

BAE Math 1 Exercises

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Exercise 1. Compute the following limits of functions, if they exist:

$$(a) \lim_{x \rightarrow +\infty} \frac{\cos(x) + 5}{x}$$

$$(h) \lim_{x \rightarrow 2} \frac{x^2 + x + 5}{x - 2}$$

$$(b) \lim_{x \rightarrow -\infty} (3x^5 - x^7 - 5x - 1)$$

$$(i) \lim_{x \rightarrow 0} \frac{\cos(x)}{x}$$

$$(c) \lim_{x \rightarrow +\infty} (e^x + x^2 + x)$$

$$(j) \lim_{x \rightarrow 0} \frac{\sin^2(3x)}{2x^3}$$

$$(d) \lim_{x \rightarrow -\infty} xe^x$$

$$k) \lim_{x \rightarrow 0} \frac{\log(1+x)(e^x - 1)}{\sin^2(x)}$$

$$(e) \lim_{x \rightarrow +\infty} \frac{x^2 - 2x - 3}{x - 3}$$

$$l) \lim_{x \rightarrow 0} \frac{\log(1 + 3x^2)}{1 - \cos(2x)}$$

$$(f) \lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}$$

$$m) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 5}}{x}$$

$$(g) \lim_{x \rightarrow 3} \frac{1}{x - 3}$$

$$n) \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1}}{x}$$

Exercise 2.

Describe the domain and find all the horizontal and vertical asymptotes, if any, for each of the following functions:

$$a) f(x) = \frac{x^3 + 1}{x^2}$$

$$b) f(x) = \frac{x^2 - 2x + 6}{2 - x^2}$$

$$c) f(x) = x^2 - 2e^{1/x}$$

$$d) f(x) = xe^{\frac{1}{x}}$$

$$e) f(x) = \log\left(\frac{3-x}{x-1}\right)$$

$$f) f(x) = e^{\frac{8x}{x^2-16}} - 2$$

$$g) f(x) = \sqrt{x^2 - 3x - 4} - x + 3$$

Exercise 3.

Determine if the following functions are continuous on their domains:

$$a) f(x) = \sin(\sqrt{x^2 + 1}) + \log(x^2 + x + 1)$$

$$b) f(x) = \sqrt{\sin(\log(|x| + 1))} + 2$$

$$c) f(x) = \begin{cases} \frac{1 - \sqrt{1 + \sin(x)}}{x} & \text{if } x < 0 \\ \frac{\sin(x) - 1}{x + 2} & \text{if } x \geq 0 \end{cases}$$

$$d) f(x) = \begin{cases} \frac{\sqrt{x^4 + x^2}}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases},$$

$$e) f(x) = \begin{cases} \frac{\sin(x)}{x} + \pi & \text{if } x < 0 \\ \pi & \text{if } x = 0 \\ \frac{(e^x - 1)(1 - \cos^2(x))}{x^3} + \pi & \text{if } x > 0 \end{cases}$$

Exercise 4.

Determine for what values of the parameter $a \in \mathbb{R}$ the following functions are continuous on \mathbb{R} :

$$a) f(x) = \begin{cases} ae^x + a^2 + 1 & \text{if } x \leq 0 \\ \frac{\log(x+1)}{x} & \text{if } x > 0 \end{cases}, \quad b) f(x) = \begin{cases} x^2 + ae^x \cos(x) + 3 & \text{if } x \leq 0 \\ \frac{3 \sin(ax) + e^x - 1}{x} & \text{if } x > 0 \end{cases}$$

$$c) f(x) = \begin{cases} e^{x+a^2} & \text{if } x < 0 \\ e^{4-x^2} & \text{if } 0 \leq x \leq 2 \\ \frac{\log(x-1)+a}{x-1} & \text{if } x > 2 \end{cases},$$