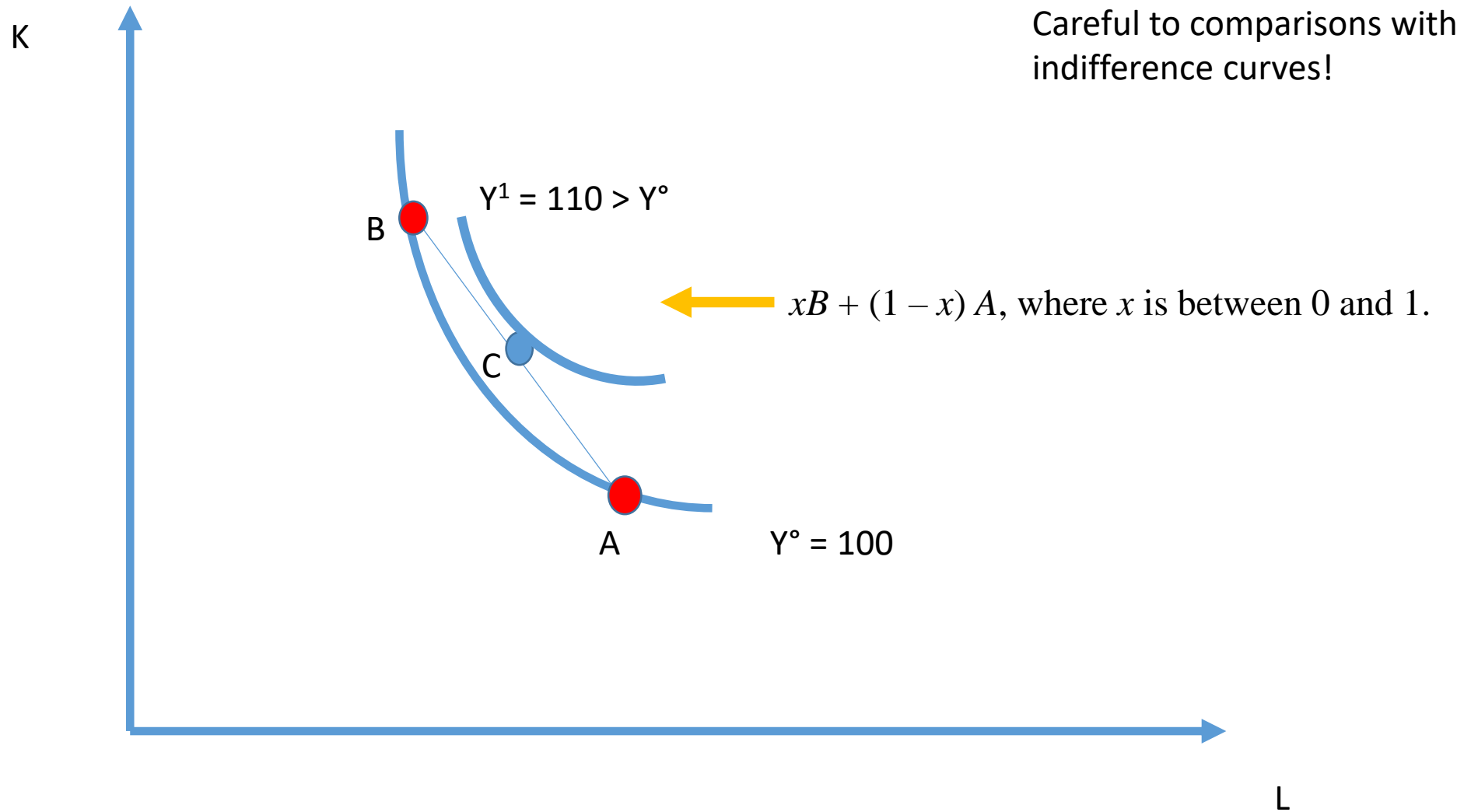




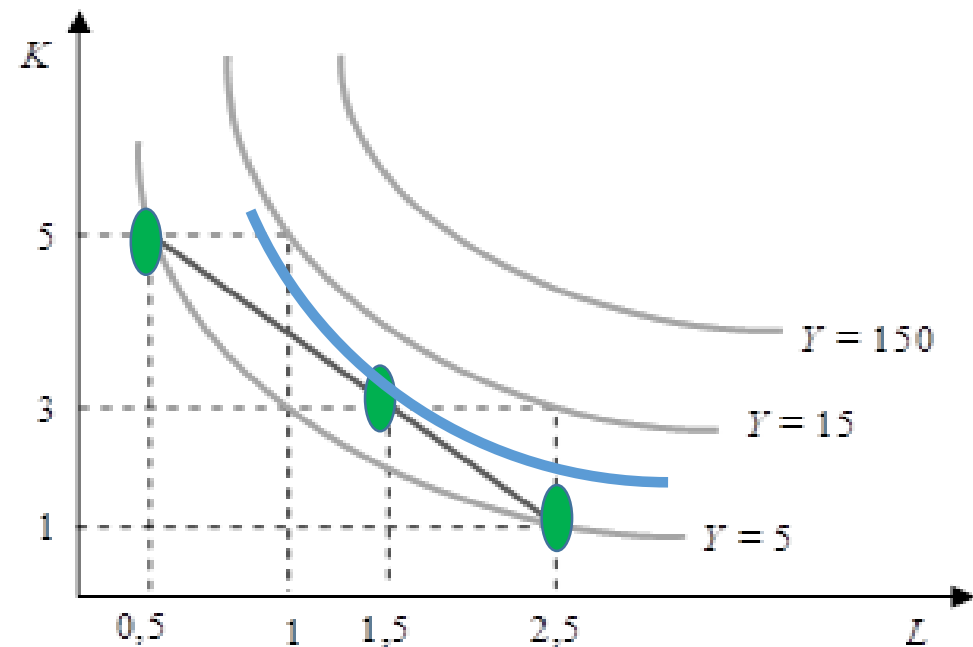
Convex Isoquants





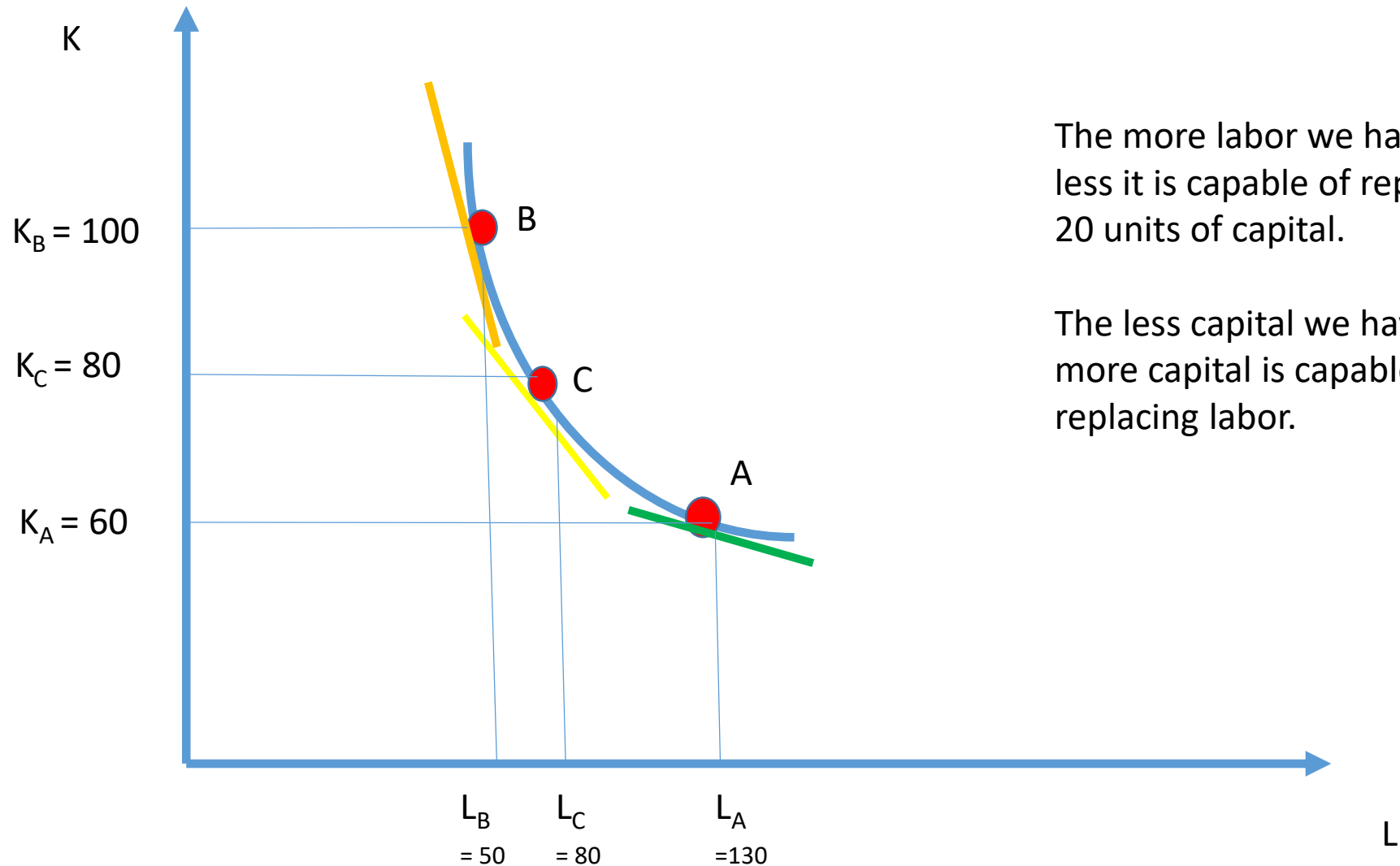
Convex?

While we can use the productive techniques $(0,5; 5)$ and $(2,5; 1)$ to produce efficiently 5 units of shirts, if we were to **combine** those two techniques, e.g. using half of the first (0,25 of L and 2,5 of K) and half of the second (1,25 of L and 0,5 of K) and so using $(1,5$ of L and 3 of K) we would obtain **greater** quantities of output.





Convex curves: the slope declines as L grows



The more labor we have, the less it is capable of replacing 20 units of capital.

The less capital we have, the more capital is capable of replacing labor.



The isoquant's slope?

$$dY = 0 = dK \times \frac{\partial Y}{\partial K} + dL \times \frac{\partial Y}{\partial L} = f^K dK + f^L dL$$

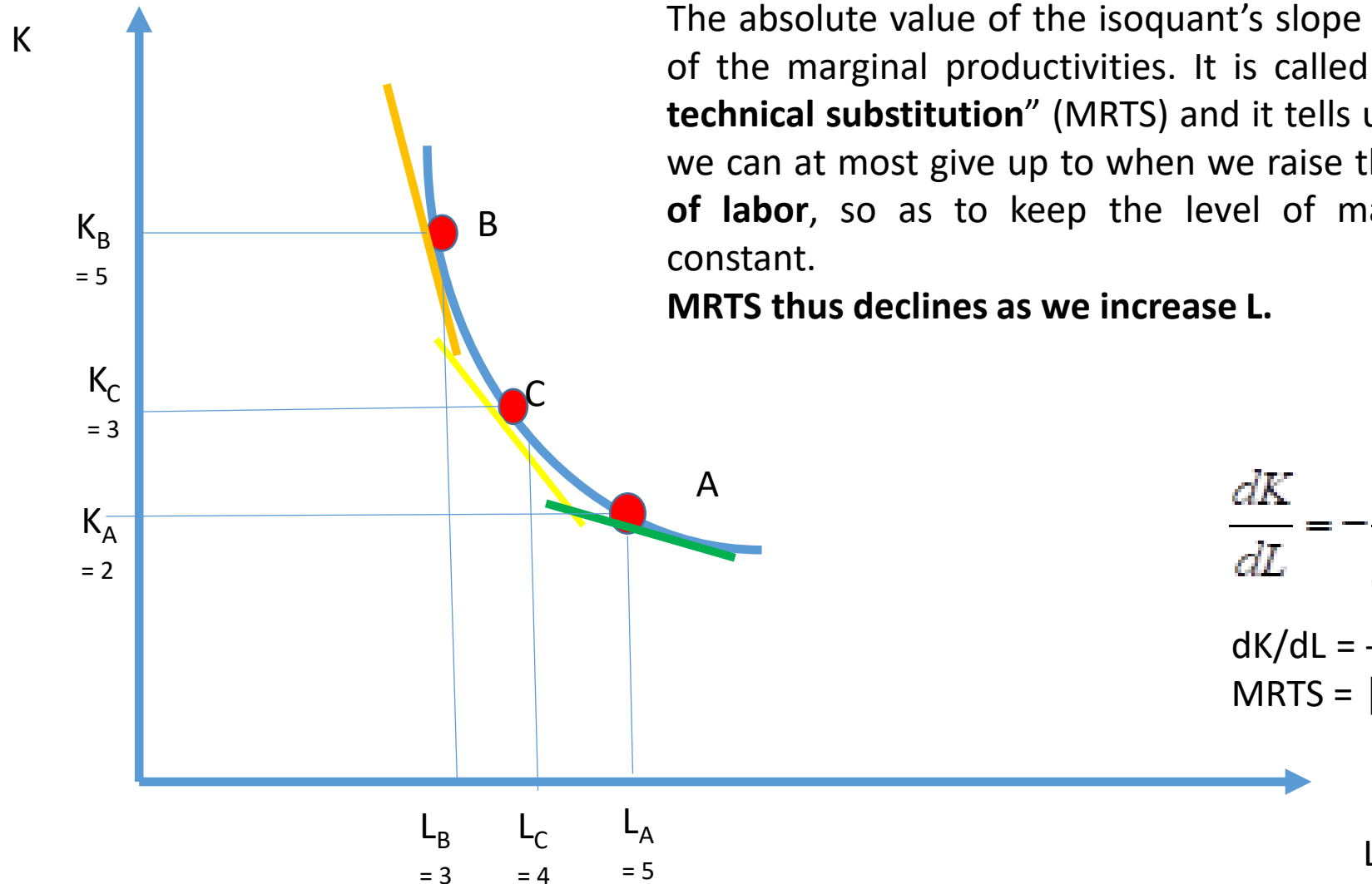
$$\frac{dK}{dL} = - \frac{f^L}{f^K} = - \frac{P_{maL}}{P_{maK}}$$

Mistake:
should read
MPL and MPK
rather than
PmaL and
PmaK

Negative or positive slope?



Convex curves: slope declining as L grows



The absolute value of the isoquant's slope is given by the ratio of the marginal productivities. It is called "**marginal rate of technical substitution**" (MRTS) and it tells us how much capital we can at most give up to when we raise the use, **by one unit, of labor**, so as to keep the level of maximum production constant.

MRTS thus declines as we increase L.

$$\frac{dK}{dL} = -\frac{f^l}{f^k} = -\frac{PmaL}{PmaK}$$

$$dK/dL = - \text{MRTS}$$

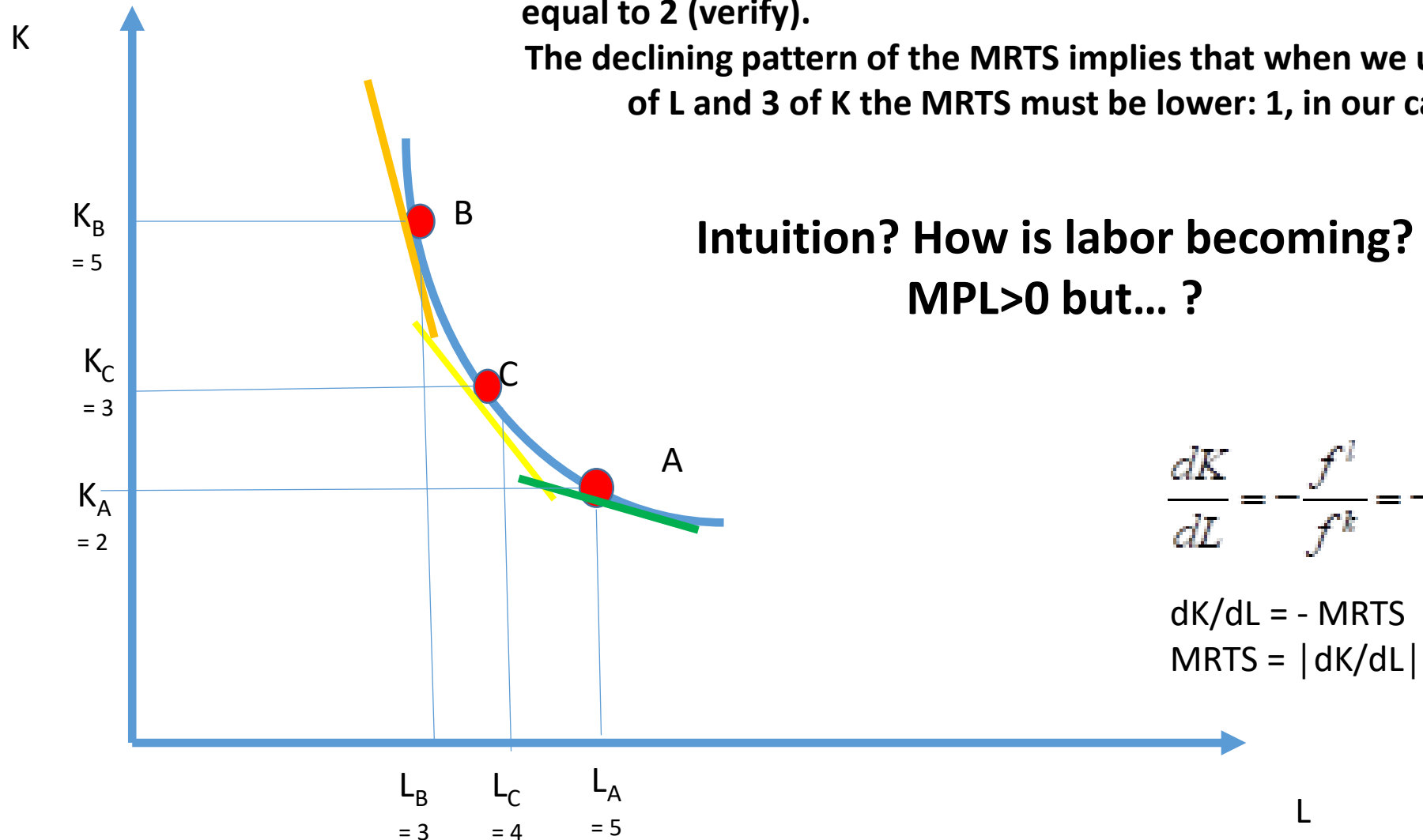
$$\text{MRTS} = |dK/dL|$$



Convex curves: slope declining as L grows

Example. When we use 3 units of L and 5 units of K the MRTS is equal to 2 (verify).

The declining pattern of the MRTS implies that when we use 4 units of L and 3 of K the MRTS must be lower: 1, in our case.



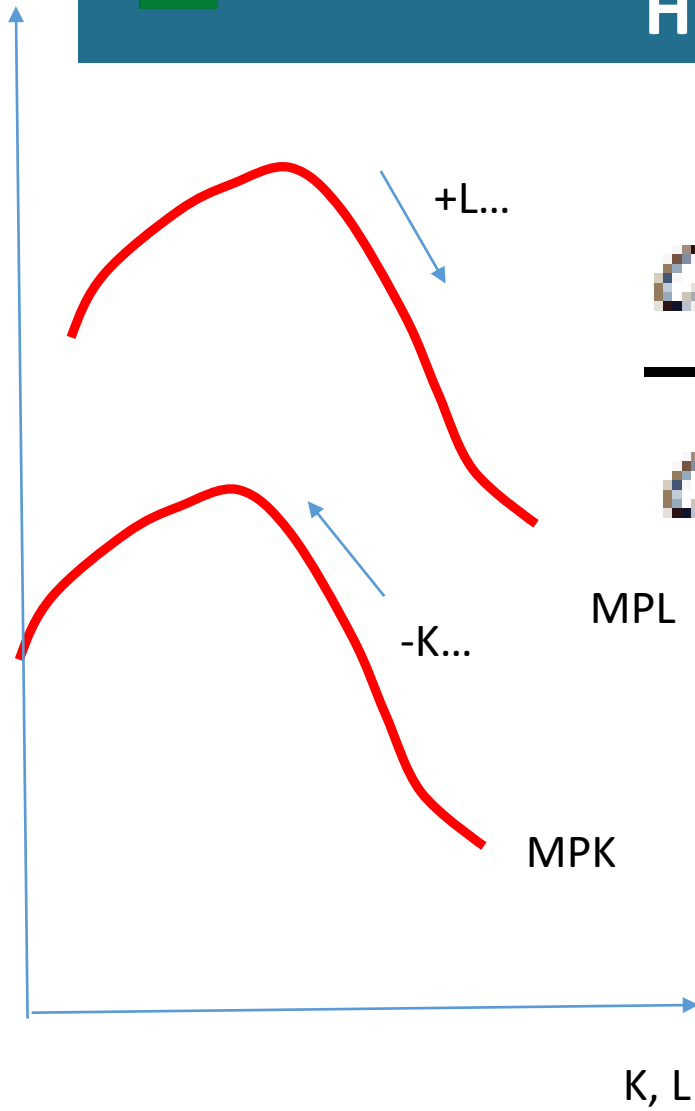
$$\frac{dK}{dL} = -\frac{f^L}{f^K} = -\frac{P_{mL}}{P_{mK}}$$

$$dK/dL = - \text{MRTS}$$

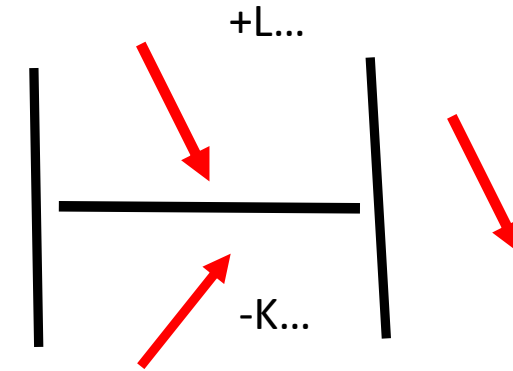
$$\text{MRTS} = |dK/dL|$$



How is the MRTS if both MP were both decreasing?



$$\frac{dK}{dL} = - \frac{f^L}{f^K} = - \frac{P_{mL}}{P_{mK}}$$



$$dK/dL = - \text{MRTS}$$

$$\text{MRTS} = |dK/dL|$$

Example. When we use 3 units of L and 5 units of K.

$$\text{MPL}(3) = 12 \text{ and } \text{MPK}(5) = 6$$

$$\text{MRTS} = 12/6 = 2$$

+ 1 L – 2K (Y constant): now you use 4 L and 3 K.

If MPL and MPK are decreasing

$$\text{MPL}(4) = 9 \text{ and } \text{MPK}(3) = 9$$

$$\text{MRTS} = 9/9 = 1$$

When we use 3 units of L and 5 units of K the MRTS is equal to 2. The decreasing pattern of the MRTS with respect to L implies that when we use more units of L (4) and less (3) of K the MRTS must be lower: 1, in our case.

The isoquant's slope? Summing up

$$\frac{dK}{dL} = - \frac{f^L}{f^K} = - \frac{P_{maL}}{P_{maK}}$$

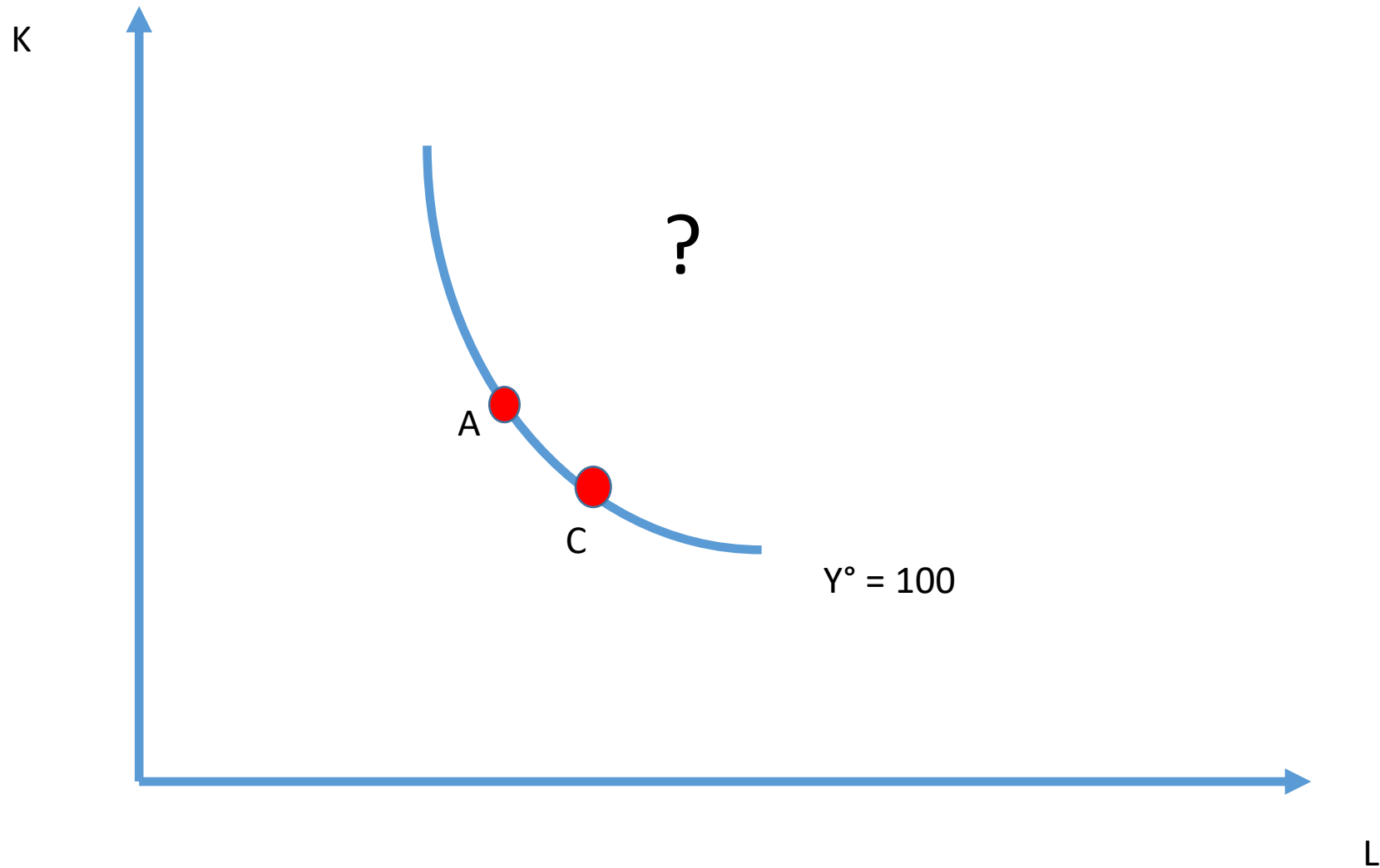
Negative slope, because MPL
and MPK are **positive**

Declining slope in absolute
value, certainly if MPL and
MPK are **both decreasing**

Mistake:
should read
MPL and MPK
rather than
P_{maL} and
P_{maK}



How to produce 100?



The economic dimension of production





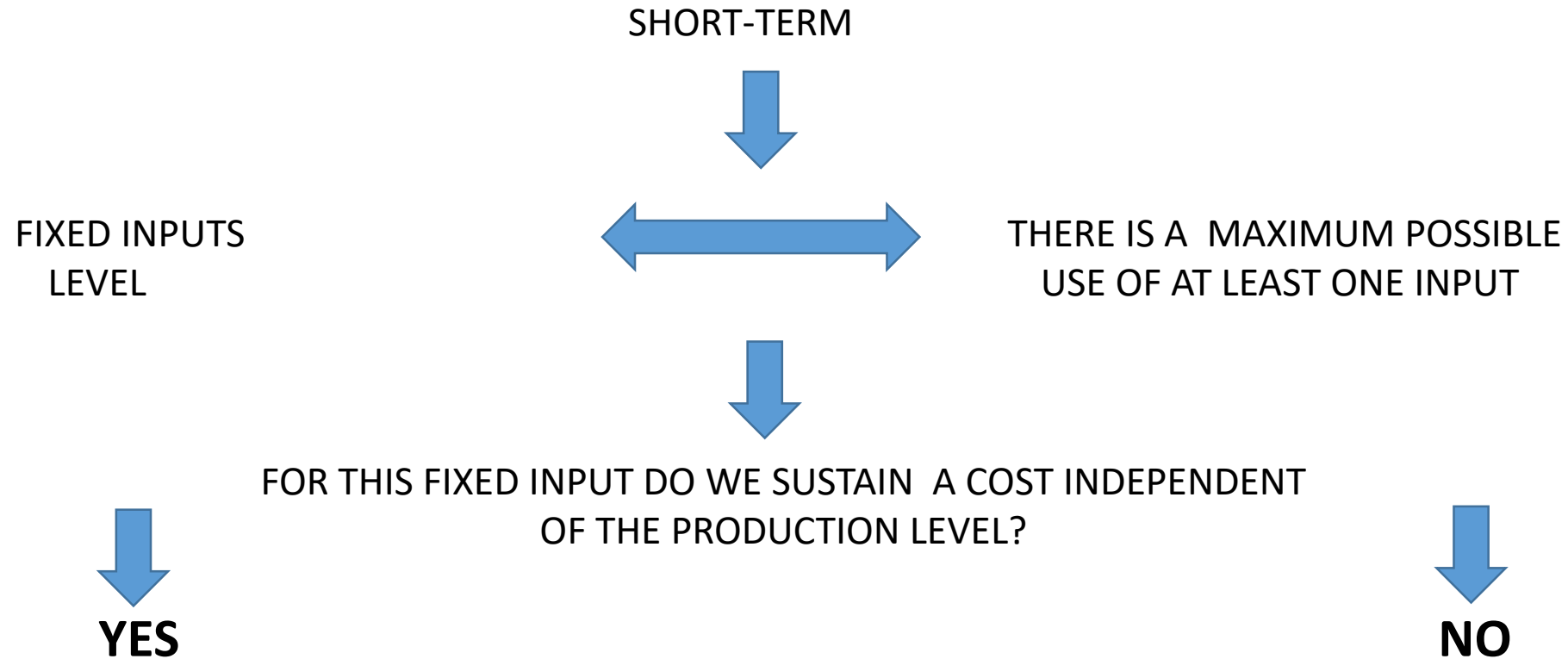
$$TC(Q) = FC + VC(Q)$$

$$TC^{\min}(Q) = FC + VC^{\min}(Q)$$

FC? What are they?



Short-term horizon





Short Term = Fixed input Costs - Examples

A) WAREHOUSE OF A GIVEN AREA IN SQUARE METERS, PAYING RENT

B) CLEANING SQUAD FOR AT MOST X HOURS, PAYING CLEANING FEES

2 FIXED INPUTS, WITH A MAXIMUM AMOUNT TO BE USED (SQ. MT., HRS.)

LET'S READ THE CONTRACT

A) CAN'T PAY THE RENTED AREA OF THE WAREHOUSE ACCORDING TO USE (NOR SUBLET IT):
FIRM NEEDS TO PAY FOR THE MAXIMUM AREA EVEN IF NOT USED.

B) CAN'T CHANGE THE QUANTITY OF CLEANING SERVICES ACCORDING TO USE (IF ONE
CLEANS LESS THE BUILDING, THE SAME AMOUNT OF HOURS, THE MAXIMUM, IS PAID)

WE HAVE A FIXED OR SUNK COST

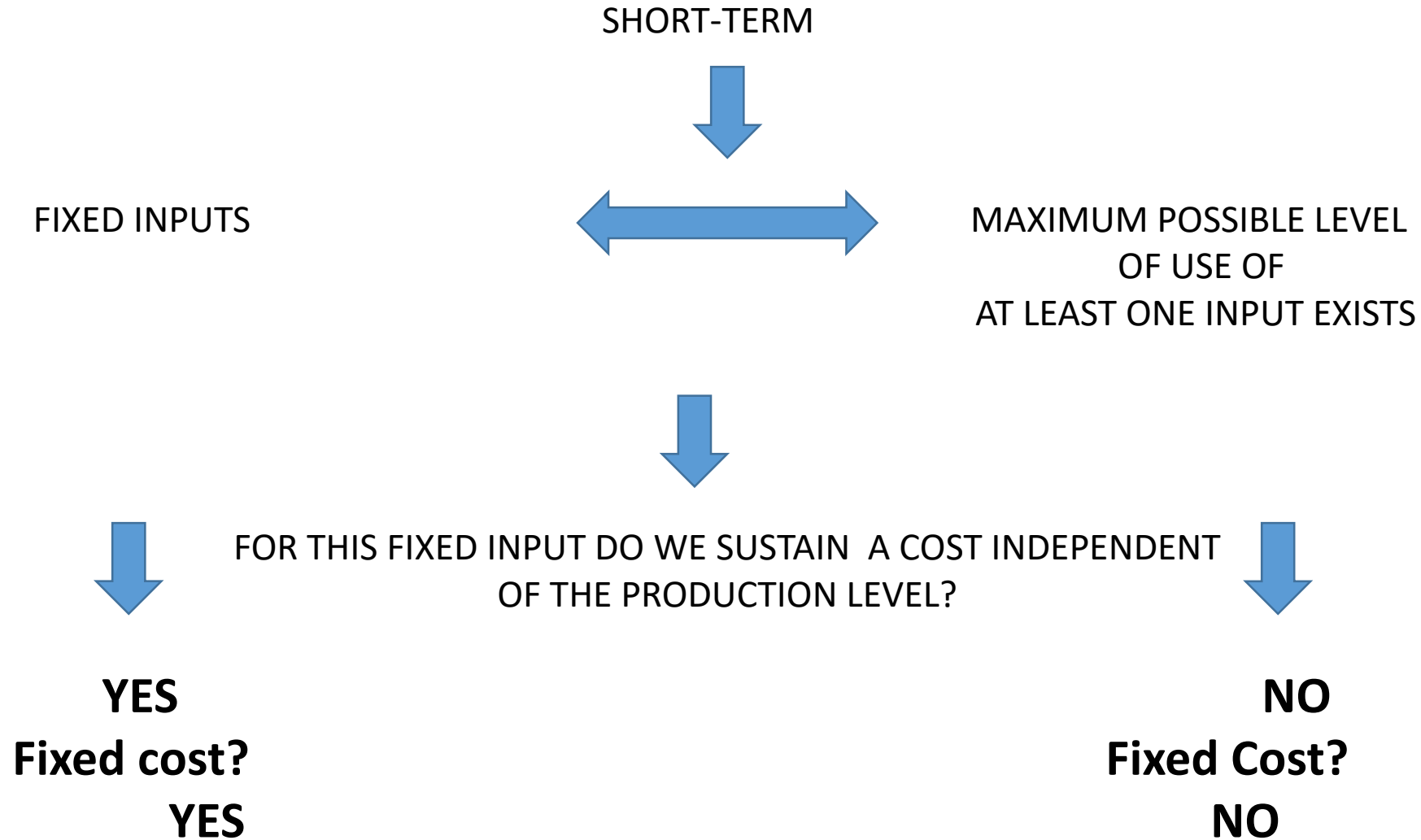
A') FIRM CAN SUBLET THE WAREHOUSE UNUSED

B') FIRM CAN PAY FOR HOURS USED WITHIN THE MAXIMUM AMOUNT

WE HAVE A VARIABLE COST



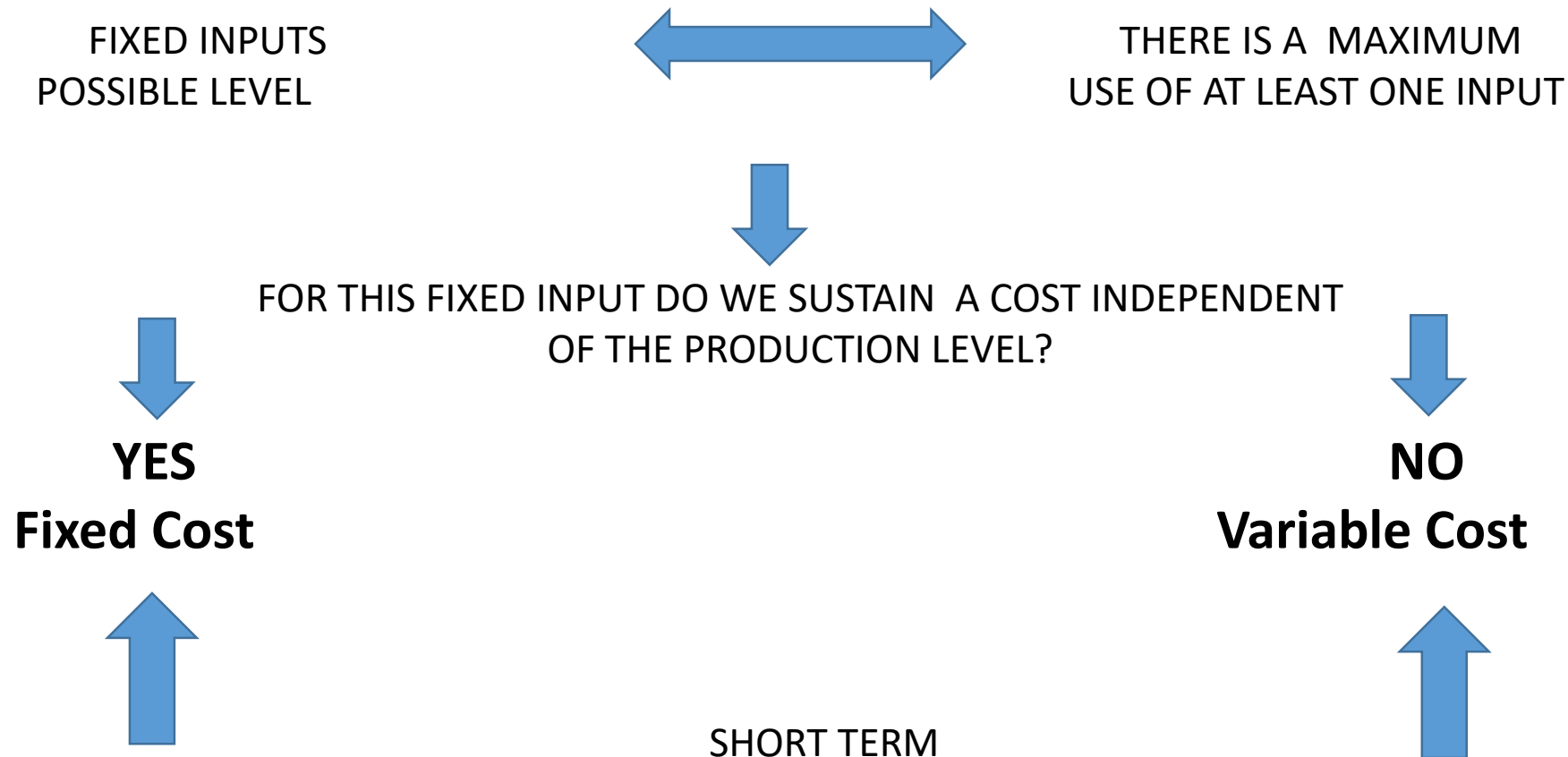
Short-term horizon



Short- AND Long-term horizon

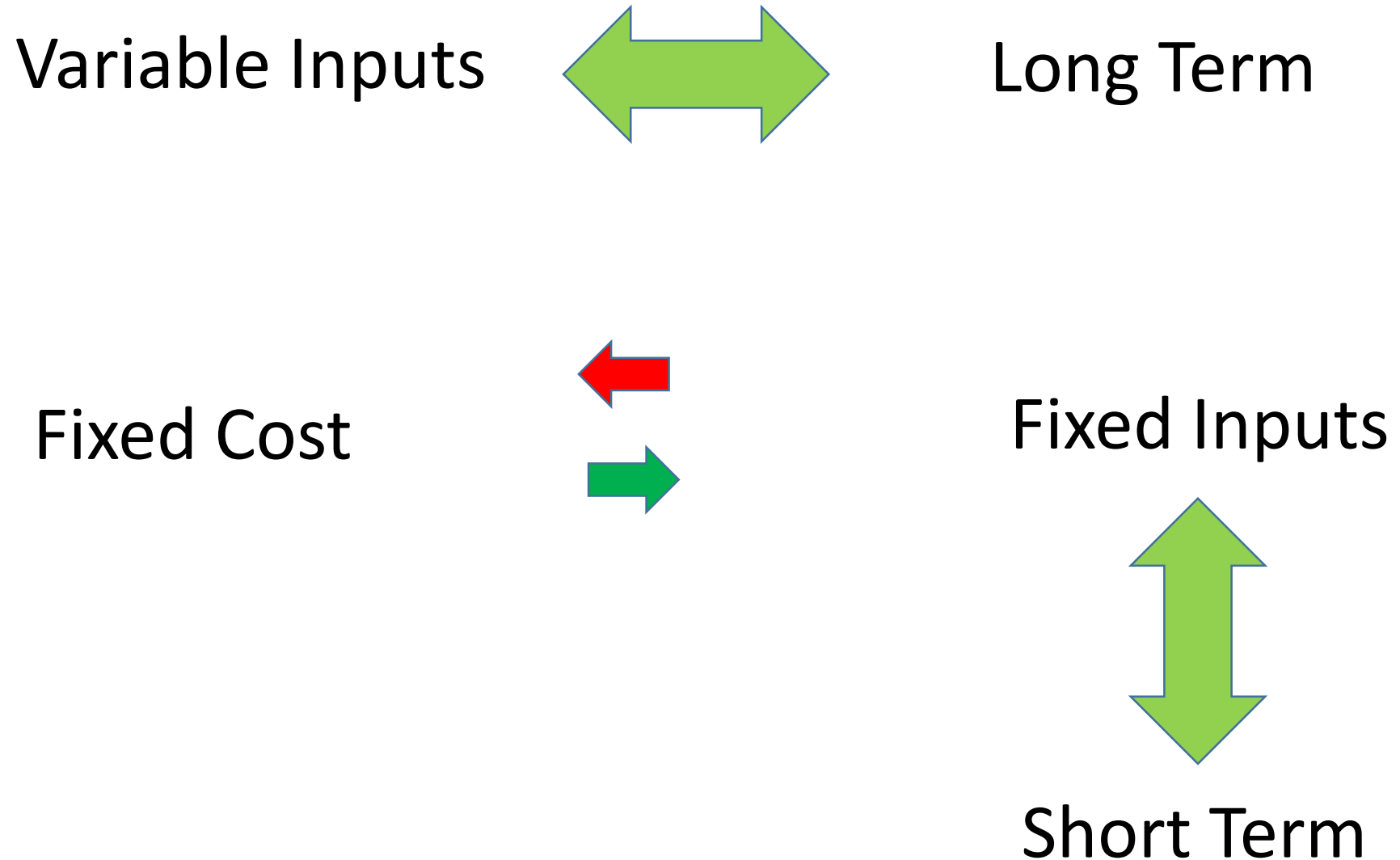
LONG TERM = all factors can be varied, no fixed cost

Today, an entrepreneur thinks also about the long-term





Summing Up





THE LONG TERM PERIOD



Given the unitary costs of the 2 factors (w° , r°) (since the firm is price-taker) cost is equal to?

$$(w^\circ L + r^\circ K)$$

We will call **isocost curve** the locus of technical productive combinations of inputs **labor-capital** all sharing **the same** total cost.

So:

$$TC^0 = w^\circ L + r^\circ K$$

represents the locus of L-K combinations with the same total cost TC^0 . We can rewrite such combinations as pertaining to the line:

Slope?
Why decreasing?

$$K = \frac{TC^0}{r^\circ} - \left(\frac{w^\circ}{r^\circ} \right) \times L$$

An isocost

$$w^{\circ} = 2000 \text{ €}$$

$$r^{\circ} = 4000 \text{ €}$$

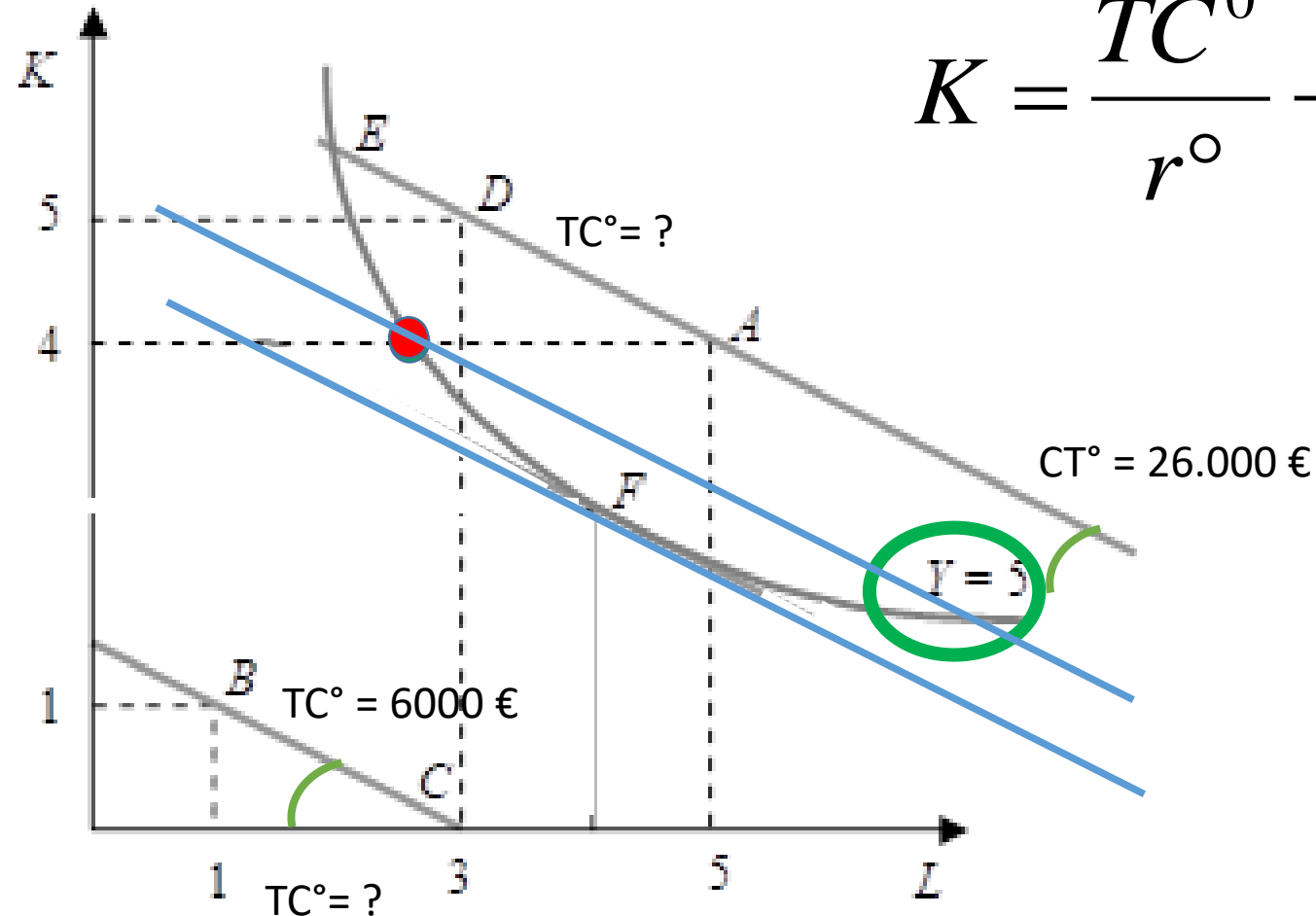
Can I produce $Y = 5$
spending 6000 euro?

Spending 26.000
euro?

D is more expensive
than E?

And what will be my
economically efficient
choice?

Or ● ?



$$K = \frac{TC^0}{r^0} - \left(\frac{w^0}{r^0} \right) \times L$$

Iso-cost slope =
 $-w^{\circ}/r^{\circ}$

To produce Y° will
you choose B?
D? or E?

Iso-cost curves
moving north-
east are?



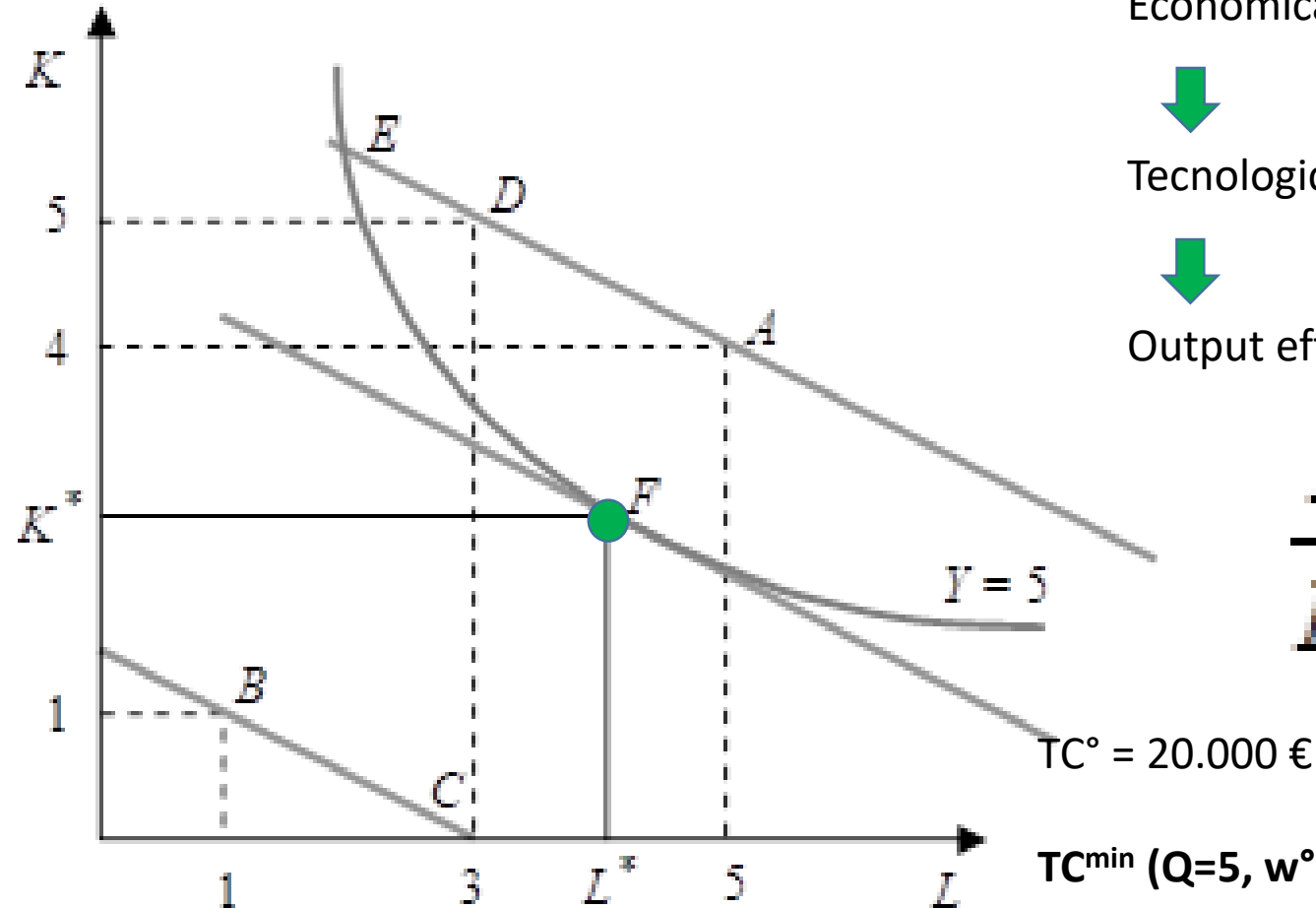
The economically efficient technique

$$w^{\circ} = 2000 \text{ €}$$

$$r^{\circ} = 4000 \text{ €}$$

$$L^* = 4$$

$$K^* = 3$$



Economically efficient



Tecnologically efficient



Output efficient

$$\frac{P_{maL}}{P_{maK}} = \frac{w}{r}$$

$$TC^{\circ} = 20.000 \text{ €}$$

$$TC^{\min} (Q=5, w^{\circ}=2000, r^{\circ}=4000) = 20.000 \text{ €}$$

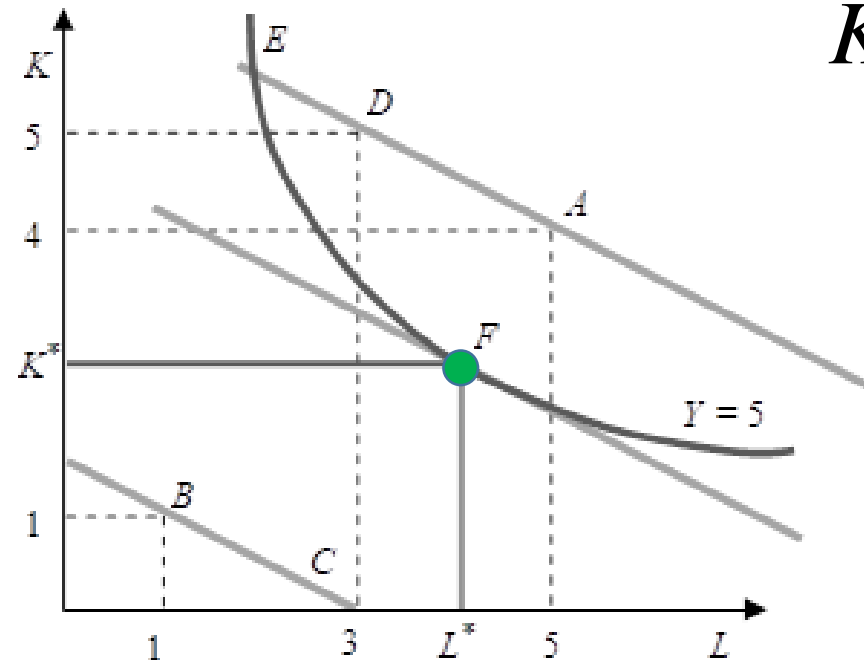
The first point of your (LT)

TC^{\min} function !



Long term costs

$$\frac{dK}{dL} = -\frac{f^l}{f^k} = -\frac{PmaL}{PmaK}$$



$$K = \frac{TC^0}{r^0} - \left(\frac{w^0}{r^0} \right) \times L$$

What if the condition does not hold?
e.g. (w/r) equal to $1/2$
(e.g. $w = 2$ and $r = 4$ euro per unit).
Marginal productivity of labor twice as
high as the one of capital.
($MPL = 2$ and $MPK = 1$).

$$(w^0/r^0) < (MPL/MPK)$$

+ 1 L and - 2 K? What happens to
cost?

$$\frac{MPL}{MPK} = \frac{w}{r}$$

$$\frac{MPL}{w} = \frac{MPK}{r}$$

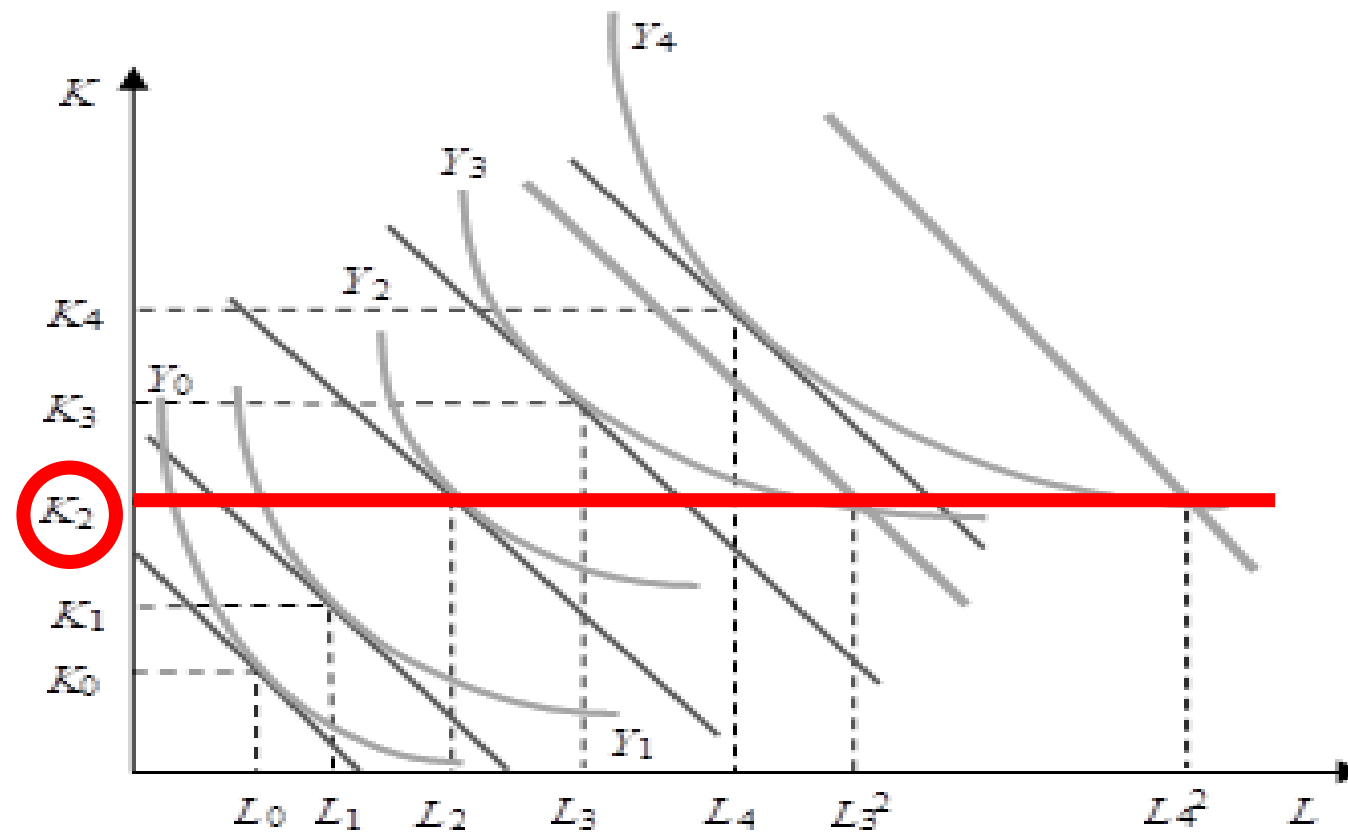
Cost function

Short Term



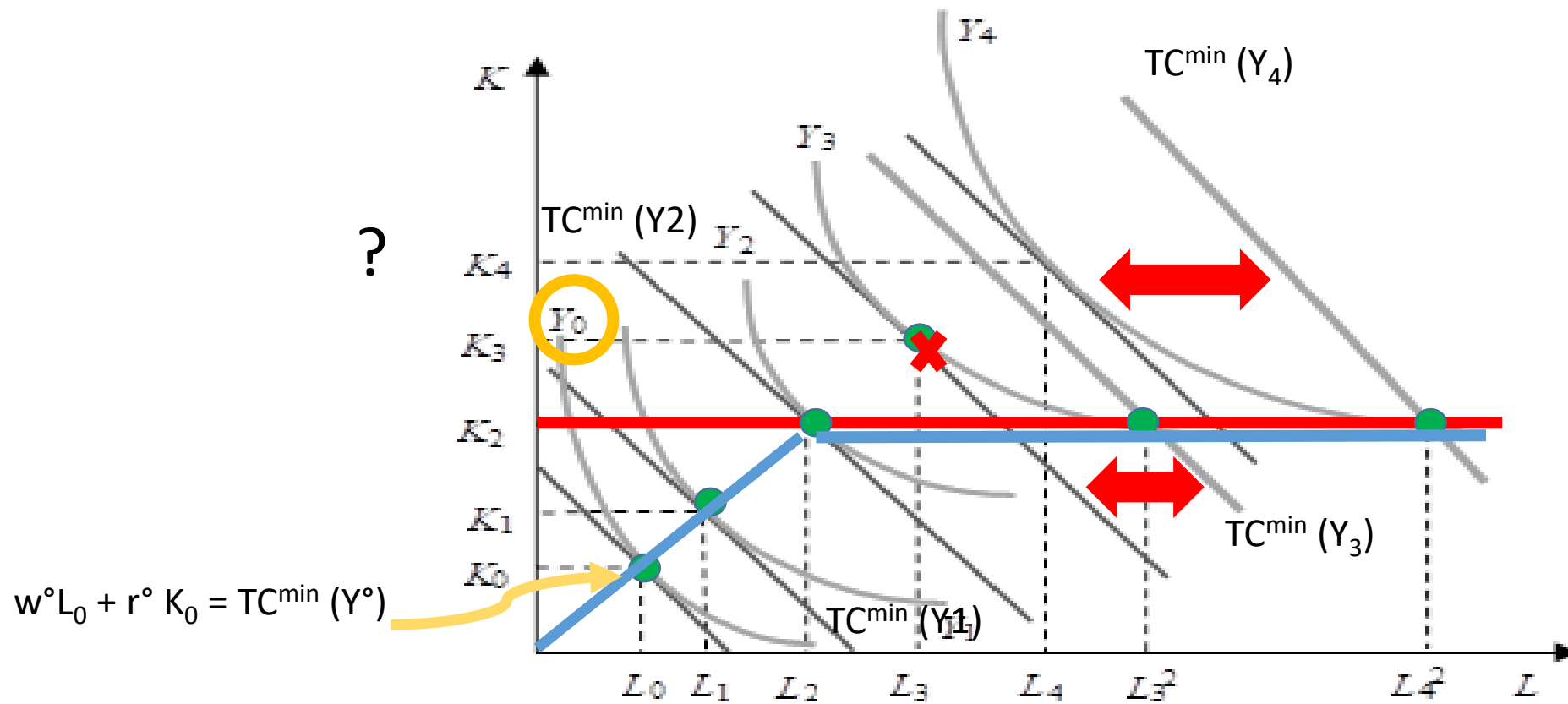


Short-term: Fixed Input (ST)

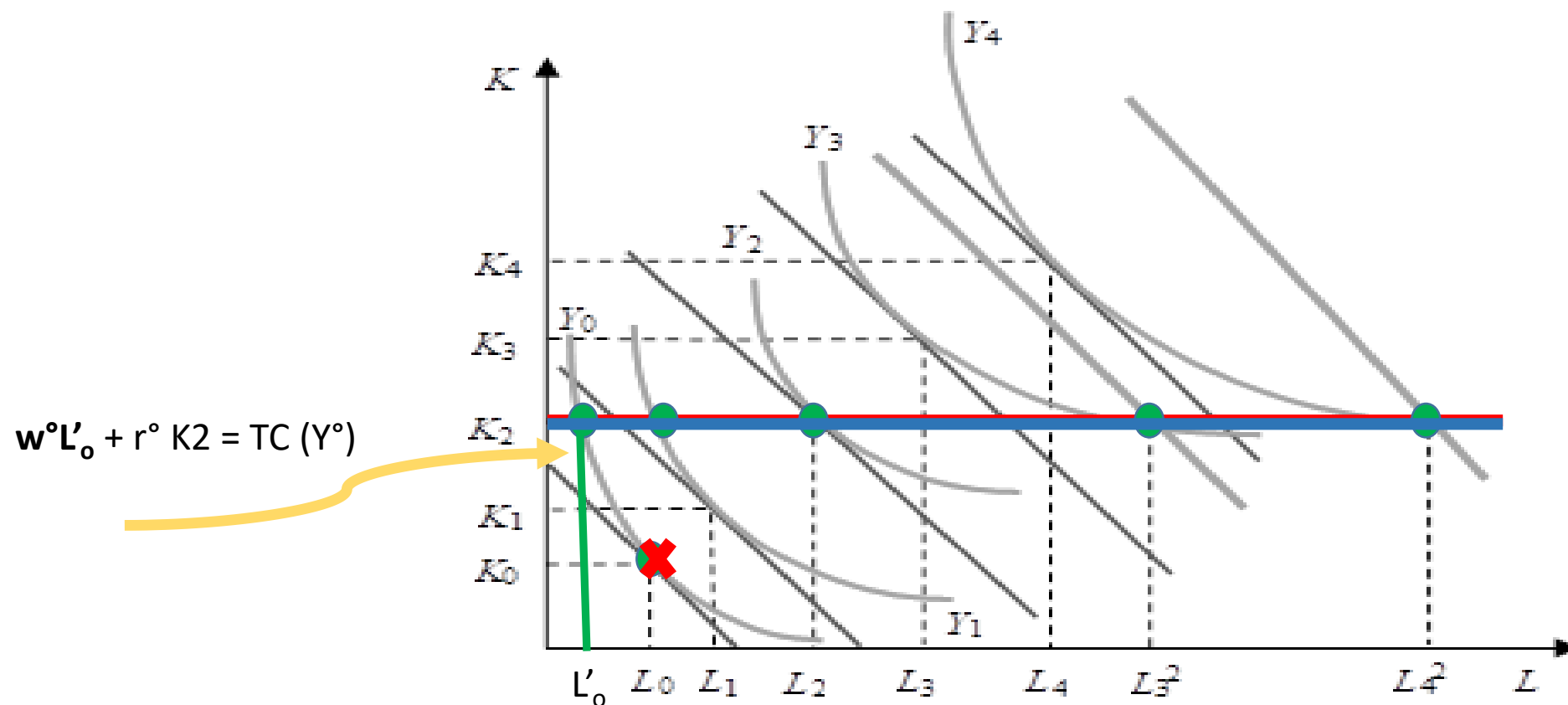


$$K \leq K_2$$

Fixed Input, **variable costs: ST technology expansion path (e.g., subletting possible)**

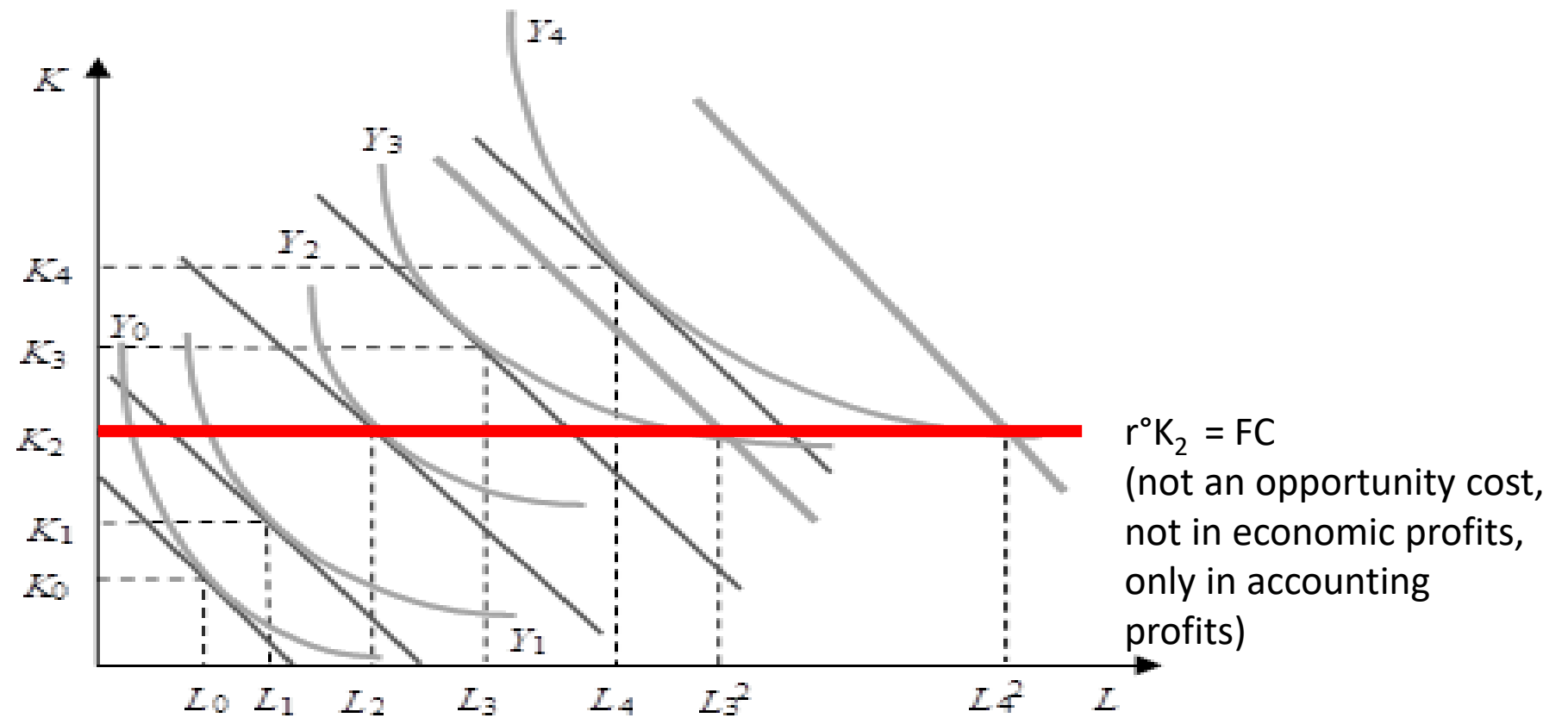


Fixed Input, Sunk Costs (e.g., subletting impossible): technology expansion path





Fixed Input, Sunk Cost = Fixed Cost



Note how the minimum total costs depend on :
 $Y, w^{\circ}, r^{\circ}K_2$

$$TC^{\min} = TC(Y, w^{\circ}, r^{\circ}, K_2)$$

What about Technology T° ?

Do (minimum) costs depend also on Technology?



Short term Cost Functions, L variable input

$$TC(Q(L, K^0), w_0, r_0, K_0) = FC + VC(Q, w_0) = r_0 K_0 + VC(Q, w_0)$$

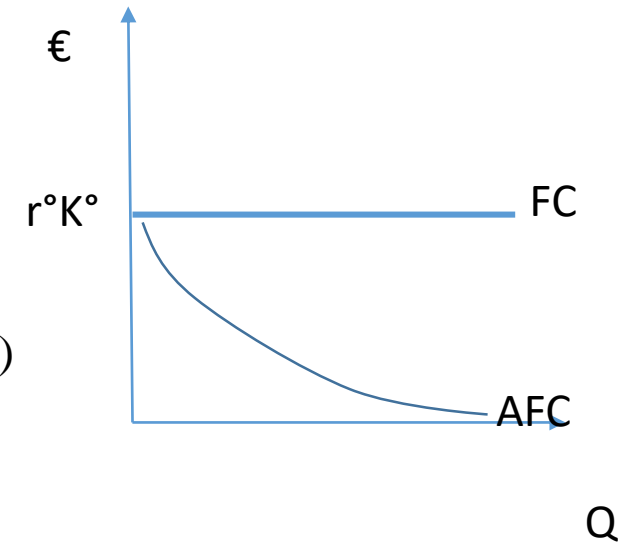
$$ATC(Q) = TC(Q)/Q$$

$$ATC(Q, w_0, r_0) = \frac{r_0 K_0}{Q} + \frac{VC(Q, w_0)}{Q} = AFC(Q, r_0) + AVC(Q, w_0)$$

PS: what
about
economic
profits?

$$\Pi(Q, w_0, r_0, K_0) = TR(Q) - TC(Q, w_0, r_0, K_0) = p(Q)Q - Q ATC(Q, w_0, r_0, K_0)$$

$$\Pi(Q, w_0, r_0, K_0) = (p(Q) - ATC(Q, w_0, r_0, K_0)) Q$$



$$\Pi^E(Q) = [p(Q) - \text{AVC}(Q, w^0)]Q$$

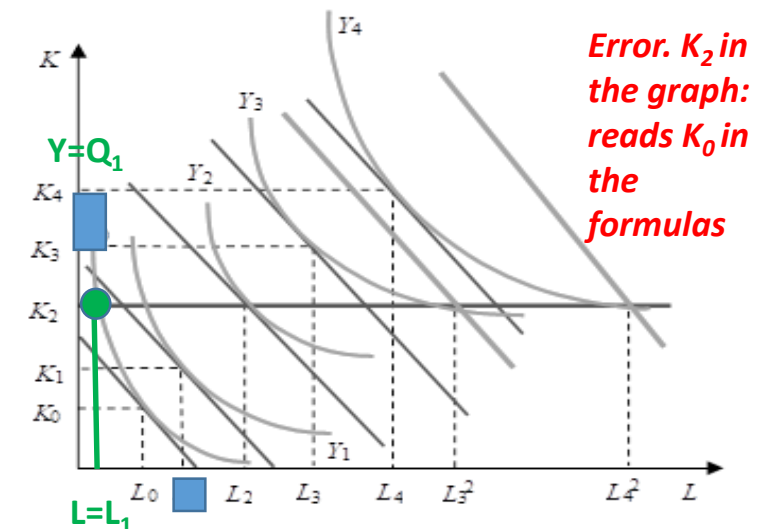
$$Q=Q_1; VC(Q_1)?$$

$$VC(Q_1, w_0) = w_0 L_1 \text{ €}$$

$$ATC(Q_1, w_0, r_0) = \frac{r_0 K_0}{Q_1} + \frac{w_0 L_1}{Q_1}$$

$$\text{With } AVC(Q_1; w_0) = [w_0 L_1]/Q_1$$

$$\text{With } ATC(Q; w_0) = [r^0 K_0 + w_0 L]/Q(L, K_0)$$





Short term Cost Functions and Technology

With $AVC(Q; w_0) = [w_0 L]/Q(L) = w_0[L/Q(L)]$ $APL(L, K_0) = \frac{Q(L, K_0)}{L}$ The **quality** of workers $(Q/L) = APL(L, K^0)$?
 $Q = L \times APL(L, K^0)$

$$AVC(Q(L, K_0), w_0) = \frac{w_0 L}{Q(L, K_0)} = \frac{w_0 \times L}{L \times APL(L, K_0)} = \frac{w_0}{APL(L, K_0)}$$

$$\Pi^E(Q) = [p(Q) - \text{AVC}(Q, w^0)]Q$$

THE TABLE ATTENDED FOR MORE THAN A CENTURY:
WHO ARE THEY? WHAT DO THEY SAY?





Average Variable Costs and economic unitary profits

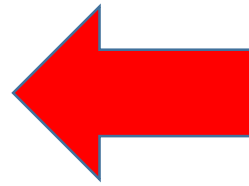
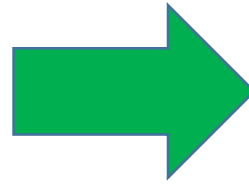
L	wL	Q	APL	AVC $wL/Q = w/APL$	PQ	Mark-up, unitary economic profit (P-AVC)	Economic Profits (P-AVC)Q
P = 5€ w= 10€							
1	10 €	2	2	5	10€	5-5	0
2	20 €	5	2,5	4	25€	5-4	1x5 €
3	30 €	15	5	2	75€	5-2	3x15 € = 45
4	40€	16	4	2,5	80€	5-2,5	2,5x16 € =40

The West in the face of China? «A given (international?) price can only be sustained with AVC such that economic profits are positive»

or...

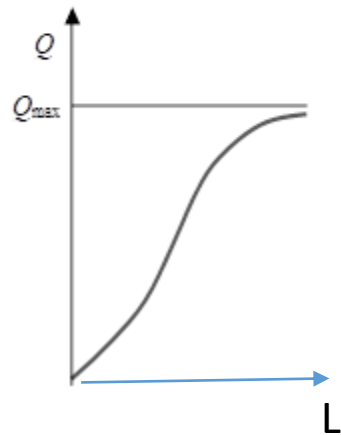
China facing the West? «A given low-price aggressive strategy wiping out competitors can only be sustained with low AVC such that economic profits are positive (in the 90s and 00's, low wages, now...) »

Who needs who?



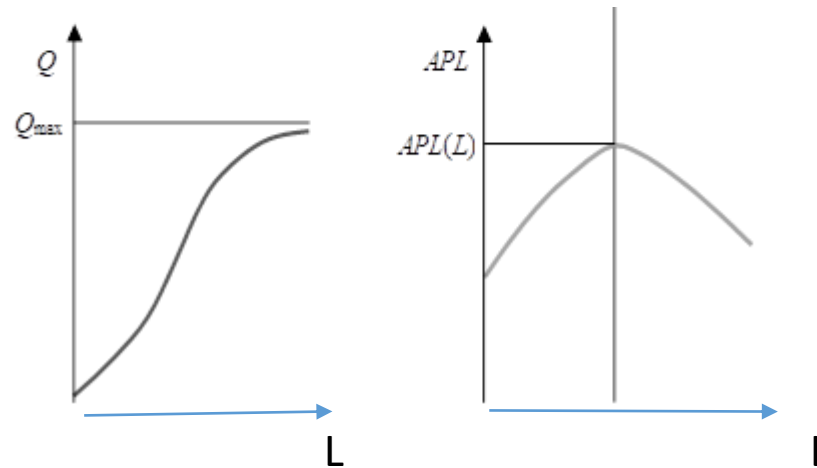
Short term Cost Function: the Role of Technology

$$Q = f(L, K^{\circ})$$





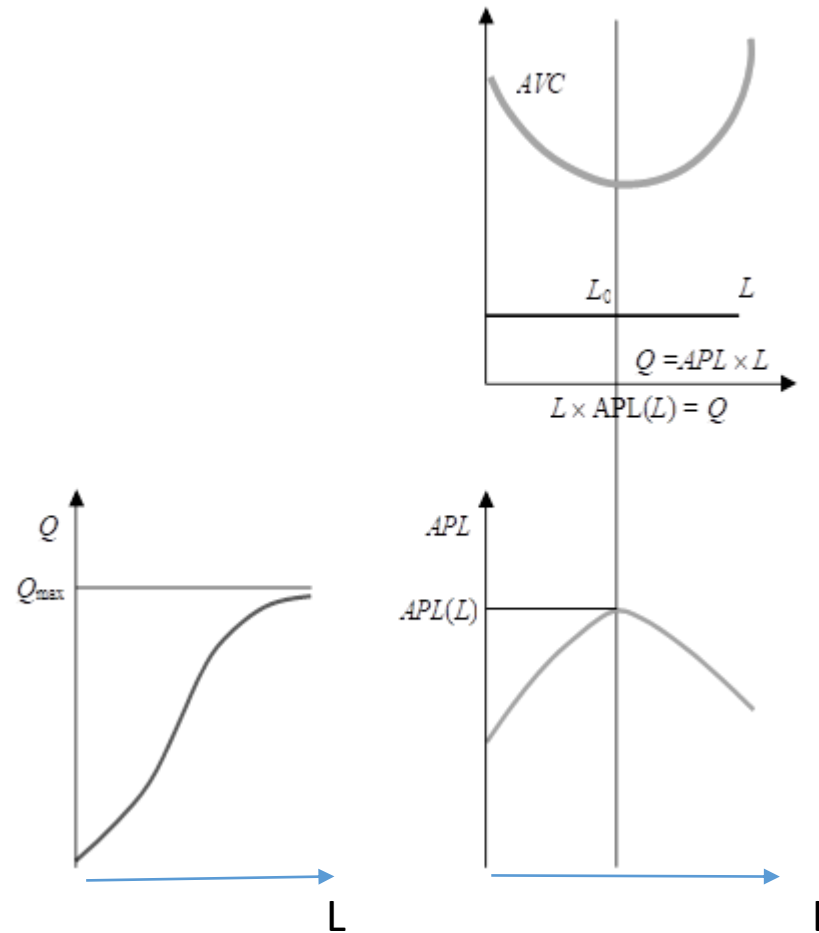
Short term Cost Function



$$AVC (Q(L,K^0),w^0) = w^0/APL (L,K^0)$$



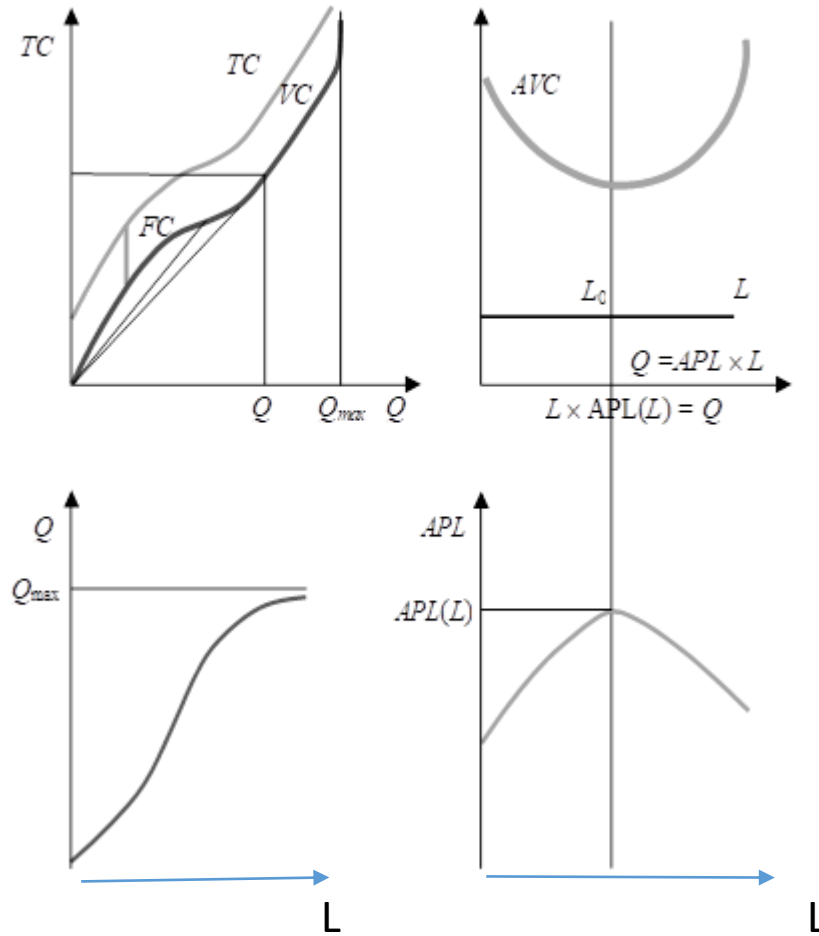
Short term Cost Function



$$AVC (Q(L,K^0),w^0) = w^0/APL (L,K^0)$$



Short term Cost Function





Short-Term: marginal costs and technology

$$MC(Q(L, K_0); w_0) \equiv \frac{\delta CT(Q(L, K_0); w_0)}{\delta Q} = \frac{\delta(w_0 L + r_0 K_0)}{\delta Q} =$$

FC?

$$= \frac{\delta(w_0 L)}{\delta Q} + \frac{\delta(r_0 K_0)}{\delta Q} = w_0 \frac{\delta L}{\delta Q} + 0 = w_0 \frac{1}{MPL(Q, K_0)} = \frac{w_0}{MPL(Q(L, K_0); K_0)}$$

$$MC(1) = \frac{CT(1) - CT(0)}{1 - 0} = CF + CV(1) - CF - CV(0) = \frac{CV(1)}{1} = AVC(1)$$



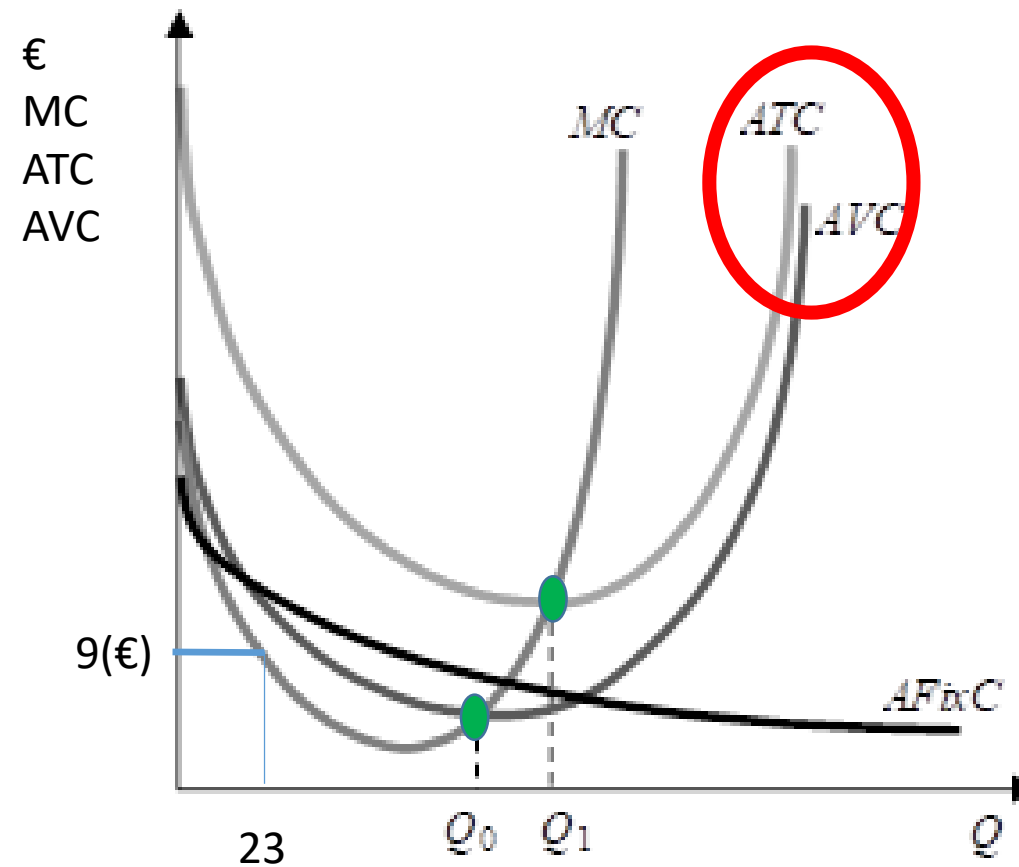
Marginal and average, again

If MPL goes up and then down, MC goes down and then up: what about AVC?

Entering the room...	Average?
1,80	1,80
1,70	1,75
1,60	1,70
1,50	1,65
1,60	1,64
1,63	1,63
1,70	1,66

Draw ATC, AVC, MC

Short term cost functions VIP!



Cost Functions Long Term





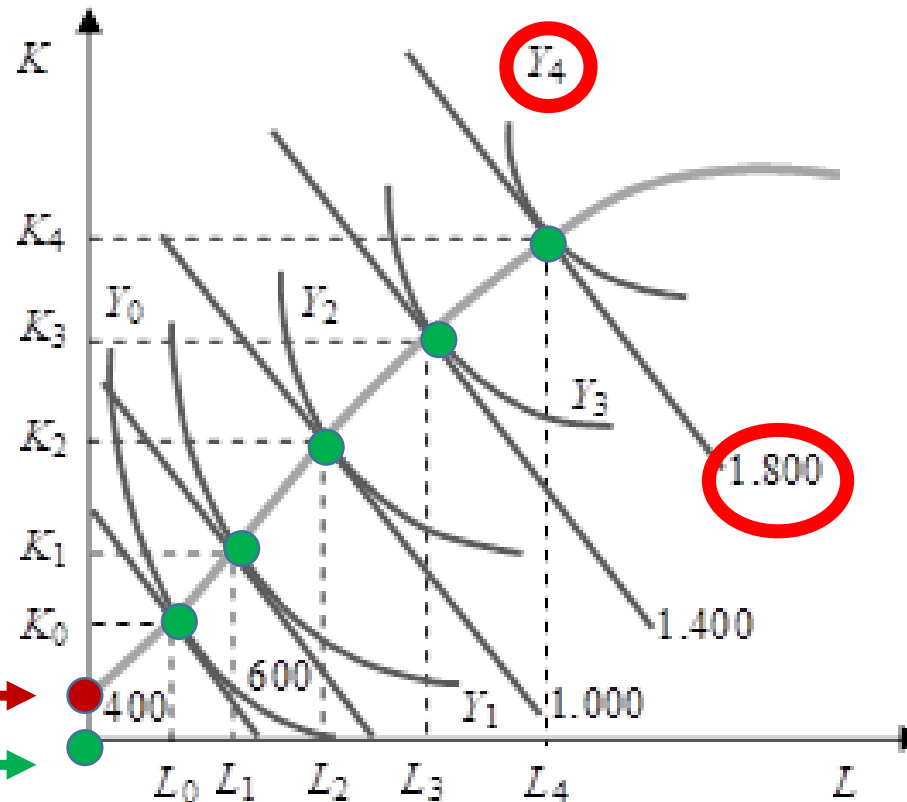
Long term technology expansion path

$$TC_{\min}^{LP}(Q(L,K); w^{\circ}; r^{\circ})$$



$$TC_{\min}^{LP}(Y_4; w^{\circ}; r^{\circ}) = 1800 \text{ €}$$

**CAREFUL MISTAKE IN
THE GRAPH! The green
point is the right one**



Technology will obviously
here too shape the long-
term cost function.

$$TC_{\min}^{LP}(Q(L,K); w^{\circ}; r^{\circ}; T^{\circ})$$

How?

The long-term: marginal «does not apply»?

We will call technologies with **constant returns to scale**, those which, if the use of all the production factors were to increase by a certain same proportion, would result in an exactly proportional increase in output.

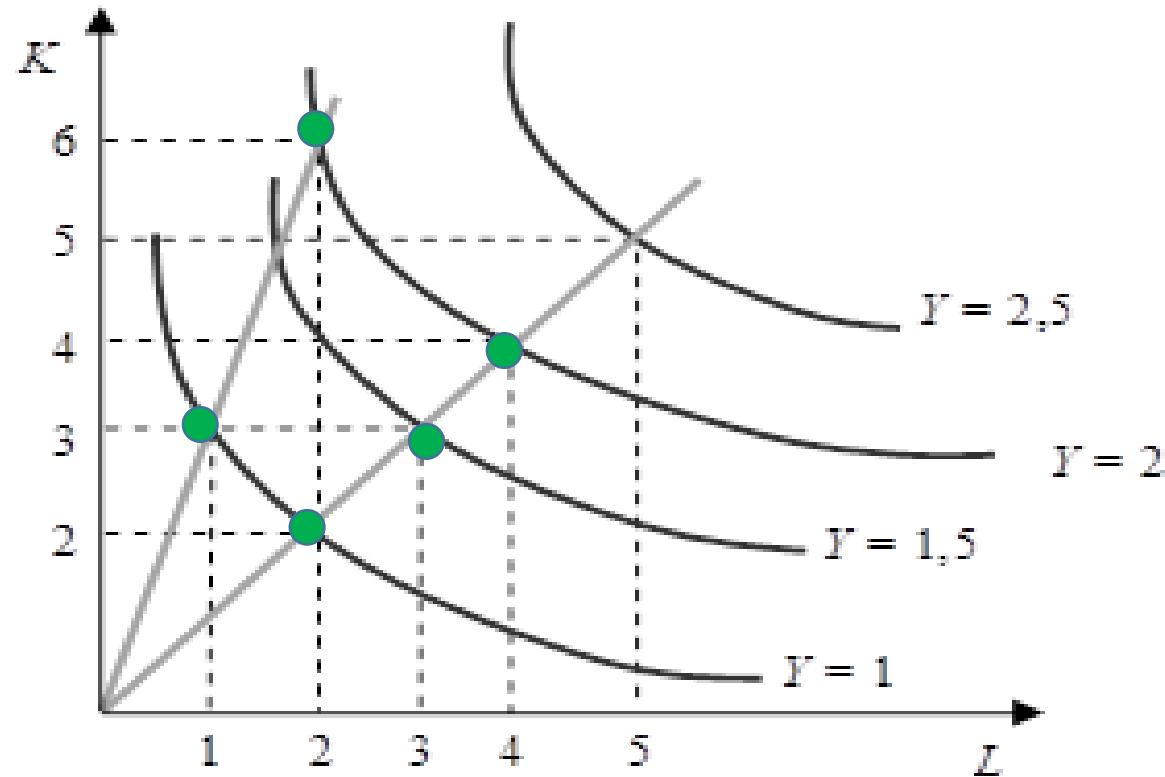
For example, at the simultaneous doubling of the use of capital and labour, the (maximum) output obtained doubles.

PS:
Marginal productivity...
Changes of one input keeping constant the other, an excellent concept of technology for the short term!
But in the long term, when all factors are changing?

It denotes a skill in producing that is not influenced by the size of the production (and therefore of the company): the ability of the firm remains the same independent of size.



The long-term: constant returns to scale



1 ice cream?
1h of L and 200g
of milk

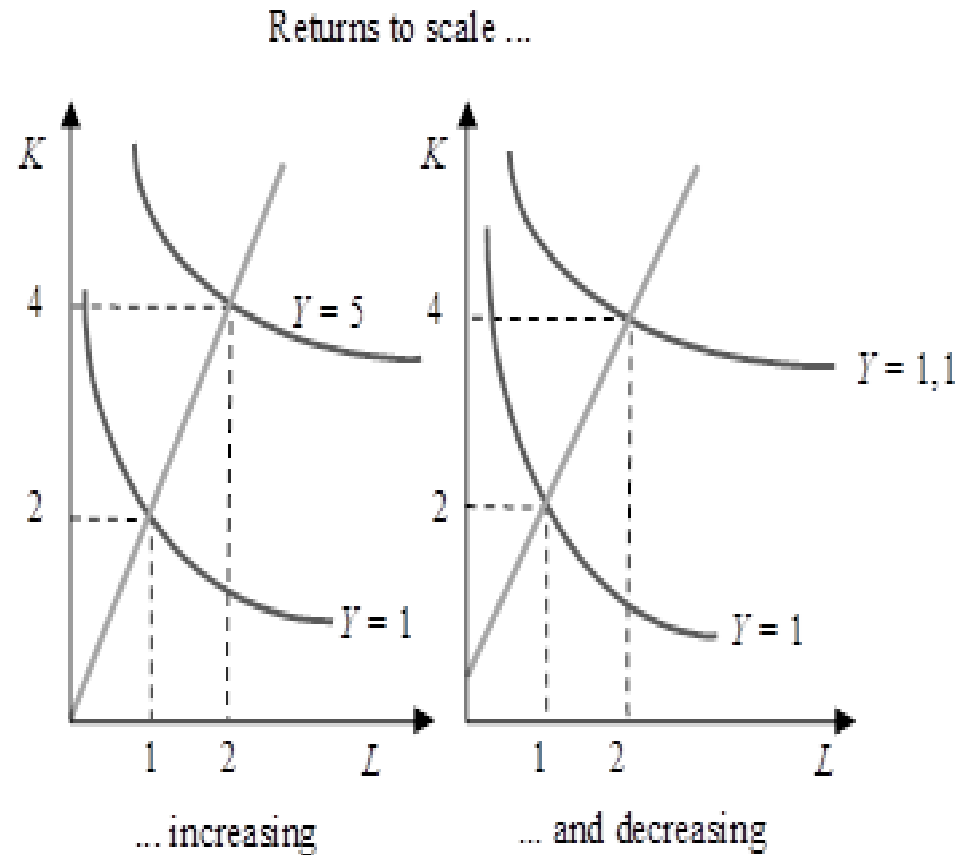
2 ice creams ?
2h of L and
400g of milk

«Good» always
in the same way,
independently
of the size of the
firm



Long term: returns to scale

Companies that transport oil via pipelines: as the diameter of the pipeline **doubles** in size, and with it the materials used to transport oil, the section of the pipeline will **quadruple**, thus increasing more than twice the amount of oil transported.



The «strange»
case of
decreasing
returns to scale

1 ice cream?
1h of L and 200g
of milk

2 ice creams ?
3h of L and
600g of milk

How can you
«forget»?

The
«fascinating»
case of
increasing
returns to scale

1 ice cream?
1h of L and 200g
of milk

2 ice creams ?
1,5 h of L and
300 g of milk

What are you
«learning by
doing»?

Long term: cost and technology, again!

Production function with **constant returns to scale**.

Hypothesis: to produce **a unit** of good, when the factor costs are fixed and equal to (w°, r°) the minimum cost is equal to $TC(1, w^\circ, r^\circ)$ which corresponds to the use of the optimal production technique for 1, $(K1, L1)$: $w^\circ L1 + r^\circ K1$.

We know that by doubling the quantity produced to **two** output units, the most efficient way to produce it will be to **double** the quantity of production factors, i.e. using $K2 = 2K1$ and $L2 = 2L1$.

Therefore, note that the minimum cost to produce the quantity 2, $TC(2, w^\circ, r^\circ)$, will necessarily be equal to double the cost of producing a unit of product: $w^\circ(2L1) + r^\circ(2K1) = 2 CT(1, w^\circ, r^\circ)$.

This would not change if we decided to produce **n units** of product therefore, given the particular property of returns to scale: the total costs would be equal to n times the costs of producing one unit.

Long term: cost and technology, CRSc!

Production function with **constant returns to scale**.

Considering the definition we have given of the **average cost function**, namely the ratio between the minimum total costs of producing a given quantity and the quantity itself, how are:

$ATC(Y, w^{\circ}, r^{\circ})$ and $ATC(nY, w^{\circ}, r^{\circ})$?

$$ATC(w_0, r_0, Y) = \frac{TC(w_0, r_0, Y)}{Y}$$

$$ATC(w_0, r_0, nY) = \frac{TC(w_0, r_0, nY)}{nY} = \frac{nTC(w_0, r_0, Y)}{nY}$$

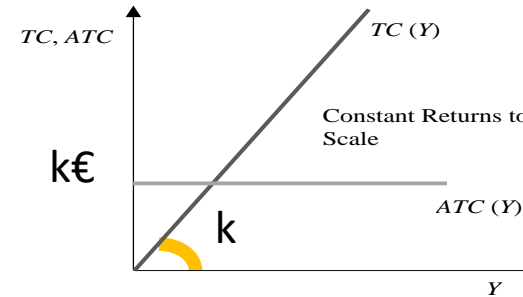
$$ATC(Y, w^{\circ}, r^{\circ}) = ATC(nY, w^{\circ}, r^{\circ}) \text{ for any } n \geq 0$$

Draw $ATC(Q)$ and $TC(Q)$!



Long term: cost and average cost functions

Constant returns to scale: technology that denotes an ability to produce that is not affected by the size of production (and therefore of the company).



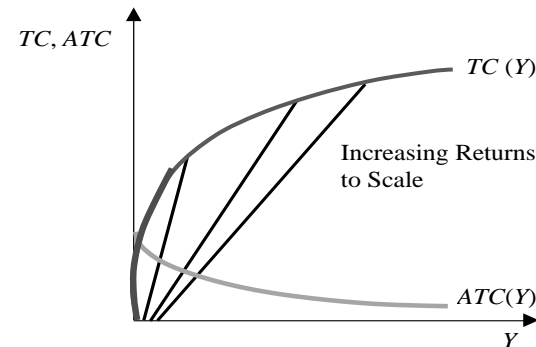
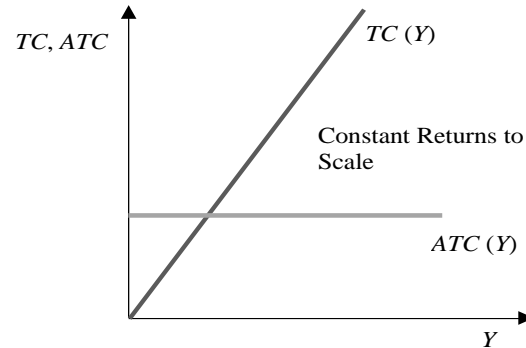


Long term: Increasing Returns to Scale

$$ATC(w_0, r_0, Y) = \frac{TC(w_0, r_0, Y)}{Y}$$

$$ATC(w_0, r_0, nY) = \frac{TC(w_0, r_0, nY)}{nY} = \frac{(n-x) TC(w_0, r_0, Y)}{nY}$$

$ATC(Y, w^\circ, r^\circ) > ATC(nY, w^\circ, r^\circ)$ for all $n \geq 1$
IRTS generate **economies** of scale



Higher wages, lower turnover,
lower average costs, higher
profits, higher wages...



Economies of scale: the «virtuous» circle of
growing. Specialization, learning by doing:
the more you grow the more competitive you
become:

Dehumanizing assembly lines

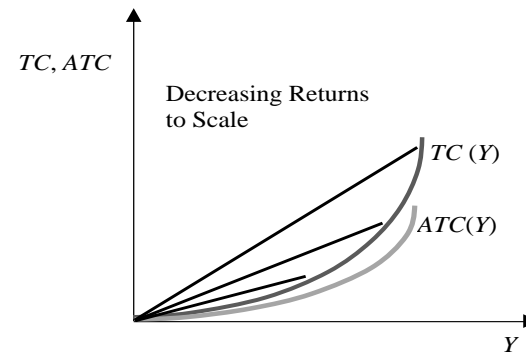
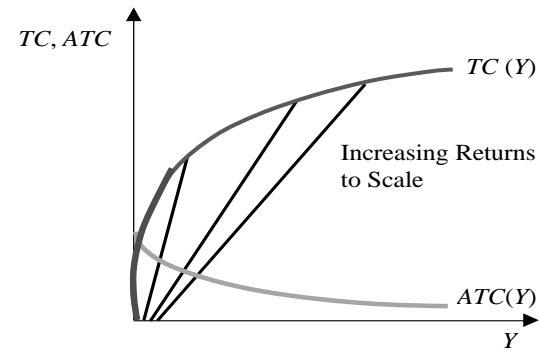
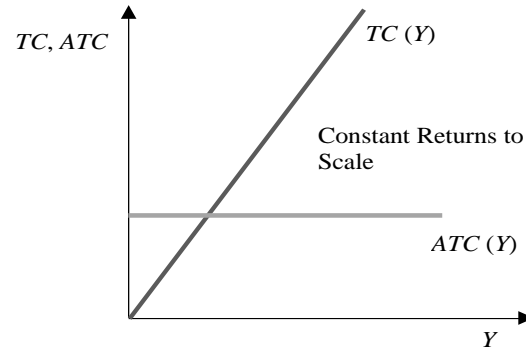


$$\pi^E(Q) = [p(Q) - \text{ATC}(Q, r^\circ, w^\circ)]Q$$



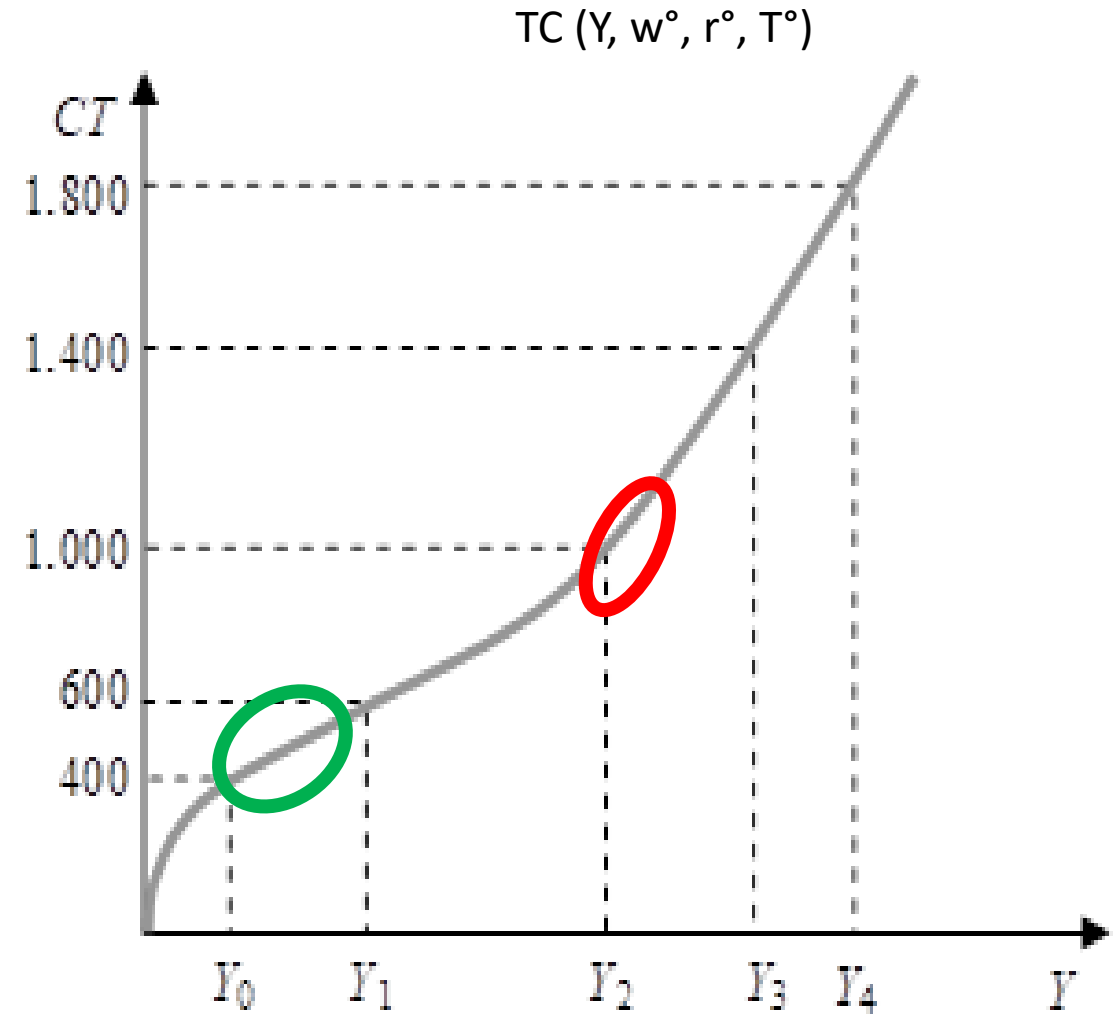
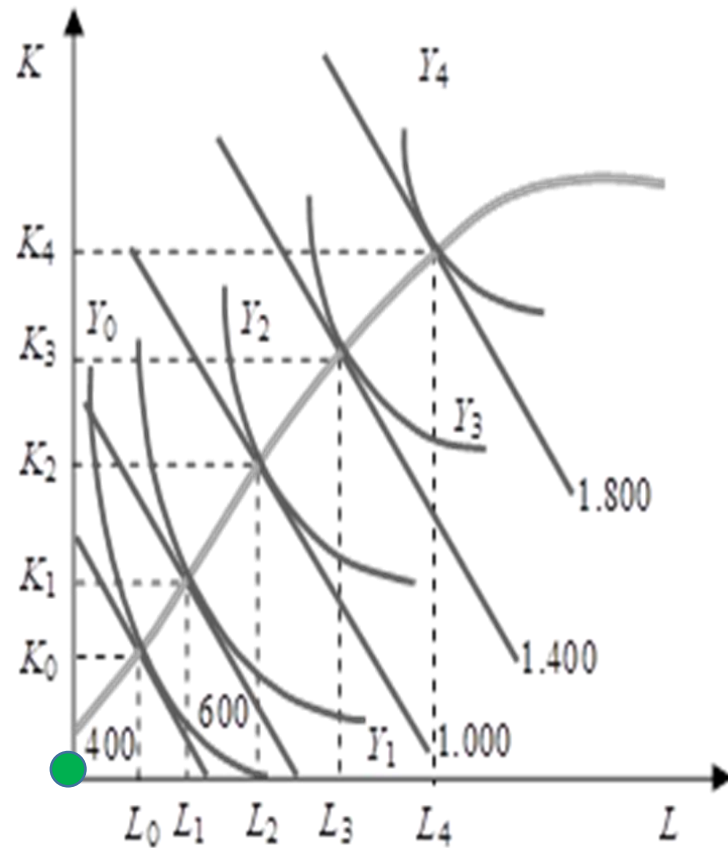


Long term: cost and average cost functions





Long Term: technology and costs. Change along the curve





How do Cost Curves Shift?

Cost curves are drawn for a given technology (production function) and for given unit costs of factors of production.

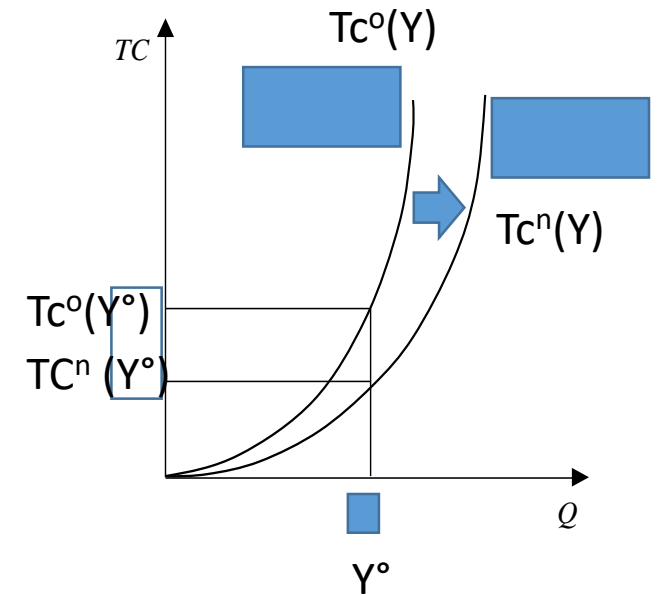
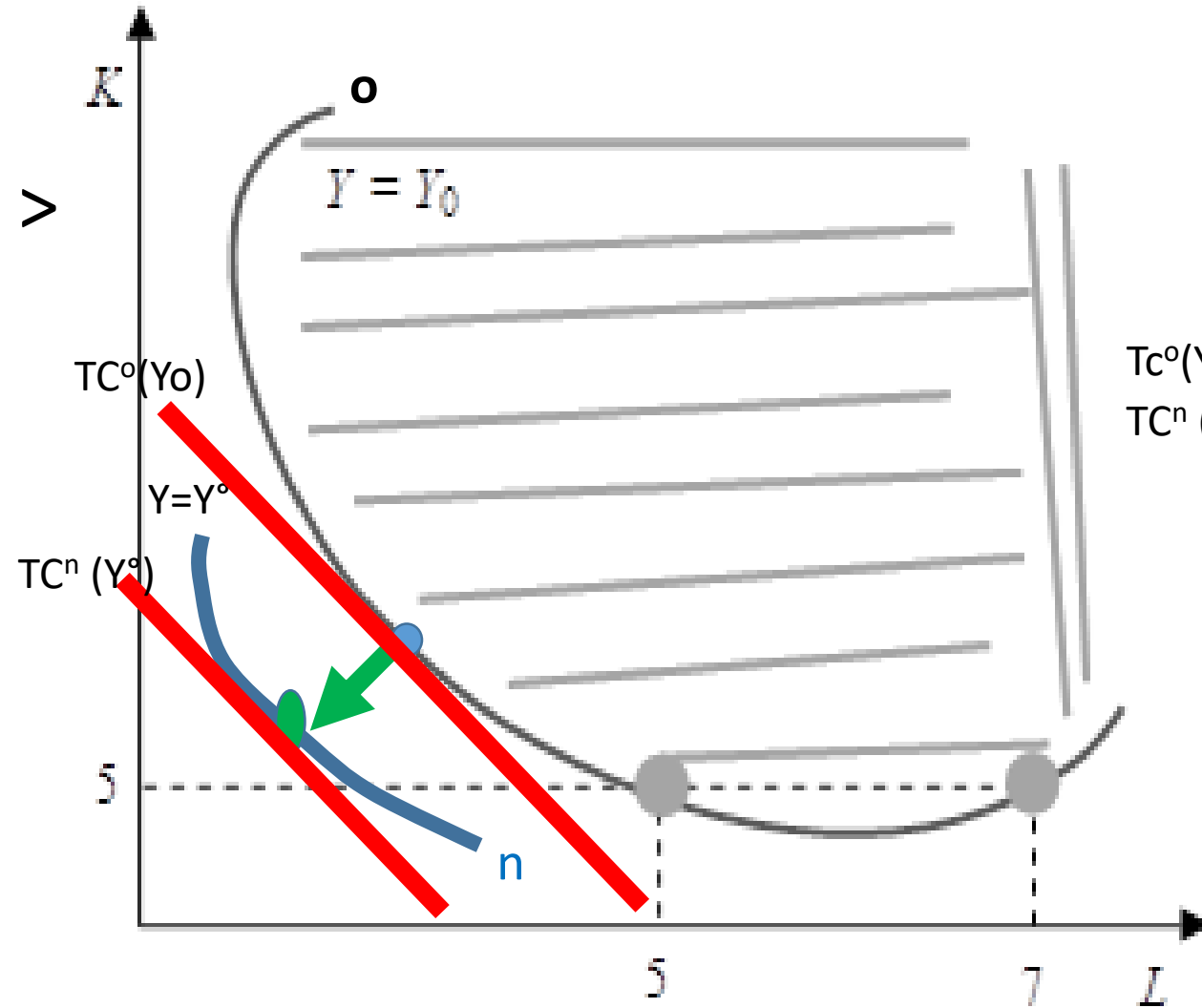
So far, we have moved **along a given cost curve.**

If **technology or **unit costs** change, cost functions **change:**
shift of the curve, not change along the curve.**



PS: technological progress

$$TC^o(Y^o) = TC_0 \text{ €} > \\ TC^n(Y^o) = TC_1 \text{ €}$$





From w° to w'' with $w'' > w^\circ$

TC?

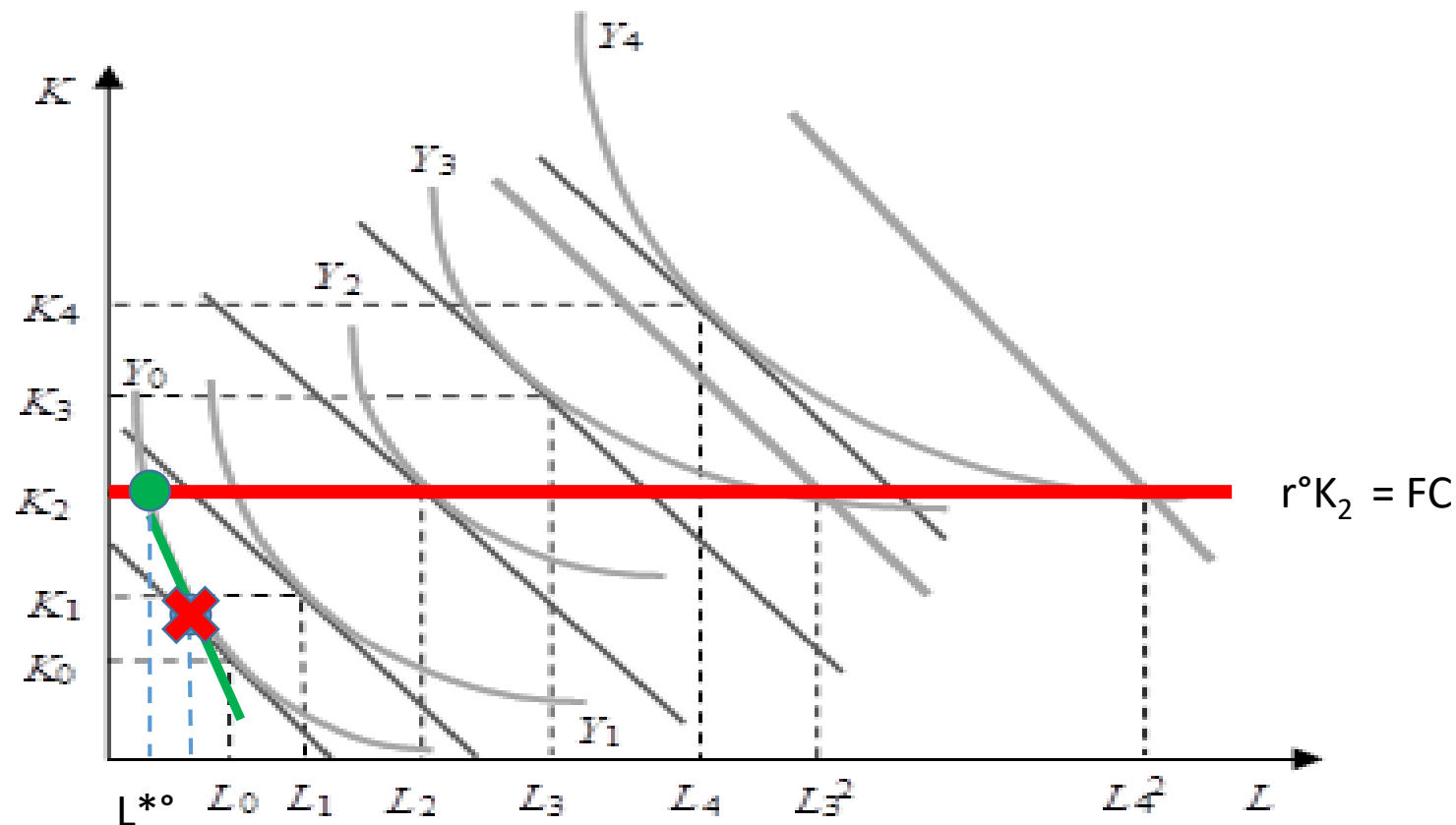
Before:

$$TC = w^\circ L^{\circ*} + r^\circ K_2$$

Now?

$$TC = w'' L^{\circ*} + r^\circ K_2$$

↗ The minimum cost rises and the cost function shifts north west.





ST, variable cost or LT: cost and factor price changes

Case A

Case a: w and r rise by the same proportion, 10%

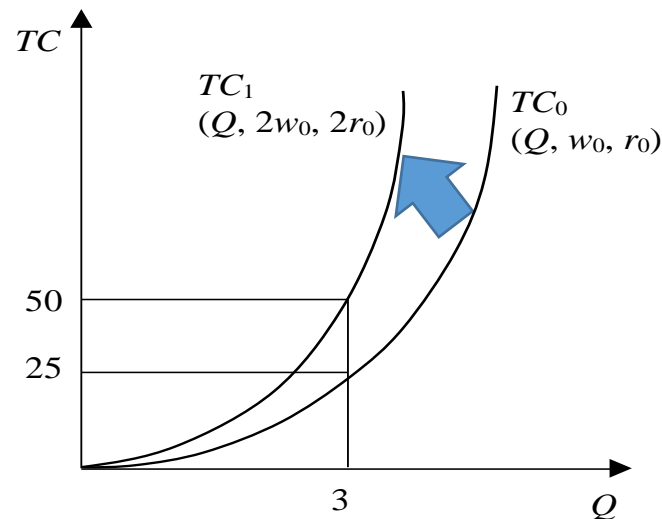
$TC(Q^0; w^0; r^0) = w^0 L^0 + r^0 K^0$ rises to?

$TC(Q^0; w^0 \times 1,1; r^0 \times 1,1) = w^0(1,1) L + r^0(1,1) K$ **What new (L, K) ?**

$TC(Q^0; 1,1 w^0; 1,1 r^0) = w^0(1,1) L^0 + r^0(1,1) K^0$ **The same!**

$TC(Q^0; 1,1 w^0; 1,1 r^0) = 1,1 TC(Q^0; w^0; r^0)$

Cost function moves northwest





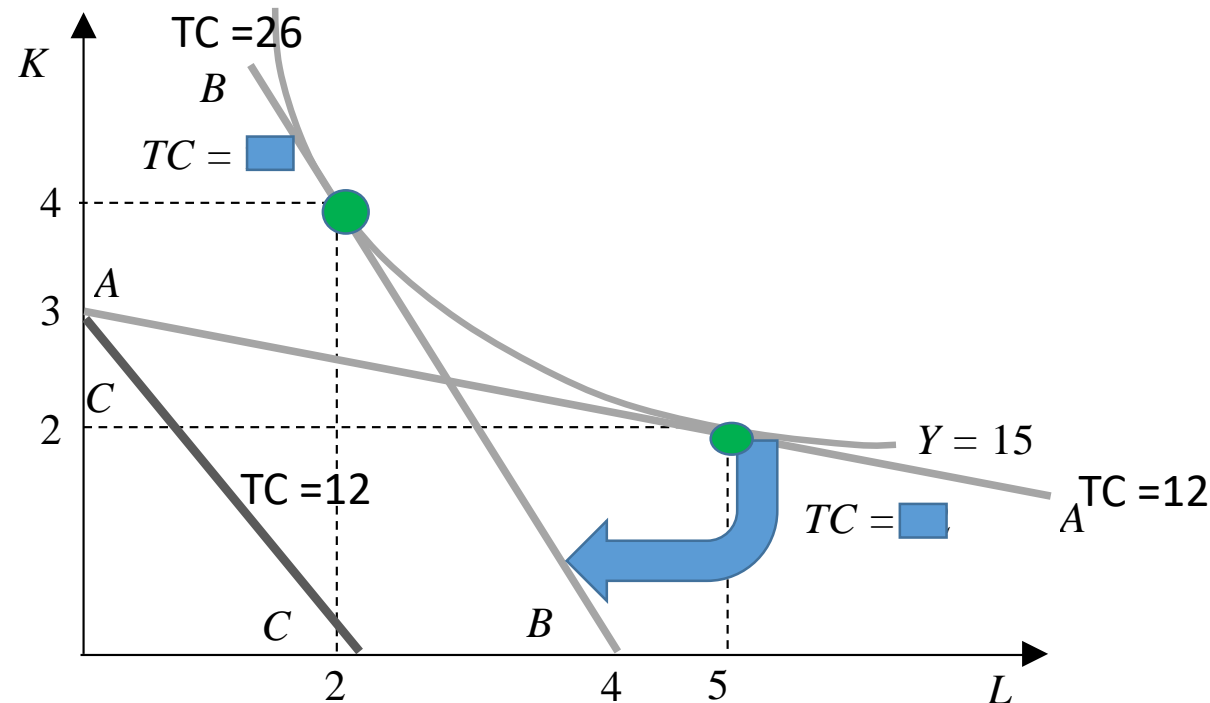
ST, variable cost or LT: case B

$TC(15, w^\circ=4/5, r^\circ=4) = ?$
 $<$
 $TC(15, w'=5, r^\circ=4) = ?$

Do cost rise (above 12€)?

Which techniques now
cost 12 € at the new
unitary costs w' and r° ?

Any of the old ones on
AA?



From AA:
 $w^\circ = (4/5) \text{ €}$

$r^\circ = 4 \text{ €}$

To BB:

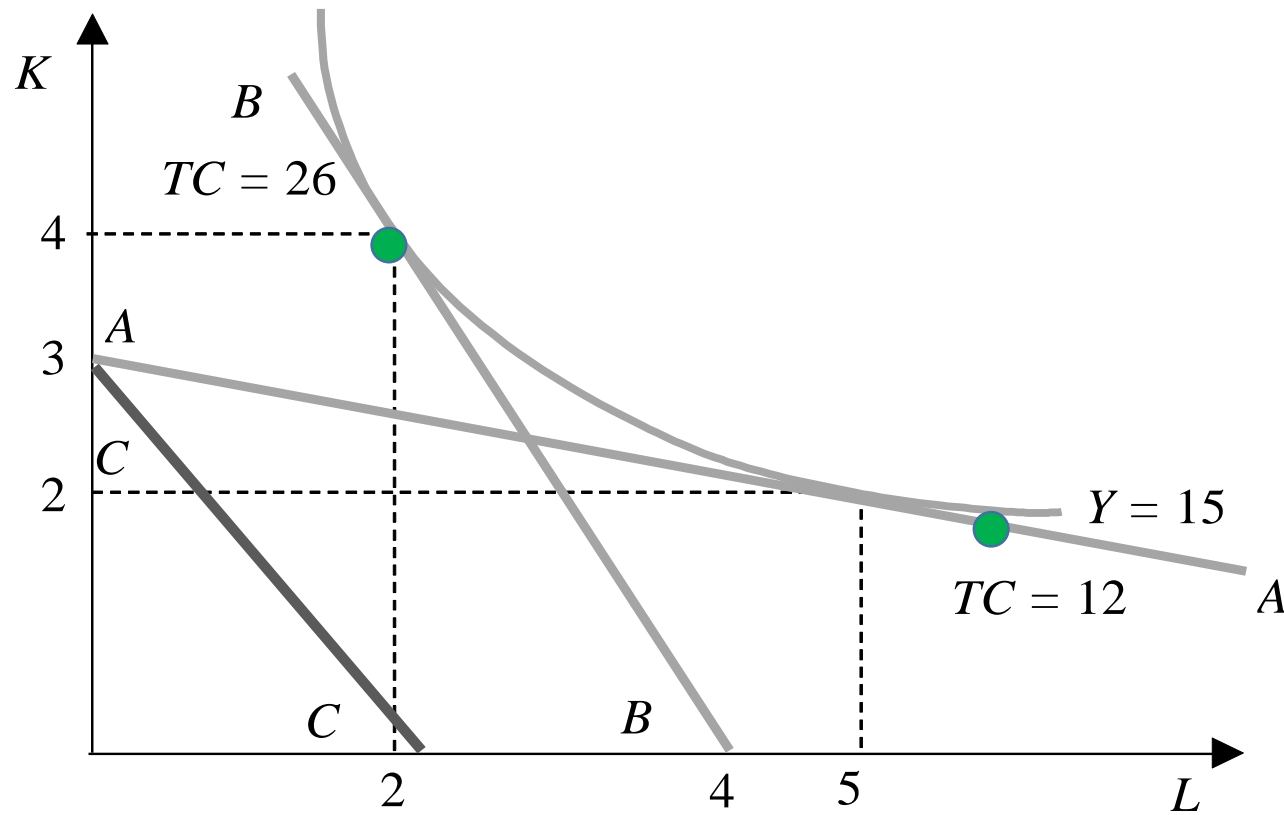
$w' = 5 \text{ €}$

$r^\circ = 4 \text{ €}$

Substituting toward
the cheaper factor of
production



ST, variable cost or LT: case B

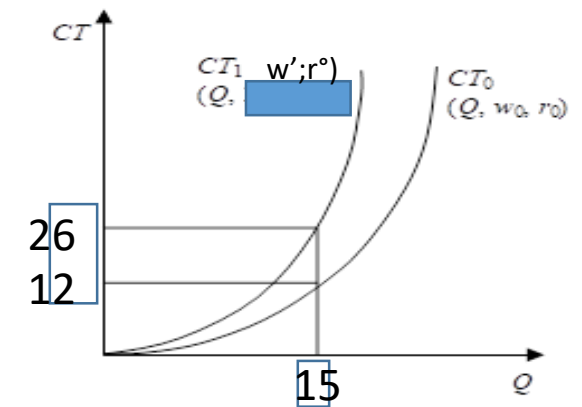


$$w^{\circ} = (4/5) \text{ €}$$

$$r^{\circ} = 4 \text{ €}$$

$$w' = 5 \text{ €}$$

$$r^{\circ} = 4 \text{ €}$$



The cost function goes north-west.



Short term and Long term cost functions

