

PRACTICE 5 - MICROECONOMICS

Bachelor Degree in Global Governance

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PRODUCTION FUNCTION AND ISOQUANTS

- **Production function:** it is the set of points, the geometric locus, that associates any available combination of inputs with the maximum obtainable level of output. Thus, it is a function where the quantity produced Y depends on the factors of production, such as capital K and labor L :

$$Y = f(L, K)$$

- **Isoquant:** it is a function that represents all combinations (L, K) of inputs that provide in an output-efficient manner a certain level of output, q . The isoquant therefore describes all combinations of capital and labor, all production techniques, that allow producing output q in an output-efficient manner (all combinations that have as output-efficient $Y = q$):

$$q = Y = f(L, K)$$

MARGINAL PRODUCTS AND MARGINAL RATE OF TECHNICAL SUBSTITUTION

- **Marginal product:** it shows the variation of the output related to an infinitesimal variation of the factor of production, with all other factors kept constant. As in the utility maximization problem, to maximize production, we need to set the first derivative equal to zero. Therefore, $dY = 0 \rightarrow \frac{dY}{dL} \cdot dL + \frac{dY}{dK} \cdot dK = 0 \rightarrow f_L dL + f_K dK = 0$. Note that $f_L = MP_L$ and $f_K = MP_K$ are the two marginal products.
- **Marginal Rate of Technical Substitution (MRTS):** it shows how the inputs can substitute for each other in the production function and it is calculated as the ratio of the marginal product of one input to the marginal product of the other input. From a geometrical perspective, it can be thought of as the absolute value of the slope of the isoquant. Solving the preceding equation,

$$f_L dL + f_K dK = 0 \rightarrow \frac{f_L}{f_K} = -\frac{dK}{dL} \rightarrow \frac{MP_L}{MP_K} = -\frac{dK}{dL}.$$

Thus, since $\frac{MP_L}{MP_K} = \left| \frac{dK}{dL} \right| = MRTS$, the MRTS is equal to the absolute value of the ratio of total derivatives.

ISOCOSTS

- **Total cost:** it represents the minimum cost of producing any quantity Q . We can divide the firm's total cost into **fixed costs**, those costs that the entrepreneur must incur regardless of the quantity produced, which will thus be represented mathematically by a constant, and **variable costs**, those costs that vary depending on the quantity produced, which will thus be represented mathematically by a function of quantity.

$$TC(Q) = FC + VC(Q)$$

- **Isocost:** it is the geometric locus of combinations of labor and capital factor productive techniques all characterized by the same cost for the entrepreneur. In the case of the two production factors already introduced in the production function, L and K , defining their remuneration respectively w the wage, the salary, of workers and r the interest rate of capital, we can write:

$$\bar{c} = TC(K, L) = rK + wL \rightarrow K = -\frac{w}{r}L + \frac{\bar{c}}{r}$$

EXERCISE

Given the production function $f(L, K) = L^{\frac{1}{4}}K^{\frac{3}{4}}$:

- Find the general equation of an isoquant, and the isoquant relative to the level $\bar{q} = 100$;
- Calculate the marginal productivity of labor and capital and compute the marginal rate of technical substitution;
- Find the equation of the isocost relative to the input prices $w = 1$ and $r = 2$ and the level of total cost $\bar{c} = 5$.

COST MINIMIZATION IN THE SHORT RUN

- In the short run, we consider one of the two inputs as fixed (usually, capital). To minimize the total cost in order to produce a certain amount of output (\bar{q}), the entrepreneur faces the following problem:

$$\min rK + wL \text{ s.t. } \bar{q} = f(K, L), \text{ s.t. } K = \bar{K} \rightarrow \min rK + wL \text{ s.t. } \bar{q} = f(\bar{K}, L)$$

- **Exercise:** Given that $f(L, K) = L^{\frac{1}{2}}K^{\frac{1}{2}}$, $w = 16$, $r = 4$, $\bar{K} = 100$, $\bar{q} = 200$:
 1. Carry out the cost minimization problem, assuming a short run time horizon,
 2. Find the optimal input bundle,
 3. Compute the total cost sustained by the producer.

COST MINIMIZATION IN THE LONG RUN

- In the short run horizon, we were setting a certain level of capital $K = \bar{K}$, while in the long run horizon, we can optimize the level of both productive inputs.
- We start from an analogous optimization problem we have already faced: the utility maximization:

$$\max U(x_1, x_2) \text{ s.t. } I = p_1 x_1 + p_2 x_2.$$

To solve it, we started from the system:

$$\begin{cases} MRS = \frac{p_1}{p_2} \rightarrow \text{tangency condition} \\ I = p_1 x_1 + p_2 x_2 \rightarrow \text{budget constraint} \end{cases}$$

COST MINIMIZATION IN THE LONG RUN

- In the case of cost minimization, the entrepreneur wants to produce a certain quantity of output (fixed) minimizing the total cost to produce it:

$$\min wL + rK \text{ s.t. } \bar{q} = f(L, K).$$

Graphically, she wants to find the tangency point between the isoquant corresponding to output level \bar{q} and the lowest isocost line. Analytically, she solves the system:

$$\begin{cases} MRTS = \frac{w}{r} \rightarrow \text{tangency condition} \\ \bar{q} = f(L, K) \rightarrow \text{isoquant} \end{cases}$$

EXERCISE

- Given the production function $f(L, K) = L^{\frac{1}{4}}K^{\frac{1}{4}}$, $w = 30$, $r = 30$, $\bar{q} = 5$:
 1. Compute the value of L and K that minimize cost;
 2. Calculate the corresponding total cost.