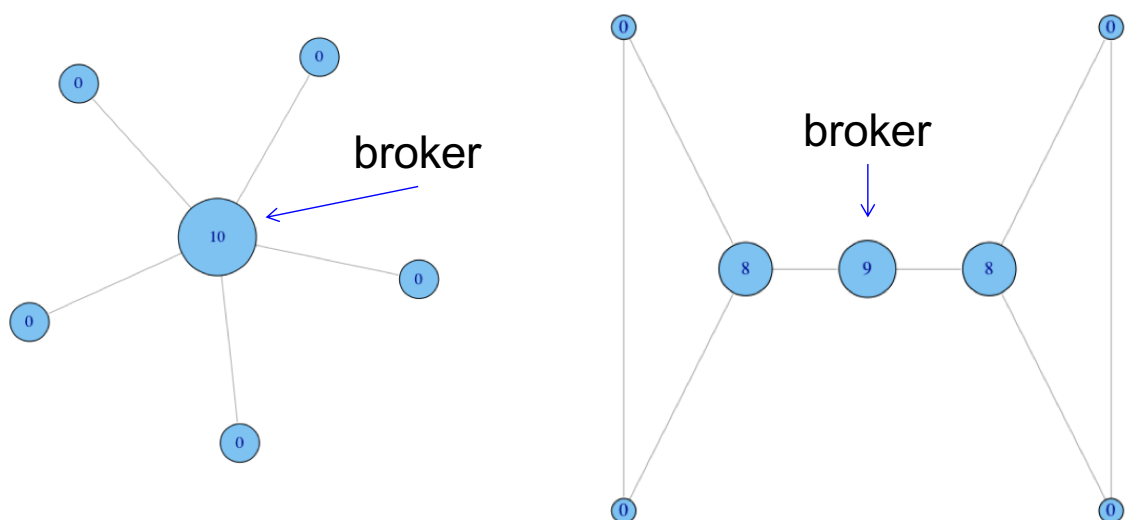
$$C_B(i) = \sum_{i \neq j \neq k \in V} \frac{\sigma_{jk}(i)}{\sigma_{jk}}$$

Betweenness: example



- $C_B(A) = C_B(E) = 0$: A and E never act as bridges
- $C_B(B) = 3$: B acts as a bridge 3 times along the shortest paths between A and C, A and D, A and E
 - Note: only one shortest path per pair, so denominator is 1
- $C_B(C) = 4$: C acts as a bridge 4 times along the shortest paths between A and D, A and E, B and D, B and E
- Highest betweenness: most strategic position for control or influence

Betweenness: example

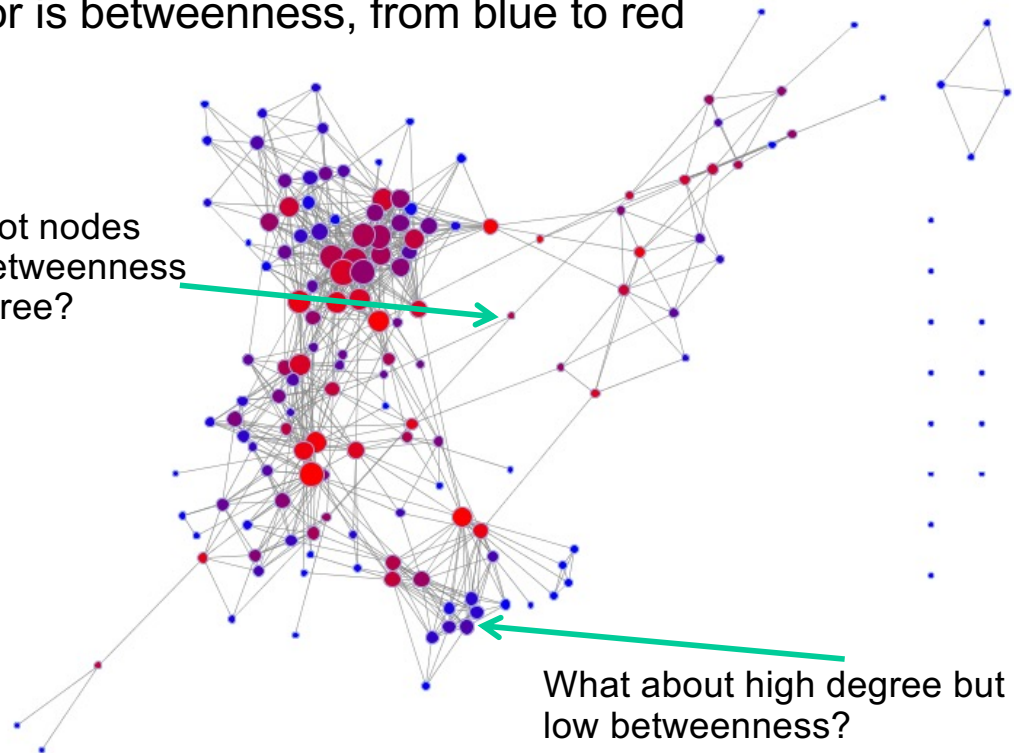


- If broker is removed, the global connection would completely collapse, and you will notice separated subgroups

Betweenness: example

- Size is degree
- Color is betweenness, from blue to red

Can you spot nodes with high betweenness but low degree?



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Closeness: another centrality measure

- Sometimes you don't need many friends (degree) or to act as a broker (betweenness)
- You just want to be “in the middle”, close to everyone else
- Closeness centrality:
 - Measures how close a node is to all other nodes
 - High closeness → node can reach others quickly

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Closeness: definition

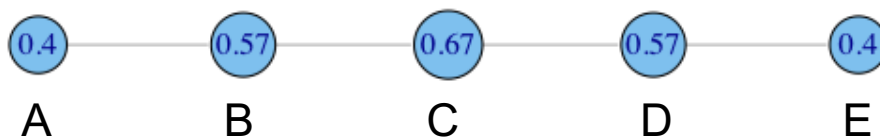
- Closeness of a node is based on the **average length of the shortest paths between that node and all other nodes in the network**
 - The more central a node is, the closer it is to all other nodes
- $C_C(i)$, closeness for node i :
 - $d(i,j)$: length of the shortest path between i and j

$$C_C(i) = \left(\sum_{j=1}^N d(i, j) \right)^{-1}$$

- $C'_C(i)$, normalized closeness for node i

$$C'_C(i) = (N - 1)C_C(i) = \frac{N - 1}{\sum_{j=1}^N d(i, j)}$$

Closeness: example

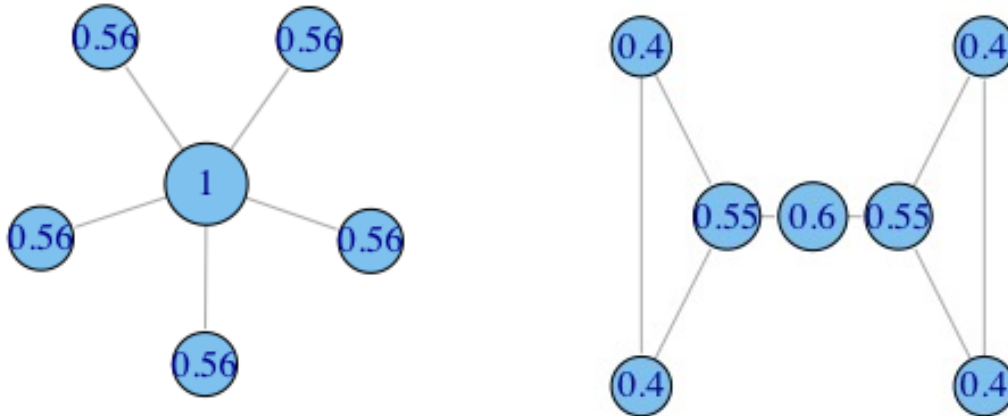


$$C'_C(A) = \frac{N - 1}{\sum_{j=1}^N d(i, j)} = \frac{5 - 1}{0 + 1 + 2 + 3 + 4} = \frac{4}{10} = 0.4$$

$$C'_C(B) = \frac{N - 1}{\sum_{j=1}^N d(i, j)} = \frac{5 - 1}{1 + 0 + 1 + 2 + 3} = \frac{4}{7} = 0.57$$

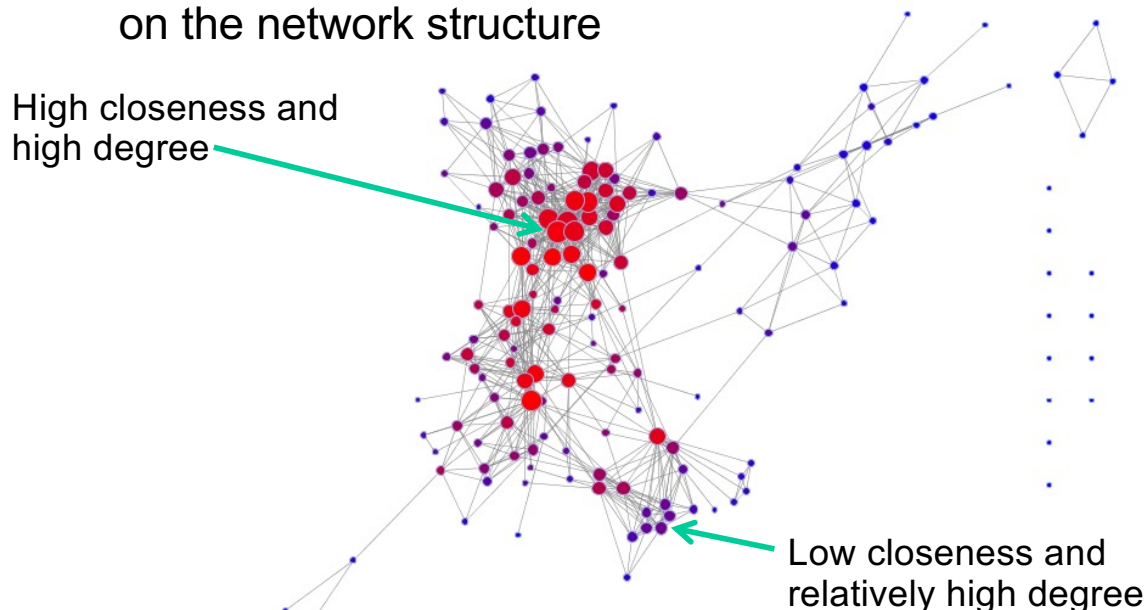
$$C'_C(C) = \frac{N - 1}{\sum_{j=1}^N d(i, j)} = \frac{5 - 1}{2 + 1 + 0 + 1 + 2} = \frac{4}{6} = 0.67$$

Closeness: examples



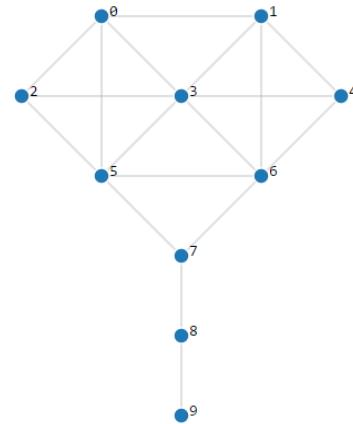
How closely does degree correspond to closeness?

- Size is degree
- Color is closeness, from blue to red
- Positively correlated, but loosely and highly dependent on the network structure



And closeness and betweenness?

- Again, it depends on network structure
 - Positive correlation in centralized networks (star)
 - The center node is both the closest to everyone and sits on every path
 - Negative/weak correlation in networks with communities
 - A node can be a powerful bridge but is physically far from the majority
- Krackhardt kite graph
 - Maximum degree: node 3
 - Maximum betweenness: node 7
 - Maximum closeness: nodes 5 and 6

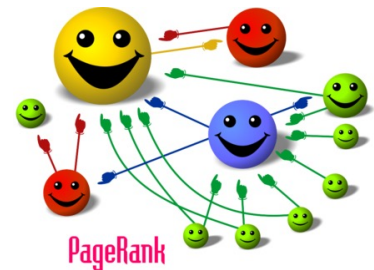


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PageRank as centrality measure

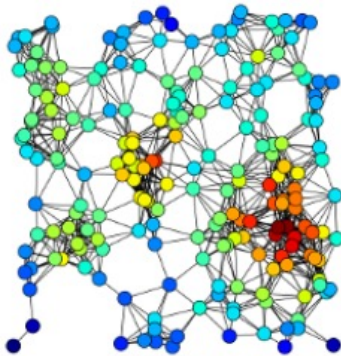
- PageRank: *a node is important if it is linked from other important, or if it is highly linked*
- PageRank as centrality measure
 - Difference from other centrality measures: accounts for **link direction**
 - How it works
 - Each node is assigned a score using the PageRank algorithm
 - Scores depend on the number and quality of incoming links: links from higher-scoring nodes contribute more
 - The influence “flows” through the network: nodes linked from influential nodes gain part of that influence
 - Result: nodes with many incoming links are influential, and their neighbors inherit some of that influence



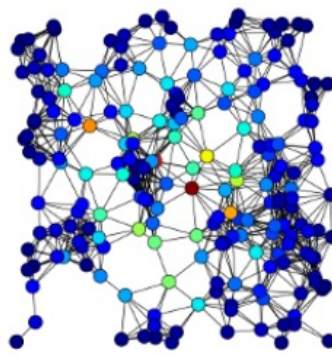
Wrap-up on centrality

- We have examined popular centrality measures, each capturing a different notion of “importance” in a network

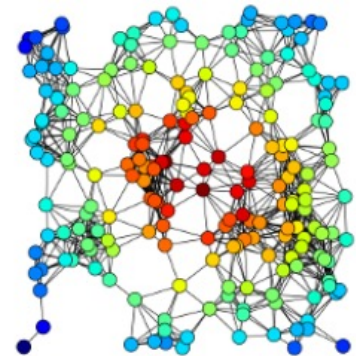
"There is certainly no unanimity on exactly what centrality is or on its conceptual foundations, and there is little agreement on the proper procedure for its measurement." (Freeman, 1979)



Degree



Betweenness

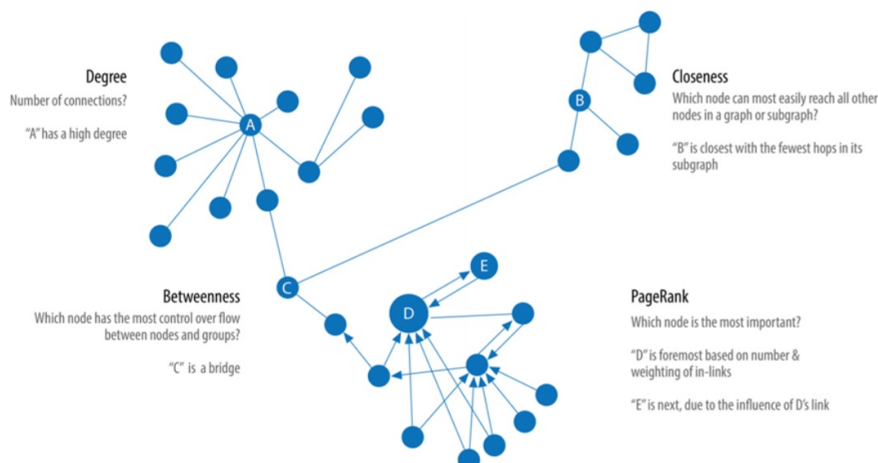


Closeness

Color from minimum (blue) to maximum (red)

Wrap-up on centrality

- Centrality depends on the network structure and the aspect of “importance” you want to capture
 - Degree: local connectivity
 - Betweenness: control of information flow
 - Closeness: reachability
 - PageRank: influence through directed links



Wrap-up on centrality

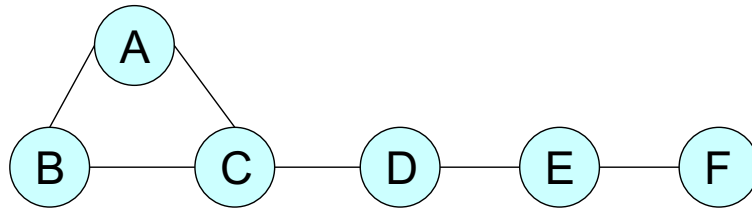
- Degree centrality
 - *What it tells us*: the number of links each node has to other nodes in the network
 - *When to use it*: to find very connected or popular individuals, those likely to hold the most information, or those who can quickly connect with the wider network
- Betweenness centrality
 - *What it tells us*: which nodes acts as “bridges” between other nodes; measures how often a node lies on the shortest path connecting other nodes
 - *When to use it*: to find individuals who influence the flow of information or control connectivity across the network

Wrap-up on centrality

- Closeness centrality
 - *What it tells us*: how close a node is to all other nodes, based on the sum of shortest path lengths
 - *When to use it*: to find individuals best placed to quickly influence the entire network
- PageRank
 - *What it tells us*: node's importance based on the importance of its neighbors (and their neighbors), capturing influence that extends beyond direct links
 - *When to use it*: to understand authority, prominence, or influence in networks with directed links, such as citations or hyperlinks

Exercise

- Consider this network



- Find the diameter and the average shortest path
- Find which is the most important node using:
 - Degree centrality
 - Betweenness centrality
 - Closeness centrality
- How do values change if we add a new node G connected only to F?

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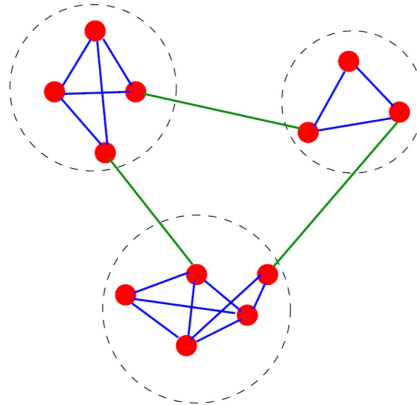
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Community in networks

- Set of nodes that share something:
 - Affiliation: friends, colleagues, club, ...
 - Similar interests: users in tagging systems, ...
 - Similar contents: movies, books, products, ...
- Connection with network structure?
- Intuition: **more densely connected inside than outside**

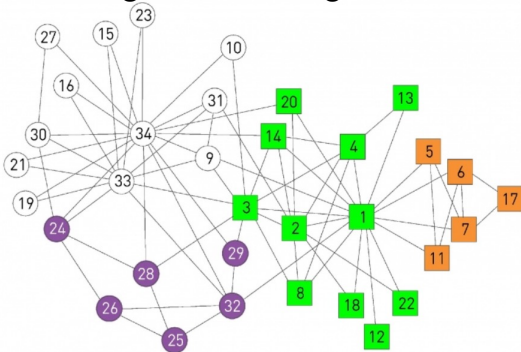
SNA: Community detection

- Goal: identify communities with more links between nodes within the group than between groups

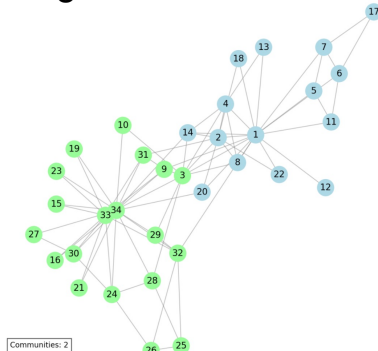


Community detection: example

Using Louvain algorithm



Using Girvan-Newman algorithm



- Zachary observed 34 members of a karate club over 2 years
- Links connect pairs of members who interacted outside club activities
- During observation, a conflict arose between the club president (node 1) and instructor (node 34)
- The club split into two groups, with one group leaving to form a new club

https://en.wikipedia.org/wiki/Zachary%27s_karate_club

Community detection: Louvain algorithm

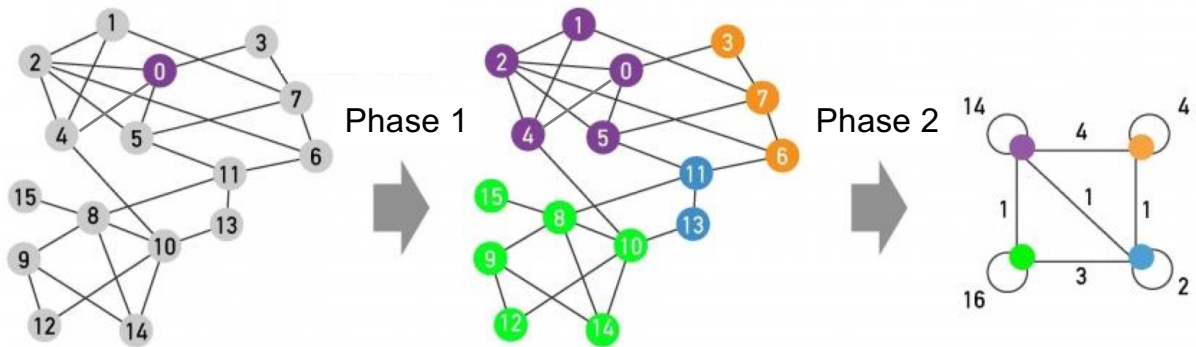
- **Louvain algorithm:** greedy optimization method that maximizes modularity
 - **Modularity Q :** measures the **density** of links inside communities vs. between communities
 - Q is between -1 and 1
- Idea: 2 phases repeated iteratively
 - Phase 1: Local moving
 - Phase 2: Community aggregation
- Running time: $O(N \log N)$
 - N is the number of nodes

Community detection: Louvain algorithm

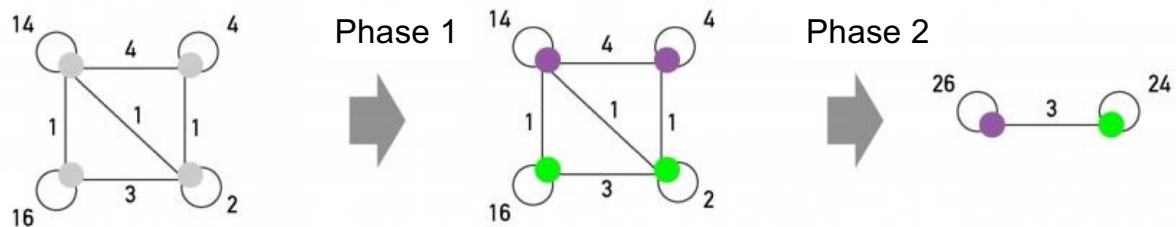
- Phase 1: local moving
 - Assign each node to its own community
 - For each node i :
 - Compute change in modularity (ΔQ) by moving i to the community of each neighbor j
 - Move i to the community that maximizes ΔQ
- Phase 2: community aggregation
 - Group nodes in the same community into a single “super-node”
 - Build a new network of these super-nodes
 - Repeat Phase 1 on the new network
- Stop:
 - When modularity reaches maximum and no further improvement is possible

Community detection: Louvain algorithm

1ST PASS



2ND PASS



References

- D. Easley and J. Kleinberg, Networks, Crowds, and Markets: Reasoning About a Highly Connected World, chapters 1 and 2, Cambridge University Press, 2010.
<http://www.cs.cornell.edu/home/kleinber/networks-book>
- A.-L. Barabási, Network Science, chapters 1, 2, 3, and 9, Cambridge University Press, 2016. <https://networksciencebook.com>