Math Review for the Microeconomics Course

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Powers: definitions and basic rules

- Given a number x, its n-th power x^n is the product of x multiplied by itself n times.
- Example: $2^3 = 2 \cdot 2 \cdot 2 = 8$.

Basic rules:

- For each $x > 0$: $\quad x^0 = 1$.
- For each x: $\quad x^1 = x$.
- For $n > 0$: $\quad x^{-n} = \dfrac{1}{x^n}$.

Exercises: compute the following powers

1. 6^2
2. 3^3
3. 2^4
4. 6^{-2}
5. 3^{-3}
6. 2^{-4}

Radicals and Powers

- The n-th root of x is the number which, raised to the power n, gives back x.
- Example: $\sqrt[3]{8} = 2$.

Relationship between radicals and powers:

$$\sqrt[n]{x} = x^{\frac{1}{n}}, \qquad \sqrt[n]{x^m} = x^{\frac{m}{n}}.$$

Note on the domain:

- Even index $\Rightarrow$ (in the reals) the argument must be ≥ 0.

Useful properties of powers

$$x^n x^m = x^{n+m}, \qquad \frac{x^n}{x^m} = x^{n-m}, \qquad (x^n)^m = x^{nm}.$$

Products and quotients with the same exponent:

$$x^n y^n = (xy)^n, \qquad \frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n.$$

Exercises – Write in power / radical form

1. $\sqrt[4]{3}$
2. $\sqrt[3]{3^5}$
3. $2^{1/2}$
4. $3^{4/5}$
5. $\sqrt[6]{10^6}$
6. $2^{-1/2}$

Exercises – Simplify using the properties of powers

1. $5^3 \cdot 5$
2. $2^4 \cdot 2^{0.25}$
3. $\dfrac{6^5}{6}$
4. $\dfrac{300}{30}$
5. 6^{-1}
6. $7^{1/2}$

Linear equations in one variable

- Equalities of polynomials of degree ≤ 1.
- The degree is the highest exponent of the unknown x.

Equivalent transformations:

- Add/subtract the same number to/from both sides.
- Multiply/divide both sides by the same (non-zero) number.

Example:

$$2x + 3 = 1 \;\Rightarrow\; 2x = -2 \;\Rightarrow\; x = -1.$$

Exercises – Solve the following equations

1. $3x+5-7x=4x+1$
2. $3+80x=-3+6x$
3. $2+\dfrac{x+1}{4}=5$

Functions: definition and representation

- A function is a relation between a **domain** set and a **codomain** set.
- In one variable: $y = f(x)$.
- The domain is the set of admissible values of x; the codomain concerns the values of y.

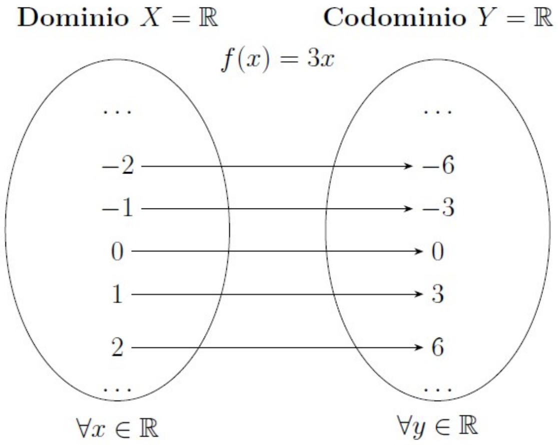
Example: $f(x) = 3x$ with $\forall x \in \mathbb{R}$.

In the Cartesian plane:

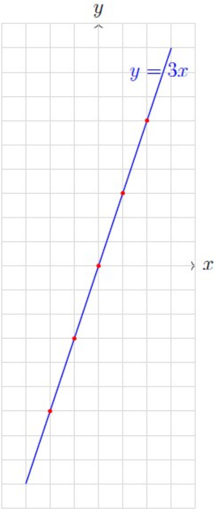
- x on the horizontal axis, y on the vertical axis.
- In microeconomics we often work in the first quadrant ($x \geq 0$, $y \geq 0$).

Figure 1: graph of $y = 3x$

Functions: Domain and Codomain



Example: graph of $y = 3x$



The straight line

A straight line has the form:

$$y = mx + q$$

where:

- m = slope (angular coefficient): positive if increasing, negative if decreasing, zero if horizontal;
- q = y-intercept (value of y when $x = 0$).

Note: vertical lines have equation $x = k$ and cannot be written in the form $y = mx + q$.

Exercises – Slope and intercept

Identify the slope and intercept; represent the lines in a Cartesian plane:

1. $5 = 3x + 2y$
2. $3y = 18 + 6x$
3. $25 = 5y + 10x$
4. $y = 5x + 10 - 2x + 3$

Line through two points

Given $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ with $x_2 \neq x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Then find q by substituting one point into $y = mx + q$ and solving for q.

Example: $P_1 = (3, 8)$, $P_2 = (4, 5)$

$$m = \frac{5-8}{4-3} = -3, \qquad 8 = -3 \cdot 3 + q \Rightarrow q = 17.$$

Therefore:

$$y = -3x + 17.$$

Exercises – Line through two points

Find the equation of the line through the given points and represent it graphically:

1. $(5,2)$ and $(3,1)$
2. $(5,2)$ and $(5,1)$
3. $(6,1)$ and $(4,1)$
4. $(8,3)$ and $(7,2)$

Intersection of two lines: linear systems

The intersection point of two lines is found by solving a system (typical method: substitution).

Example:

$$\begin{cases} 3x+2y=5 \\ x+y=7 \end{cases} \quad \Rightarrow \; y=7-x \Rightarrow \; 3x+2(7-x)=5 \Rightarrow \; x=-9, \; y=16.$$

Exercises – Intersection point of two lines

1. $y = 3x + 2 \quad \text{and} \quad 5 = 2y + 3x$
2. $25 = 3x + 2y \quad \text{and} \quad 2y = 4x + 6$
3. $20x = 100 \quad \text{and} \quad 25 = 5y + 10x$

Exponentials and logarithms: definitions

Exponential with base a:

- maps every $x \in \mathbb{R}$ to the power a^x, with $a > 0$ and $a \neq 1$.

Logarithm base a of $b > 0$:

$$\log_a(b) = x \iff a^x = b.$$

Natural logarithm: $\ln(b) = \log_e(b)$.

Inverse: e^x is the inverse of $\ln(x)$.

Exponential equations (intuition) and graphs

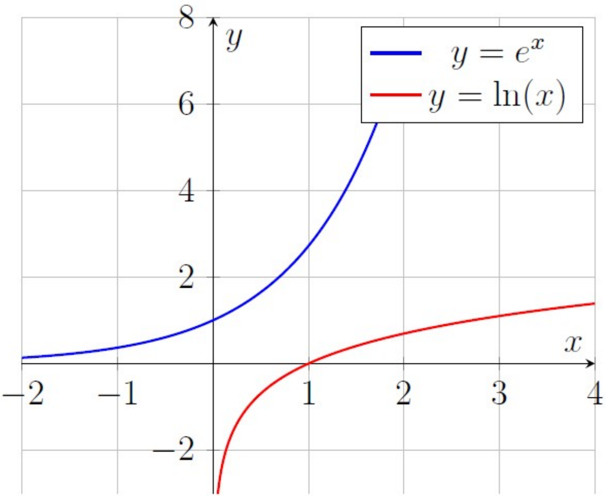
An equation of the type $a^x = b$ is solved by:

- rewriting b as a power of the same base (when possible), or
- using logarithms: $\log_a(b) = x$.

Graphical intuition:

- $\ln(x)$ is defined for $x > 0$; it has a "barrier" near 0^+.
- e^x grows rapidly for large x; it intersects the y-axis at 1 ($e^0 = 1$).

Graph of $y = e^x$ and $y = \ln(x)$

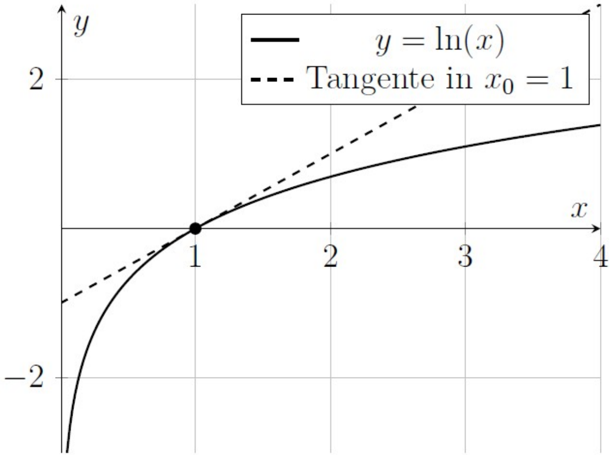


First derivative: meaning

The first derivative $f'(x)$ measures:

- the slope of the tangent line to the graph of f at a point;
- in economics: marginal variation (how much y changes when x increases by one infinitesimal unit).

Example: $y = \ln(x)$ and tangent at $x_0 = 1$ (slope $1/x_0 = 1$)



Differentiation rules 1

Function	**Derivative**
$f(x) = k$	$f'(x) = 0$
$f(x) = x$	$f'(x) = 1$
$f(x) = k\,h(x)$	$f'(x) = k\,h'(x)$
$f(x) = kx$	$f'(x) = k$
$f(x) = x^n$	$f'(x) = nx^{n-1}$
$f(x) = g(h(x))$	$f'(x) = g'(h(x))\,h'(x)$
$f(x) = \ln(x)$	$f'(x) = \dfrac{1}{x}$
$f(x) = \log_a(x)$	$f'(x) = \dfrac{1}{x\ln(a)}$

Differentiation rules 2

Function	**Derivative**
$f(x) = e^x$	$f'(x) = e^x$
$f(x) = a^x$	$f'(x) = a^x \ln(a)$
$f(x) = h(x) + g(x)$	$f'(x) = h'(x) + g'(x)$
$f(x) = h(x)g(x)$	$f'(x) = h'(x)g(x) + g'(x)h(x)$
$f(x) = \dfrac{h(x)}{g(x)}$	$f'(x) = \dfrac{h'(x)g(x) - g'(x)h(x)}{[g(x)]^2}$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$

Exercises – Differentiate the following functions

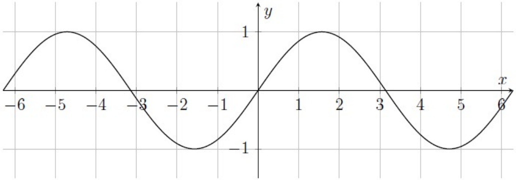
1. $f(x) = 10x^3 + 100$
2. $f(x) = x^2 + 4x + 20$
3. $f(x) = -200x - x^2 - 20x - 200$
4. $f(x) = x + 10 + x^2$
5. $f(x) = -x^2 + x^3 + 3281$

Derivative and monotonicity

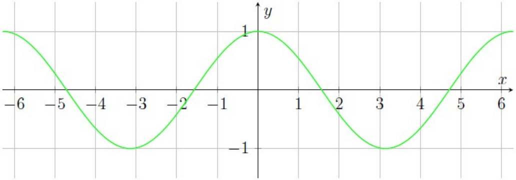
- If $f'(x) < 0$, the function is decreasing.
- If $f'(x) > 0$, the function is increasing.
- If $f'(x) = 0$, the point is stationary (horizontal tangent).

Example: $f(x) = \sin(x)$ and $f'(x) = \cos(x)$

$f(x) = \sin(x)$



$f'(x) = \cos(x)$



Second derivative and shape of the function

The second derivative $f''(x)$ is obtained by differentiating $f'(x)$.

- $f''(x) < 0$: **concave** function (curvature downward).
- $f''(x) > 0$: **convex** function (curvature upward).

Optimization:

- typical local maximum: $f'(x^*) = 0$ and $f''(x^*) < 0$;
- typical local minimum: $f'(x^*) = 0$ and $f''(x^*) > 0$.

Quadratic example: concave vs convex

Consider:

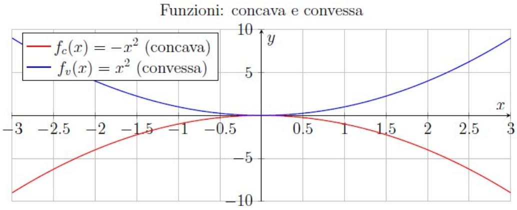
$$f_c(x) = -x^2 \text{ (concave)}, \qquad f_v(x) = x^2 \text{ (convex)}.$$

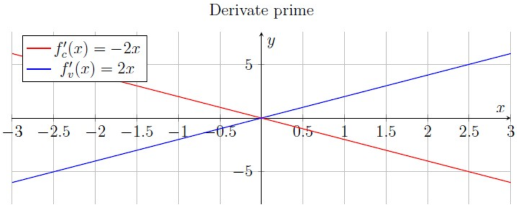
Derivatives:

$$f_c'(x) = -2x, \quad f_v'(x) = 2x; \qquad f_c''(x) = -2, \quad f_v''(x) = 2.$$

Graph: functions $-x^2$ and x^2 Graph: first derivatives $-2x$ and $2x$ Graph: second derivatives -2 and 2

Graph of Concavity and Convexity





Exercises – Maximise concave functions

1. $f(x) = 50x - x^2 + 28$
2. $f(x) = -3x^2 + 3$
3. $f(x) = -4x^2 + 5x - 45$

Exercises – Minimise convex functions

1. $f(x) = x + 10 + \frac{10000}{x}$
2. $f(x) = 4x^2 - 8x - 21$
3. $f(x) = -2x + \frac{16}{x}$

General idea: functions of two variables

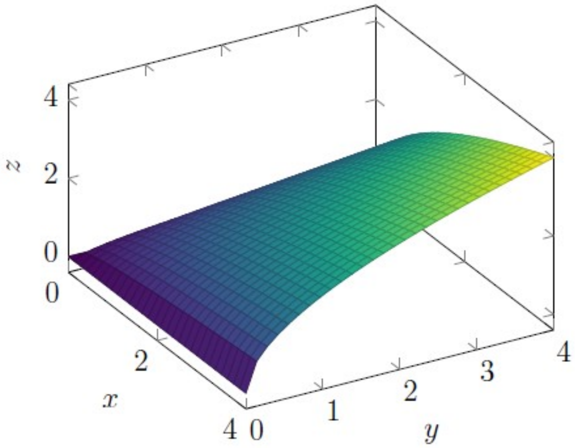
A function $Z = f(X, Y)$ assigns a value Z to every pair (X, Y).

It is represented in three-dimensional space.

Example:

$$Z = x^{\frac{1}{2}} y^{\frac{1}{2}}.$$

Example: 3D graph of $z = x^{\frac{1}{2}} y^{\frac{1}{2}}$



Level curves

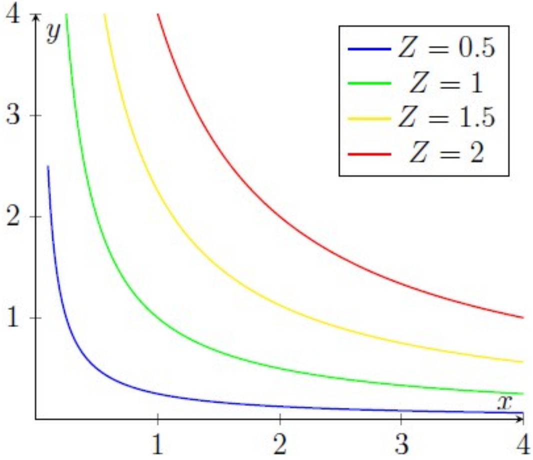
Given a value Z_0, the level curve is the set of points (X, Y) such that:

$$f(X, Y) = Z_0.$$

Example: if $Z = X^2 Y^3$ and $Z_0 = 8$:

$$X^2 Y^3 = 8 \Rightarrow Y^3 = \frac{8}{X^2} \Rightarrow Y = \left(\frac{8}{X^2}\right)^{1/3}.$$

Level curves for $Z = x^{1/2}y^{1/2}$



Partial derivatives: meaning and computation

$\dfrac{\partial f(X,Y)}{\partial X}$ measures the effect of X on Z, holding Y constant; $\dfrac{\partial f}{\partial Y}$ analogously for Y.

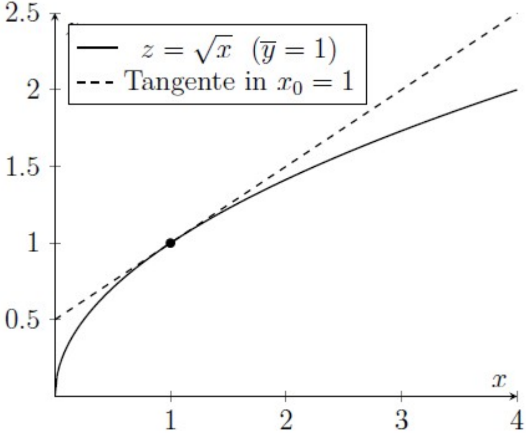
To compute them: differentiate treating the other variable as a constant.

Geometric interpretation (intuition): fix a value $y = \bar{y}$ and differentiate the cross-section $Z = f(x, \bar{y})$ with respect to x.

Example: for $Z = X^2 Y^3$

$$\frac{\partial Z}{\partial X} = 2XY^3, \qquad \frac{\partial Z}{\partial Y} = 3X^2Y^2.$$

Partial Derivatives: graphical intuition



$z = x^{\frac{1}{2}} y^{\frac{1}{2}}$. Cross-section with $y = 1$ and tangent at $x_0 = 1$

Exercises – Level curves

Find the equation of the level curve for the following functions, given Z_0:

1. $Z = x^2y^3$ with $Z_0 = 3$
2. $Z = x^2y$ with $Z_0 = 5$
3. $Z = x^2y^2$ with $Z_0 = 8$
4. $Z = x^3y^3$ with $Z_0 = 27$

Exercises – Partial derivatives

Find the partial derivatives of the functions from the previous exercise.