Exercise 1 (Short Term)

In a market under the assumption of perfect competition, 50 firms operate, each with the following total cost function:

$$CT(Q_i) = 2Q_i^2 + 8$$

The demand function characterizing this market is given by the following function:

$$Q_d = 400 - 10p$$

Calculate:

1. The short-term supply function of a single firm
2. The short-term supply function of the industry
3. The market equilibrium price and quantity
4. The production level and profit realized by a single firm in the short term

Solution

1. To calculate the short-term supply function, it is necessary to find the short-term equilibrium for a generic firm i and express the quantity as a function of the price:

$$p = MC(Q_i) \rightarrow p = 4Q_i \rightarrow Q_i = \frac{p}{4}$$

2. To calculate the supply function for the entire industry, we sum the supply functions of the individual firms linearly:

$$Q_s = 50\left(\frac{p}{4}\right) = 12.5p$$

3. To calculate the market equilibrium, we need to solve a system of two equations with two unknowns:

$$\begin{cases} Q_d(p) = 400 - 10p \\ Q_s(p) = 12.5p \end{cases}$$

By equating the two functions:

$$400 - 10p = 12.5p \rightarrow 22.5p = 400 \rightarrow p = \frac{400}{22.5} \approx 17.78$$

Substituting the found price into one of the functions:

$$Q_s(p) = 12.5 \cdot 17.78 \approx 222.25$$

The equilibrium condition is:

$$(Q_E = 222.25, p_E \approx 17.78)$$

4. With 50 firms in perfect competition, each will produce one-fiftieth of the demanded quantity:

$$Q_{E_i} = \frac{Q_E}{50} \approx \frac{222.25}{50} \approx 4.45$$

Substituting the equilibrium price and quantity for each firm in the profit formula:

$$\pi_i = p_E \cdot Q_{E_i} - CT(Q_{E_i}) = 17.78 \cdot 4.45 - (2 \cdot 4.45^2 + 8) \approx 79.17 - 48.12 \approx 31.05$$

In this case, the firms have a profit of about 31.05.

Exercise 2 (Short Term)

In a market under the assumption of perfect competition, 60 firms operate, each with the following total cost function:

$$CT(Q_i) = 3Q_i^2 + 5$$

The demand function characterizing this market is given by the following function:

$$Q_d = 600 - 30p$$

Calculate:

1. The short-term supply function of a single firm
2. The short-term supply function of the industry
3. The market equilibrium price and quantity
4. The production level and profit realized by a single firm in the short term

Solution

1. To calculate the short-term supply function, it is necessary to find the short-term equilibrium for a generic firm i and express the quantity as a function of the price:

$$p = MC(Q_i) \rightarrow p = 6Q_i \rightarrow Q_i = \frac{p}{6}$$

2. To calculate the supply function for the entire industry, we sum the supply functions of the individual firms linearly:

$$Q_s = 60\left(\frac{p}{6}\right) = 10p$$

3. To calculate the market equilibrium, we need to solve a system of two equations with two unknowns:

$$\begin{cases} Q_d(p) = 600 - 30p \\ Q_s(p) = 10p \end{cases}$$

   By equating the two functions:

$$600 - 30p = 10p \rightarrow 40p = 600 \rightarrow p = \frac{600}{40} = 15$$

   Substituting the found price into one of the functions:

$$Q_s(p) = 10 \cdot 15 = 150$$

   The equilibrium condition is:

$$(Q_E = 150, p_E = 15)$$

4. With 60 firms in perfect competition, each will produce one-sixtieth of the demanded quantity:

$$Q_{E_i} = \frac{Q_E}{60} = \frac{150}{60} = 2.5$$

   Substituting the equilibrium price and quantity for each firm in the profit formula:

$$\pi_i = p_E \cdot Q_{E_i} - CT(Q_{E_i}) = 15 \cdot 2.5 - (3 \cdot 2.5^2 + 5) \approx 37.5 - 23.75 \approx 13.75$$

   In this case, the firms have a profit of about 13.75.

Exercise 3 (Long Term)

In a market under the assumption of perfect competition, 40 firms operate, each with the following total cost function:

$$CT(Q_i) = 4Q_i^2 + 20$$

The demand function characterizing this market is given by the following function:

$$Q_d = 800 - 20p$$

Calculate:

1. The equilibrium price and quantity of the firm in the long term
2. The long-term equilibrium quantity of the market for the industry
3. The number of firms operating in the long term
4. The long-term profit sustained by each firm in the case where the size of the plants is not free to vary

Solution

1. To calculate the price and quantity, start from the equilibrium condition of a firm in the long term:

$$p = ATC(Q_{\min})$$

Therefore, minimize the average cost function:

$$ATC(Q) = \frac{CT(Q)}{Q} = 4Q + \frac{20}{Q}$$

$$\frac{\partial ATC(Q)}{\partial Q} = \frac{\partial(4Q + \frac{20}{Q})}{\partial Q} = 4 - \frac{20}{Q^2} = 0 \rightarrow Q^2 = 5 \rightarrow Q_{\min} = \sqrt{5}$$

After finding the minimum point for the average costs, insert them into the starting function and set it equal to the price:

$$p = ATC(\sqrt{5}) = 4\sqrt{5} + \frac{20}{\sqrt{5}} = 4\sqrt{5} + 4\sqrt{5} = 8\sqrt{5} \rightarrow p_{LP} = 8\sqrt{5}$$

2. To calculate the entire industry's supply, multiply by 40 the quantity supplied by the single firm:

$$Q_{LP} = 40 \cdot \sqrt{5} = 40 \cdot 2.24 = 89.6$$

3. The number of operating firms is given by the ratio between the market demand function at the found price:

$$Q_d(8\sqrt{5}) = 800 - 20 \cdot 8\sqrt{5} = 800 - 160\sqrt{5} \approx 800 - 357.77 = 442.23 \rightarrow Q_E \approx 442.23$$

and the maximum quantity that each single firm is willing to supply in the long term:

$$n_{LP} = \frac{Q_E}{Q_i} = \frac{442.23}{2.24} \approx 197.42$$

We can therefore notice that in the long term, the supply exceeds the demand and many firms will participate in the market.

4. Finally, calculate the profit at the equilibrium point for each single firm:

$$\pi_i = p_E \cdot Q_{E_i} - CT(Q_{E_i}) = 8\sqrt{5} \cdot \sqrt{5} - (4 \cdot 5 + 20) = 40 - 40 = 0$$

In this case, the firms have zero profit.

Exercise 4 (Long Term)

In a market under the assumption of perfect competition, 30 firms operate, each with the following total cost function:

$$CT(Q_i) = Q_i^2 + 12Q_i + 20$$

The demand function characterizing this market is given by the following function:

$$Q_d = 200 - 5p$$

Calculate:

1. The equilibrium price and quantity of the firm in the long term
2. The long-term equilibrium quantity of the market for the industry
3. The number of firms operating in the long term
4. The long-term profit sustained by each firm in the case where the size of the plants is not free to vary

Solution

1. To calculate the price and quantity, start from the equilibrium condition of a firm in the long term:

   $$p = ATC(Q_{\min})$$

   Therefore, minimize the average cost function:

   $$ATC(Q) = \frac{CT(Q)}{Q} = Q + 12 + \frac{20}{Q}$$

   $$\frac{\partial ATC(Q)}{\partial Q} = \frac{\partial(Q + 12 + \frac{20}{Q})}{\partial Q} = 1 - \frac{20}{Q^2} = 0 \rightarrow Q^2 = 20 \rightarrow Q_{\min} = \sqrt{20}$$

   After finding the minimum point for the average costs, insert them into the starting function and set it equal to the price:

   $$p = ATC(\sqrt{20}) = \sqrt{20} + 12 + \frac{20}{\sqrt{20}} = 2\sqrt{20} + 12 \rightarrow p_{LP} = 16$$

2. To calculate the entire industry's supply, multiply by 30 the quantity supplied by the single firm:

   $$Q_{LP} = 30 \cdot \sqrt{20} = 30 \cdot 4.47 = 134.1$$

3. The number of operating firms is given by the ratio between the market demand function at the found price:

$$Q_d(16) = 200 - 5 \cdot 16 = 200 - 80 = 120 \rightarrow Q_E = 120$$

and the maximum quantity that each single firm is willing to supply in the long term:

$$n_{LP} = \frac{Q_E}{Q_i} = \frac{120}{4.47} = 26.85$$

We can therefore notice that in the long term, the demand is lower than the supply and fewer firms will participate in the market.

4. Finally, calculate the profit at the equilibrium point for each single firm:

$$\pi_i = p_E \cdot Q_{E_i} - CT(Q_{E_i}) = 16 \cdot 4.47 - (4.47^2 + 12 \cdot 4.47 + 20) = 71.52 - 71.52 = 0$$

In this case, the firms have zero profit.