# BAE Math 1 Exercises

**Teacher: Christoph Lhotka**

**Teaching Assistants: Alessio Fiorentini & Isabella Valdivia**

**September 21, 2023**

## Exercises on some Math Notation, on Set Theory and on intervals

**Exercise 1.**
Establish if the following statements are True or False

- If $n \in \mathbb{N}$ is such that $n^2 - 8 \leq 0$, then $n = 0 \ \vee \ n = 1 \ \vee \ n = 2$.
- Given $x \in \mathbb{R}$, $x^2 - 1 \leq 1 \Longrightarrow 0 \leq x \leq 1$.
- If $x \in \mathbb{R}$ is such that $x^2 - 2 \leq 0$, then $x \notin \mathbb{Q}$
- $\forall x \in \mathbb{R} \ \exists n \in \mathbb{N} \ : \ \dfrac{1}{1+x^2} \leq n$.
- $\exists x \in \mathbb{R} \ : \ x^2 + 1 \geq n \ \forall n \in \mathbb{N}$.
- $\forall x \in \mathbb{R}$ and $\forall z \in \mathbb{R} \ \exists y \in \mathbb{R} \ : \ x + y = z$.

**Exercise 2.**
Write the negative of the previous statements

**Exercise 3.**
Represent the following intervals through all possible methods (inequalities, brackets or Real number line)

- All Real numbers less than $-2$ or greater or equal than $\dfrac{5}{3}$.
- All Real numbers that are greater or equal than 4 and less than 5.

**Exercise 4.**
Which of the following sets are empty

a) $\{x \,|\, x \text{ even and odd}\}$

b) $\{x \,|\, x \in \mathbb{N} \text{ and } x \text{ is a prime number less than } 2\}$

c) $\{x \,|\, x \text{ natural number multiple of 7 and 5}\}$

d) $\{x \,|\, x \in \mathbb{Z} \text{ and } x \neq 1 + x\}$

e) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 = 25\}$

f) $\{x \mid x \in \mathbb{N} \text{ and } x^2 = 25\}$

g) $\{x \mid x \in \mathbb{Z} \text{ and } x^2 = -25\}$

h) $\{x \mid x \in \mathbb{Z} \text{ and } 3x = 25\}$

**Exercise 5.**
Given the sets

$$A = \{a, e, i, o, u\} \quad B = \{b, c, d, e\} \quad C = \{f, g, h\}$$

determine, when possible, the set $X$ such that

- $X = A \cup B \cup C$
- $X \setminus A = B$
- $B \cup X = B$
- $C \cup X = B$

**Exercise 6.**
Represent on the Real number line the following sets

- $M = \{y \in \mathbb{R} \mid |y - 1| \leq 2\}$;
- $P = \{x \in \mathbb{R} \,!x^2 < 16\}$;
- $Q = \{x \in \mathbb{R} \mid 3x - 5 \geq x + 7\}$;
- $L = \{x \in \mathbb{R} \mid x^2 - 4 = (x - 2)(x + 2)\}$;
- $N = \{t \in \mathbb{R} \mid t^2 > 9\}$.

**Exercise 7.**
Establish if the following statements are True

- $-1 \in \{x \in \mathbb{R} \mid x^2 = 1\}$
- $0 \notin \{x \in \mathbb{R} \mid x^2 \neq 1\}$
- $0 \in \{x \in \mathbb{R} \mid x^2 \leq 1\}$
- $1 \in \{x \in \mathbb{R} \mid x^2 < 1\}$
- $\{x \in \mathbb{R} \mid x^2 \leq 1\} \subseteq \{x \in \mathbb{R} \mid x^2 \leq 4\}$

## Exercises on Exponential and Logarithmic equations and inequalities

**Exercise 8.**
Solve the following exponential equations and exponential inequalities

- $5^x \cdot 125 = 25$;
- $8 \cdot 16^x = 4^x \cdot 2$
- $5^x + 3 \cdot 5^x > 2 \cdot 5^x + 10$
- $2^{x+1} + 2^{x+2} = 48$
- $25^x - 5^x - 20 < 0$

**Exercise 9.**
Solve the following logarithmic equations

- $\log(x-3) + \log(x+1) = \log(4x-3)$
- $\log_3(2x-3) = 2 + \log_3(x+1)$
- $2(\log_3 x)^2 + 3\log_3 x - 2 = 0$