BAE Math 1 Exercises

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Exercise 1. Compute the following limits of sequences, if they exist:

(a) $\lim_{n\to\infty} \dfrac{\sqrt{n}+3n-n\sqrt{n}-3}{7-n}$;

(b) $\lim_{n\to\infty} \dfrac{1+n^5+n^6-n^7}{7n+2^n}$;

(c) $\lim_{n\to\infty} \dfrac{3^n+n^3}{6^n+n^6}$;

(d) $\lim_{n\to\infty} \dfrac{5e^n-3^n-2}{e^{n+1}-e^3+1}$;

(e) $\lim_{n\to\infty} \left(1+\dfrac{2}{5n}\right)^{4n}$;

(f) $\lim_{n\to\infty} \left(\dfrac{n+4}{n+3}\right)^{2n-n^2}$;

(g) $\lim_{n\to\infty} \left(\dfrac{n^2+4}{n^2-n+3}\right)^{\frac{n-n^2}{n+1}}$;

(h) $\lim_{n\to\infty} \sqrt[n]{3^n+7^n}$

(i) $\lim_{n\to\infty} \dfrac{\sin(n)-3}{n^2+3n+2}$;

(j) $\lim_{n\to\infty} \cos(\pi n)(n-2^n)$;

Exercise 2.
Determine if the following series converge and in case compute their sums:

$a)$ $\sum_{n=0}^{+\infty} \left(-\dfrac{1}{9}\right)^n$

$b)$ $\sum_{n=0}^{+\infty} \left(\dfrac{e}{2}\right)^n$

$c)$ $\sum_{n=0}^{+\infty} \left(\dfrac{e}{3}\right)^n$

$d)$ $\sum_{n=0}^{+\infty} \dfrac{1+2^n}{3^n}$

$e)$ $\sum_{n=0}^{+\infty} \dfrac{3^{2n}}{8^n}$

$f)$ $\sum_{n=1}^{+\infty} \dfrac{\pi}{3^{n+1}}$

Exercise 3. Compute the following limits of functions, if they exist:

$(a)\ \lim_{x\to+\infty} \dfrac{\cos(x)+5}{x}$

$(b)\ \lim_{x\to-\infty} (3x^5 - x^7 - 5x - 1)$

$(c)\ \lim_{x\to+\infty} (e^x + x^2 + x)$

$(d)\ \lim_{x\to-\infty} xe^x$

$(e)\ \lim_{x\to+\infty} \dfrac{x^2-2x-3}{x-3}$

$(f)\ \lim_{x\to3} \dfrac{x^2-2x-3}{x-3}$

$(g)\ \lim_{x\to3} \dfrac{1}{x-3}$

$(h)\ \lim_{x\to2} \dfrac{x^2+x+5}{x-2}$

$(i)\ \lim_{x\to0} \dfrac{\cos(x)}{x}$

$(j)\ \lim_{x\to0} \dfrac{\sin^2(3x)}{2x^3}$

$k)\ \lim_{x\to0} \dfrac{\log(1+x)(e^x-1)}{\sin^2(x)}$

$l)\ \lim_{x\to0} \dfrac{\log(1+3x^2)}{1-\cos(2x)}$

$m)\ \lim_{x\to-\infty} \dfrac{\sqrt{x^2+5}}{x}$

$n)\ \lim_{x\to0} \dfrac{\sqrt{x^2+1}}{x}$