Various Microeconomics Exercises

1 Utility Maximization

Exercise 1

Given the utility function:

$$U(x_1, x_2) = x_1^{1/2} x_2^{1/3}$$

Determine:

1. The marginal utilities;
2. The marginal rate of substitution (MRS);
3. The equation of the indifference curve for $U = 12$.

Exercise 2

A consumer has income $R = 120$, prices $p_1 = 6$, $p_2 = 3$ and utility function:

$$U(x_1, x_2) = x_1^{1/2} x_2^{1/2}$$

Determine the optimal consumption bundle.

2 Elasticity, Demand and Supply

Exercise 3

Given the price-quantity combinations:

$$A = (2, 10), \qquad B = (6, 6)$$

Determine:

1. The inverse demand function;
2. The elasticity of demand at point A;
3. The elasticity of demand at point B.

Exercise 4

Firms 1 and 2 have supply functions:

$$Q_1 = p$$

$$Q_2 = \max\{2p - 4, 0\}$$

Determine the aggregate supply.

3 Constructing a Demand Curve

Exercise 5

Consider a consumer with Cobb-Douglas utility function:

$$U(x_1, x_2) = x_1^{2/3} x_2^{1/3}$$

The consumer's income is:

$$R = 240$$

The price of good 2 is fixed at:

$$p_2 = 4$$

Determine:

1. The optimal demand for good x_1;
2. The direct demand curve for good 1;
3. The inverse demand curve for good 1;
4. The price elasticity of demand.

4 Production and Costs

Exercise 6

Given the production function:

$$Y = K^{1/2} L^{1/2}$$

Determine:

1. The marginal productivities;
2. The marginal rate of technical substitution (MRTS);
3. Returns to scale.

Exercise 7

Given the total cost function:

$$TC(Q) = Q^2 + 20Q + 50$$

Calculate:

1. Average cost;
2. Marginal cost;
3. The production level for which $MC = AC$.

5 Perfect Competition and Monopoly

Exercise 8

In a perfectly competitive market there are 20 identical firms with:

$$TC(Q_i) = Q_i^2 + 16$$

Market demand is:

$$Q^d = 240 - 10p$$

Determine:

1. The optimal output of the single firm in the long run;
2. The long-run equilibrium price;
3. The number of active firms in the long run.

Exercise 9

A monopolist faces the demand:

$$Q = 300 - 3p$$

and has costs:

$$TC(Q) = 30Q$$

Determine:

1. Monopoly quantity and price;
2. The monopolist's profit;
3. Competitive equilibrium.

Solution Exercise 1

$$MU_1 = \frac{1}{2} x_1^{-1/2} x_2^{1/3}$$

$$MU_2 = \frac{1}{3} x_1^{1/2} x_2^{-2/3}$$

$$MRS = \frac{MU_1}{MU_2} = \frac{3x_2}{2x_1}$$

For $U = 12$:

$$x_1^{1/2} x_2^{1/3} = 12$$

Solution Exercise 2

Tangency condition:

$$\frac{x_2}{x_1} = \frac{6}{3} = 2$$

Therefore:

$$x_2 = 2x_1$$

Budget constraint:

$$120 = 6x_1 + 3x_2$$

$$120 = 6x_1 + 6x_1 = 12x_1$$

$$x_1^* = 10, \qquad x_2^* = 20$$

Solution Exercise 3

Slope coefficient:

$$m = \frac{6 - 10}{6 - 2} = -1$$

Inverse demand:

$$p = 12 - Q$$

Elasticity:

$$\varepsilon = \frac{\partial Q}{\partial p} \frac{p}{Q}$$

At point A:

$$\varepsilon_A = -\frac{2}{10} = -0.2$$

At point B:

$$\varepsilon_B = -\frac{6}{6} = -1$$

Solution

For a Cobb-Douglas function:

$$U(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$$

optimal demands are:

$$x_1^d = \frac{\alpha}{\alpha+\beta}\frac{R}{p_1}$$

$$x_2^d = \frac{\beta}{\alpha+\beta}\frac{R}{p_2}$$

In our case:

$$\alpha = \frac{2}{3}, \qquad \beta = \frac{1}{3}$$

Thus:

$$\alpha + \beta = 1$$

The optimal demand for good 1 is:

$$x_1^d = \frac{2/3}{1}\frac{240}{p_1}$$

$$x_1^d = \frac{160}{p_1}$$

This is the direct demand curve:

$$Q = \frac{160}{p}$$

To obtain the inverse demand curve, isolate price:

$$p = \frac{160}{Q}$$

Demand elasticity:

$$\varepsilon = \frac{\partial Q}{\partial p}\frac{p}{Q}$$

Since:

$$Q = 160p^{-1}$$

we have:

$$\frac{\partial Q}{\partial p} = -160p^{-2}$$

Therefore:

$$\varepsilon = (-160p^{-2})\frac{p}{160p^{-1}}$$

$$\varepsilon = -1$$

Demand has constant unit elasticity at every point on the curve.

Solution Exercise 4

Aggregate supply:

$$Q^S = \begin{cases} p & \text{if } p < 2 \\ 3p - 4 & \text{if } p \geq 2 \end{cases}$$

Solution Exercise 5

$$MP_L = \frac{1}{2}K^{1/2}L^{-1/2}$$

$$MP_K = \frac{1}{2}K^{-1/2}L^{1/2}$$

$$MRTS = \frac{K}{L}$$

The sum of the exponents is:

$$\frac{1}{2} + \frac{1}{2} = 1$$

Therefore returns to scale are constant.

Solution Exercise 6

Average cost:

$$AC(Q) = Q + 20 + \frac{50}{Q}$$

Marginal cost:

$$MC(Q) = 2Q + 20$$

Setting:

$$MC = AC$$

$$2Q + 20 = Q + 20 + \frac{50}{Q}$$

$$Q^2 = 50$$

$$Q = \sqrt{50}$$

Solution Exercise 7

Average cost:

$$AC(Q) = Q + \frac{16}{Q}$$

Minimum condition:

$$1 - \frac{16}{Q^2} = 0$$

$$Q_i^{LR} = 4$$

Price:

$$p^{LR} = 4 + \frac{16}{4} = 8$$

Demand:

$$Q^d = 240 - 10(8) = 160$$

Number of firms:

$$n = \frac{160}{4} = 40$$

Solution Exercise 8

Inverse demand:

$$p = 100 - \frac{Q}{3}$$

Total revenue:

$$TR = 100Q - \frac{Q^2}{3}$$

Marginal revenue:

$$MR = 100 - \frac{2Q}{3}$$

Monopoly condition:

$$100 - \frac{2Q}{3} = 30$$

$$Q^m = 105$$

Price:

$$p^m = 100 - \frac{105}{3} = 65$$

Profit:

$$\pi = 65 \cdot 105 - 30 \cdot 105 = 3675$$

Competitive equilibrium:

$$p = MC = 30$$

$$30 = 100 - \frac{Q}{3}$$

$$Q^{cp} = 210$$