$f: D \subseteq \mathbb{R} \rightarrow \mathbb{R}$

1) INIETTIVA: $\forall x_1, x_2 \in D: x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$f: D \subseteq \mathbb{R} \rightarrow C \subseteq \mathbb{R}$

2) SURIETTIVA: $\forall y \in C \; \exists x \in D: \; y = f(x)$
SURGETTIVA

$f: D \subseteq \mathbb{R} \rightarrow C \subseteq \mathbb{R}$ $\quad C \subseteq D'$

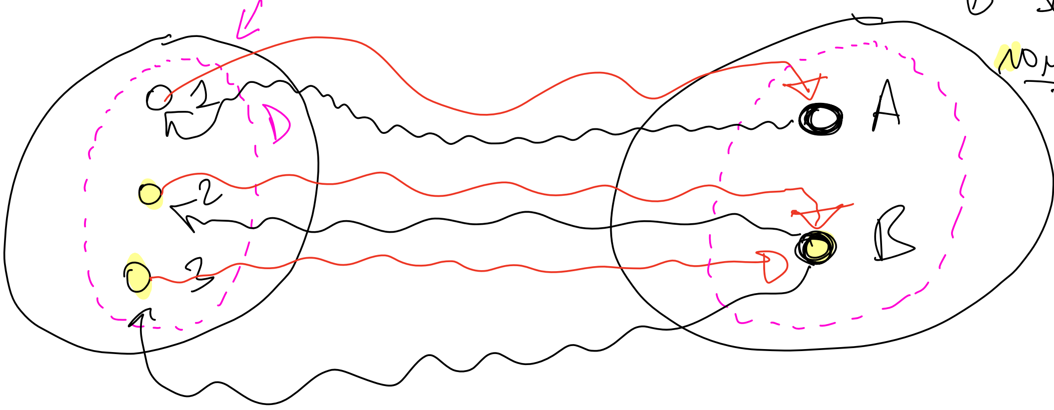
$g: D' \subseteq \mathbb{R} \rightarrow \mathbb{R}$

3) $(g \circ f)(x) = g(f(x)) \quad \forall x \in D$

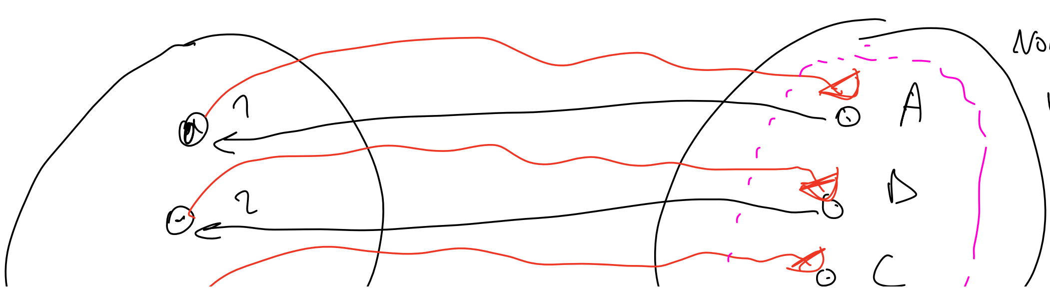
$(g \circ f)(x) = (f \circ g)(x) = x \quad \forall x \in D$

g SI CHIAMA FUNZIONE INVERSA DI f.

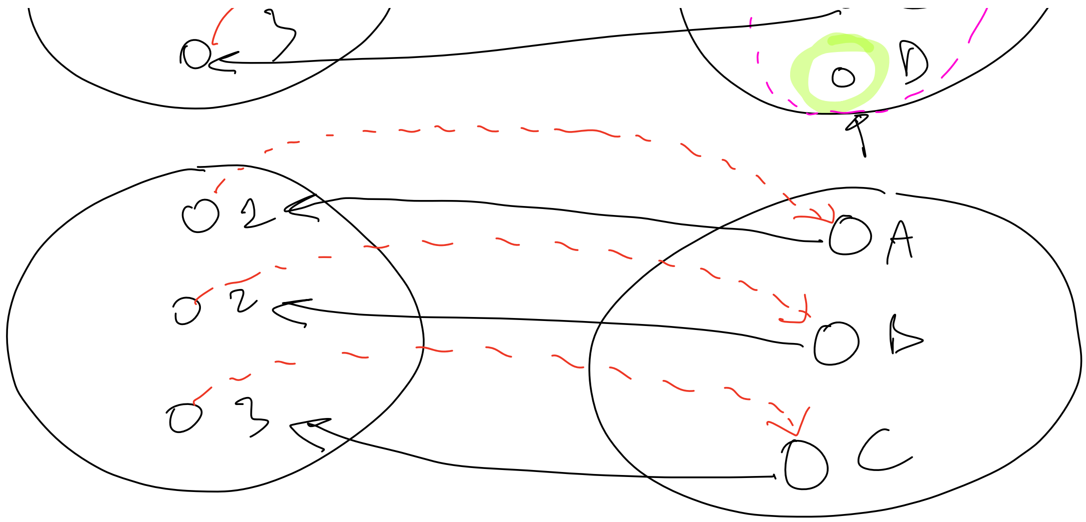
TEO: UN FUNZIONE f È INVERTIBILE SE E SOLTANTO SE f È SIA INIETTIVA SIA SURGETTIVA



SURGETTIVA
NON INIETTIVA



NON SURGETTIVA
INIETTIVA



TEO: SIA $y>0$, $y\in\mathbb{R}$. SIA $m\in\mathbb{N}=\{0,1,2,3,\dots\}$, $m>0$

ALLORA $\exists!\, x\in\mathbb{R}$ TALE CHE

$$\boxed{y = x^m}$$

TALE x SI CHIAMA RADICE m-ESIMA DI y

$$\boxed{x \equiv y^{\frac{1}{m}}} = \sqrt[m]{y}$$

$A=\{x\in\mathbb{R} \mid x^m \le y\}$ SUP(A)

DEF: $x>0$, $m\in\mathbb{N}$, $m>0$

$$x^m \equiv \underbrace{x\cdots x}_{m\text{-VOLTE}}$$

$m\in\mathbb{N}$, $m>0$

$$\rightsquigarrow x^{\left(\frac{m}{m}\right)} \equiv \left(x^{\frac{1}{m}}\right)^m = \left(x^m\right)^{\frac{1}{m}}$$

SE $q \in \mathbb{Q}$, $q<0$ $\quad x^q = \frac{1}{x^{-q}}$

IN CONCLUSIONE $\forall x>0$ E $\forall q \in \mathbb{Q}$ È BEN DEFINITO x^q

$$\boxed{x^0 = x^{q-q} = x^q \cdot x^{-q} = x^q \frac{1}{x^q} = 1}$$

$x^m \qquad x^{\frac{1}{n}} \qquad (x^n)^{\frac{1}{n}} = x^{\frac{n}{n}} = x^1 = x$

$$2^{\frac{2}{3}} = \left(2^2\right)^{\frac{1}{3}} = \left(2^{\frac{1}{3}}\right)^2 \in \mathbb{R}$$

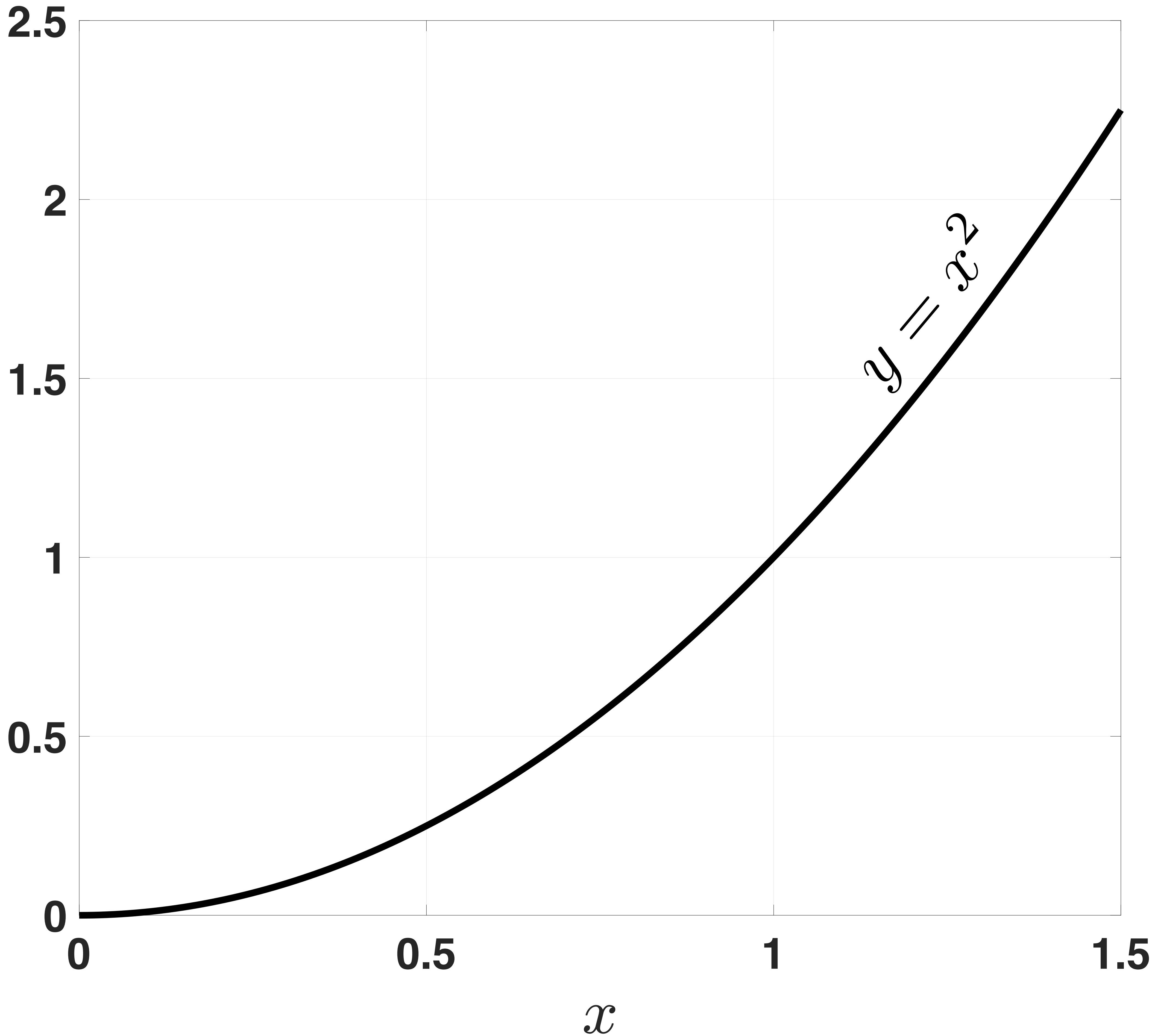
x^q È BEN POSTO $\forall x>0$ E $\forall q \in \mathbb{Q}$

$2^{\sqrt{2}} = ? \qquad 3^{\sqrt{2}}$

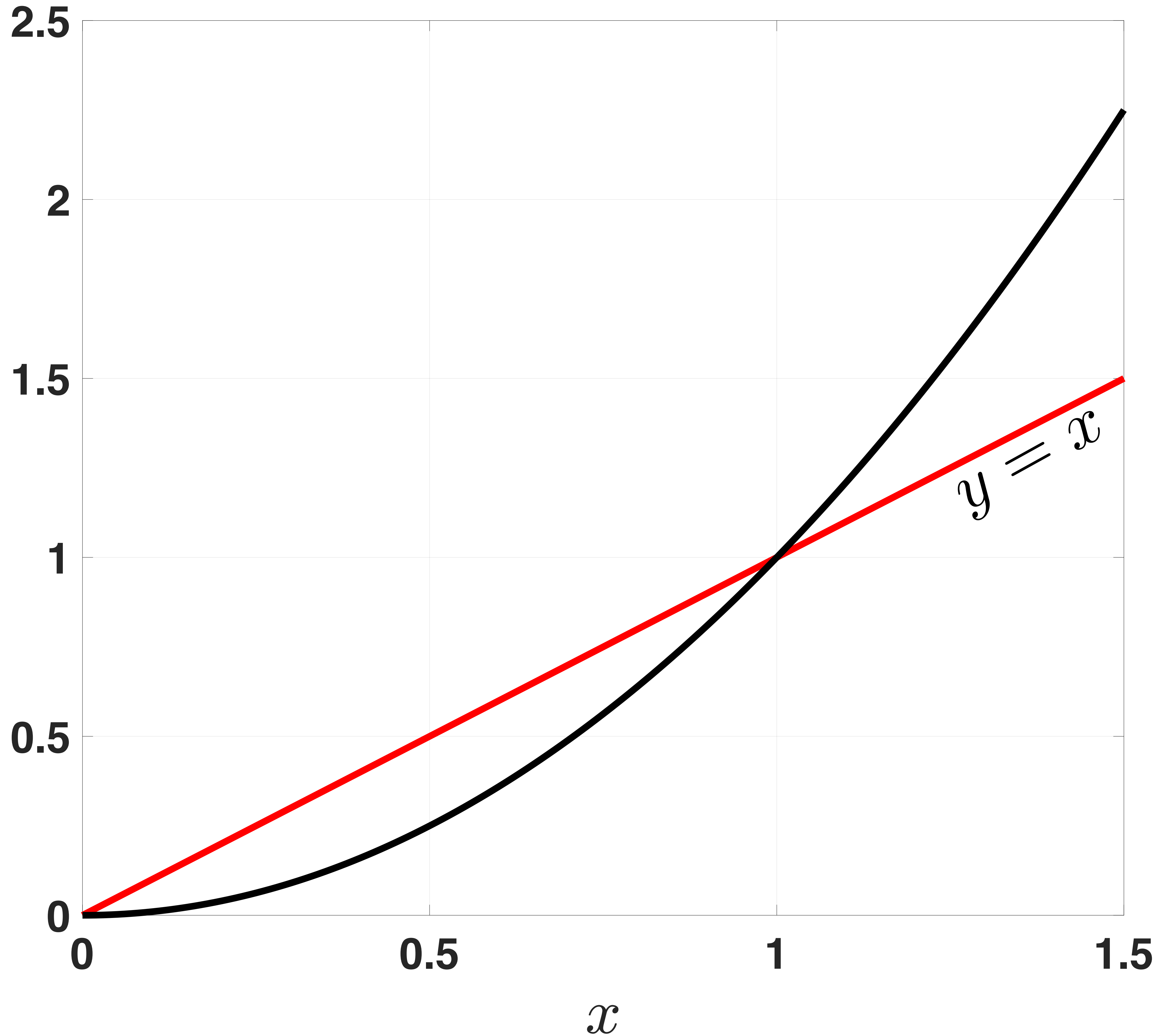
$\sqrt{2} \neq \frac{n}{m} \qquad \left(2^{2^{\frac{1}{2}}}\right)$

$\downarrow$

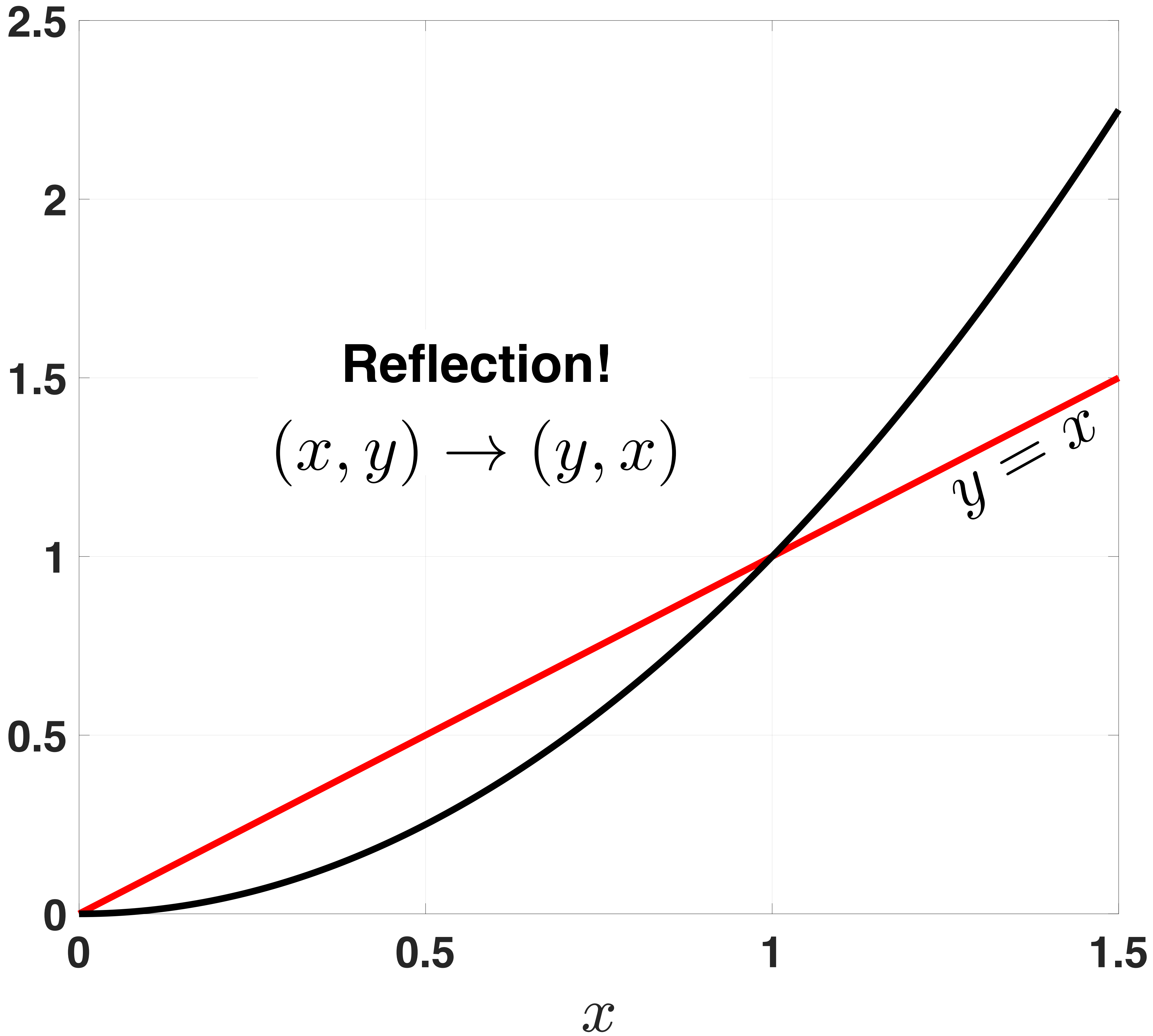
$\sqrt{2} = 1{,}4142135623 \ldots$ ← INFINITE →

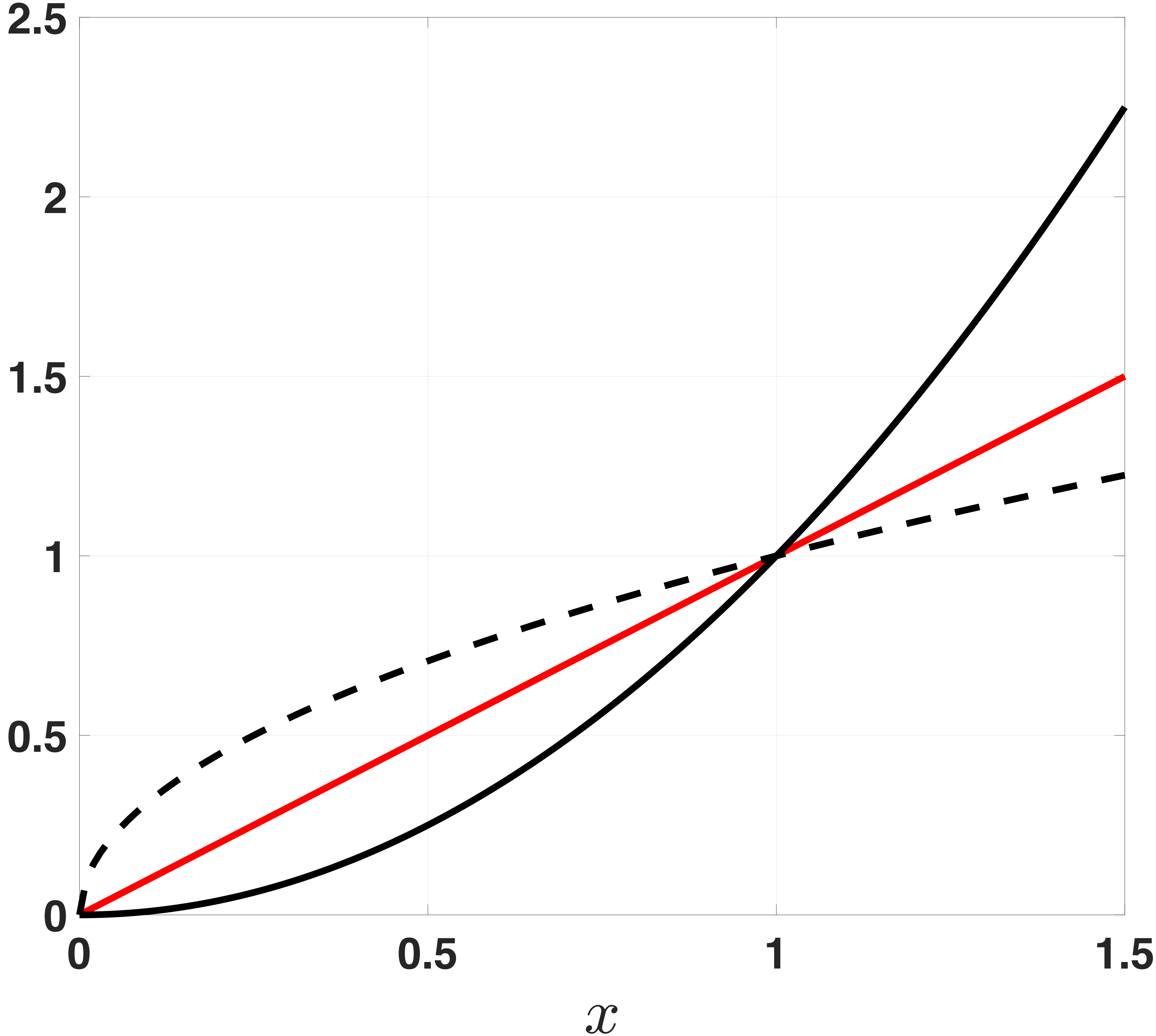


2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
x
$y = x^2$

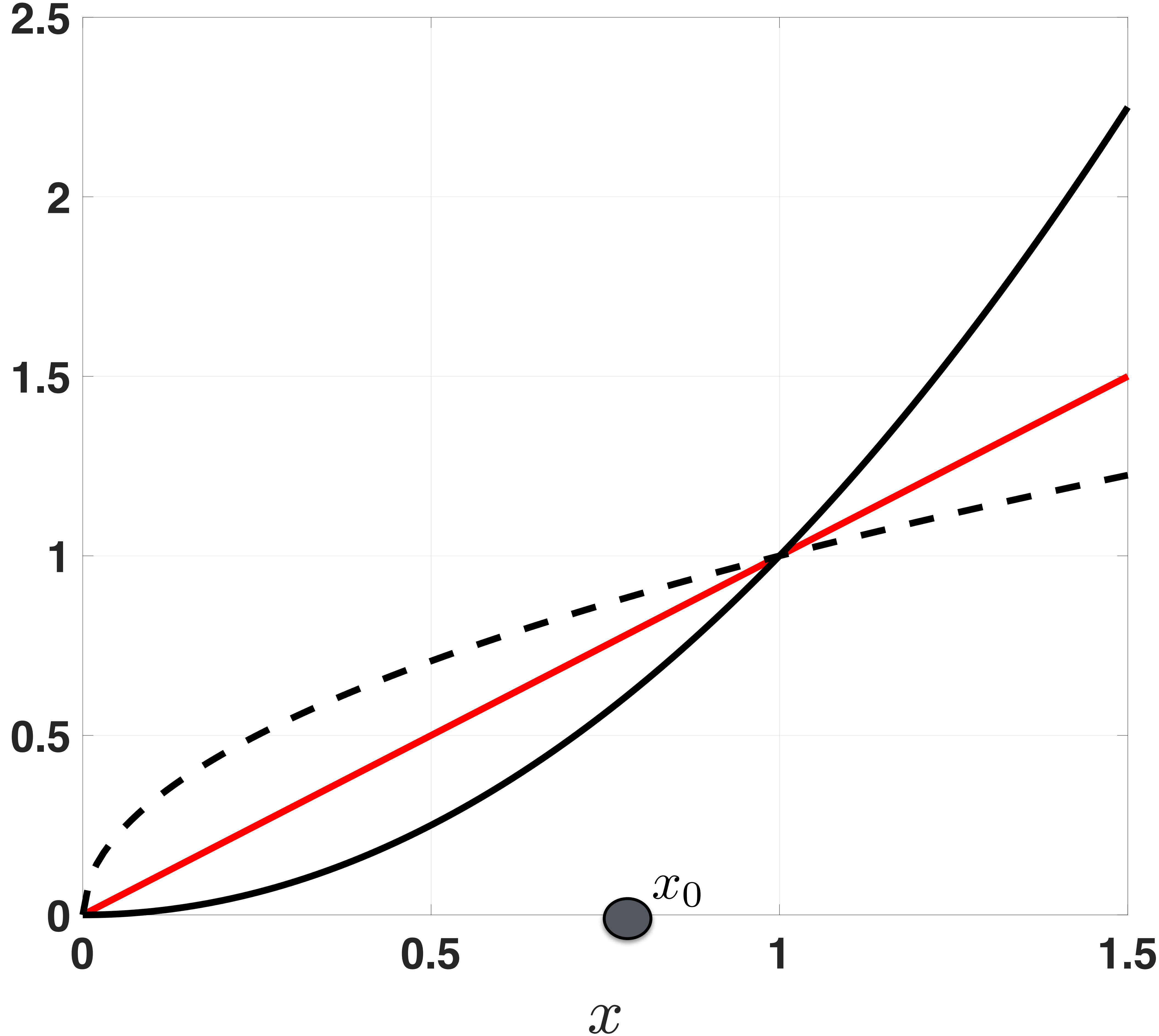


2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
x
$y = x$

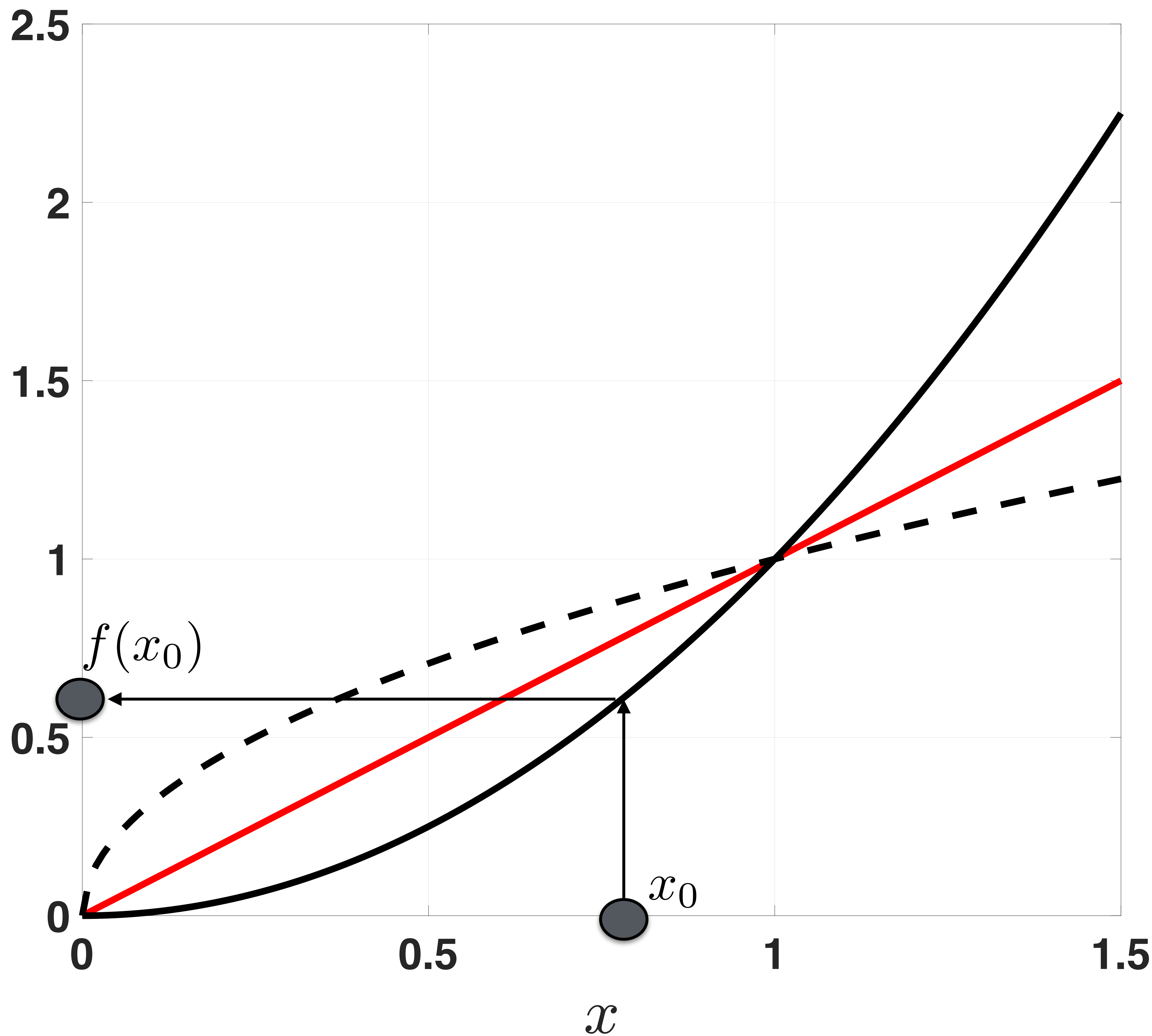

Reflection!
$(x, y) \to (y, x)$
$y = x$
x
0
0.5
1
1.5
2
2.5



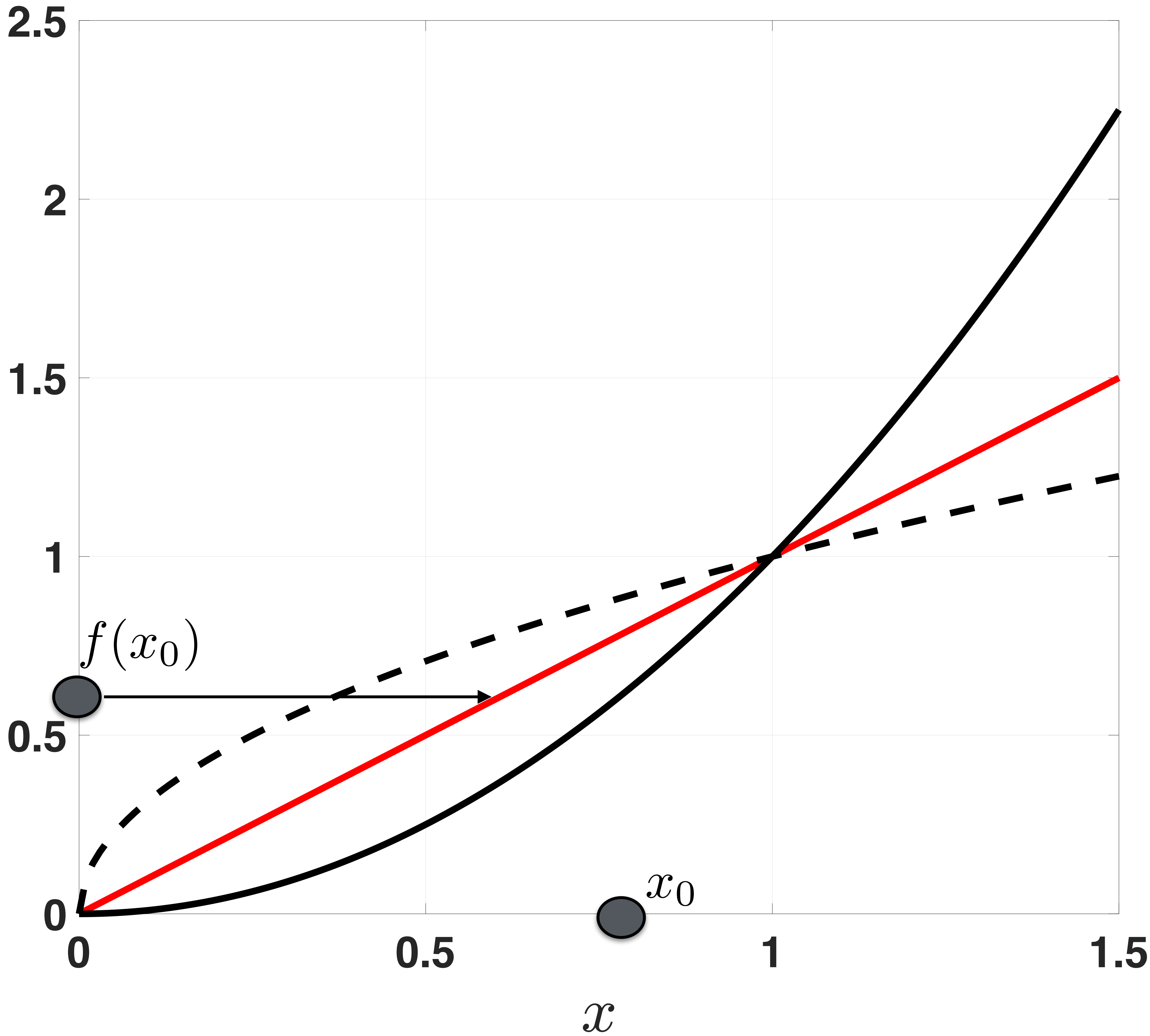
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
x



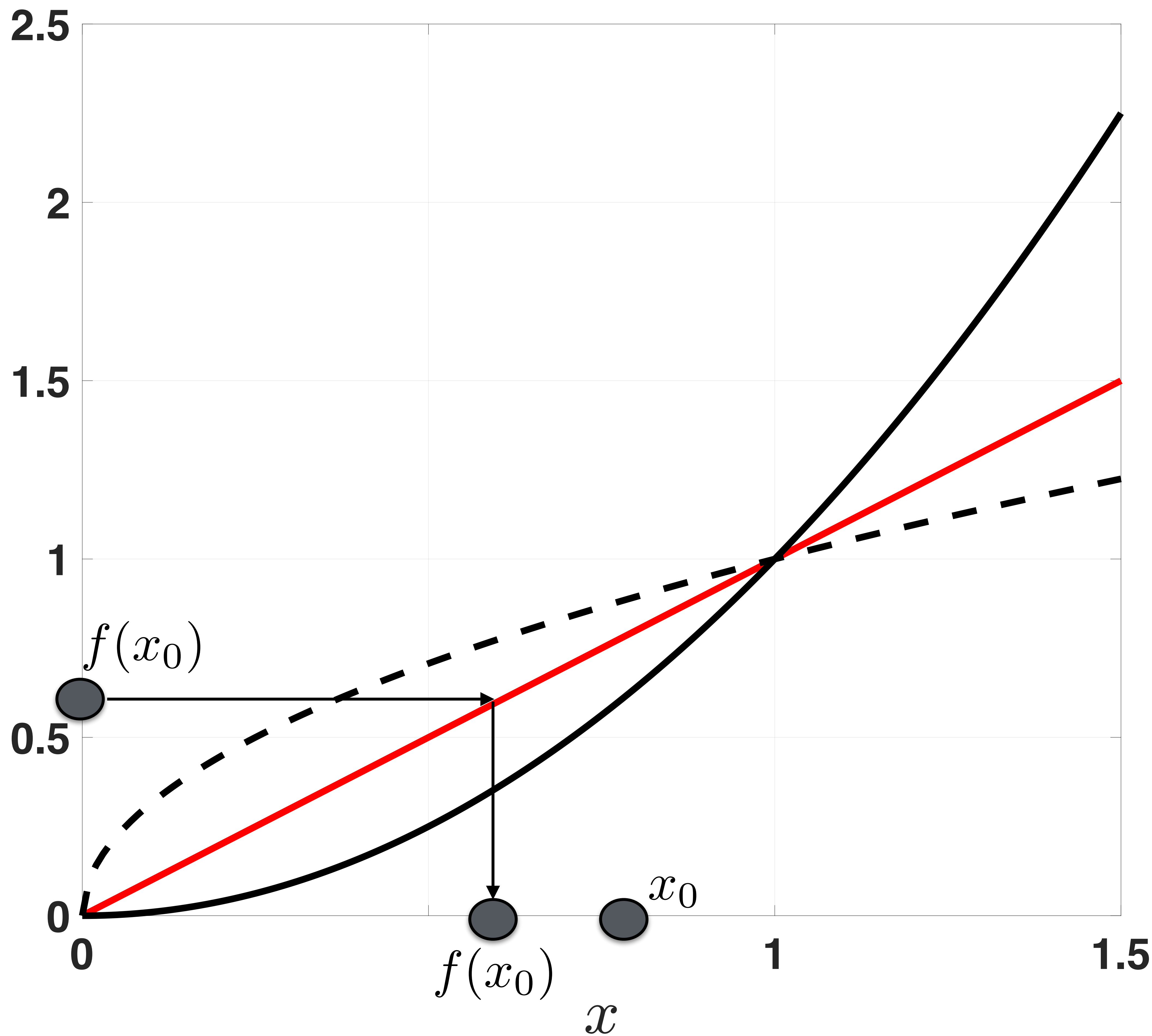
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
x_0
x

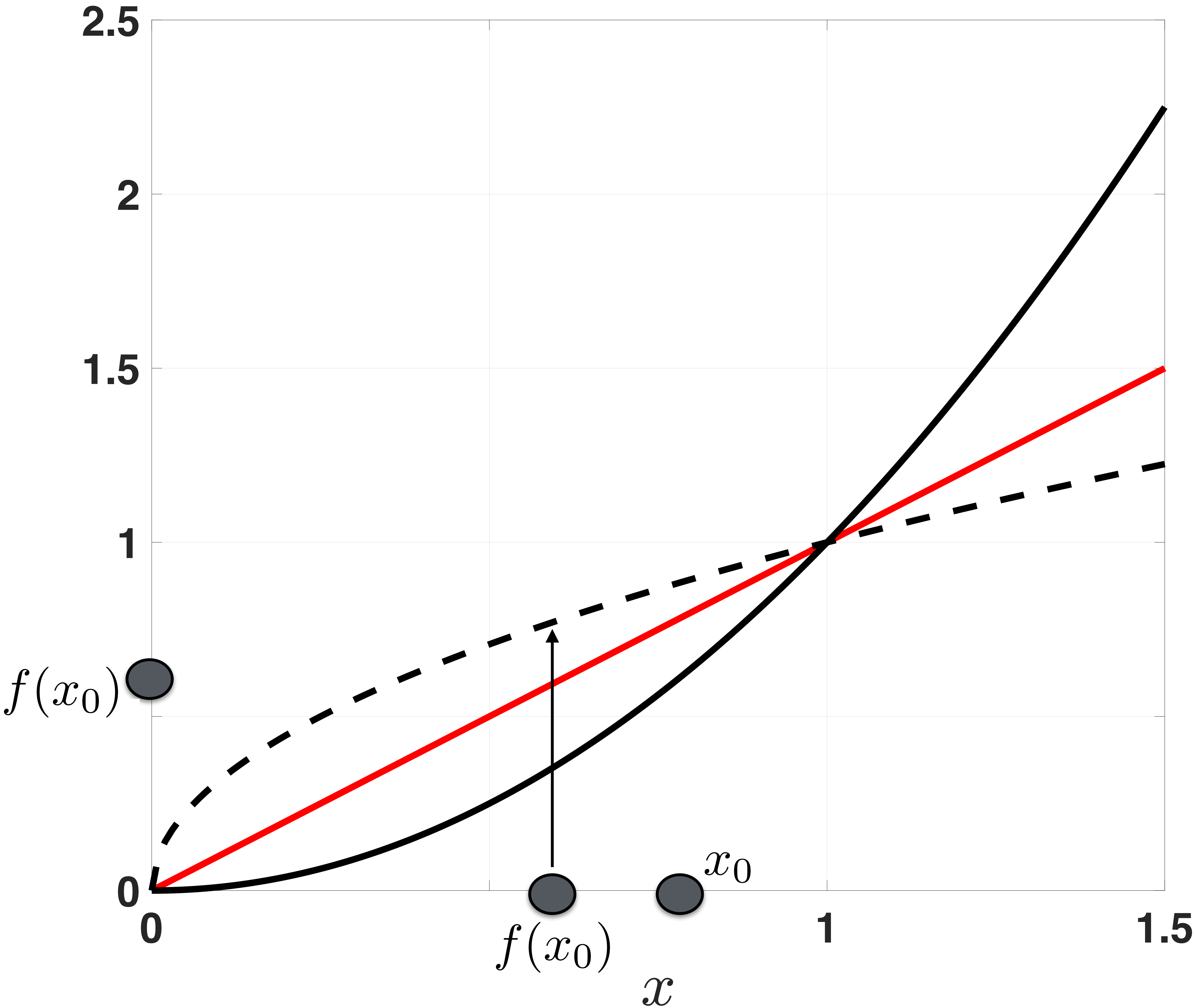


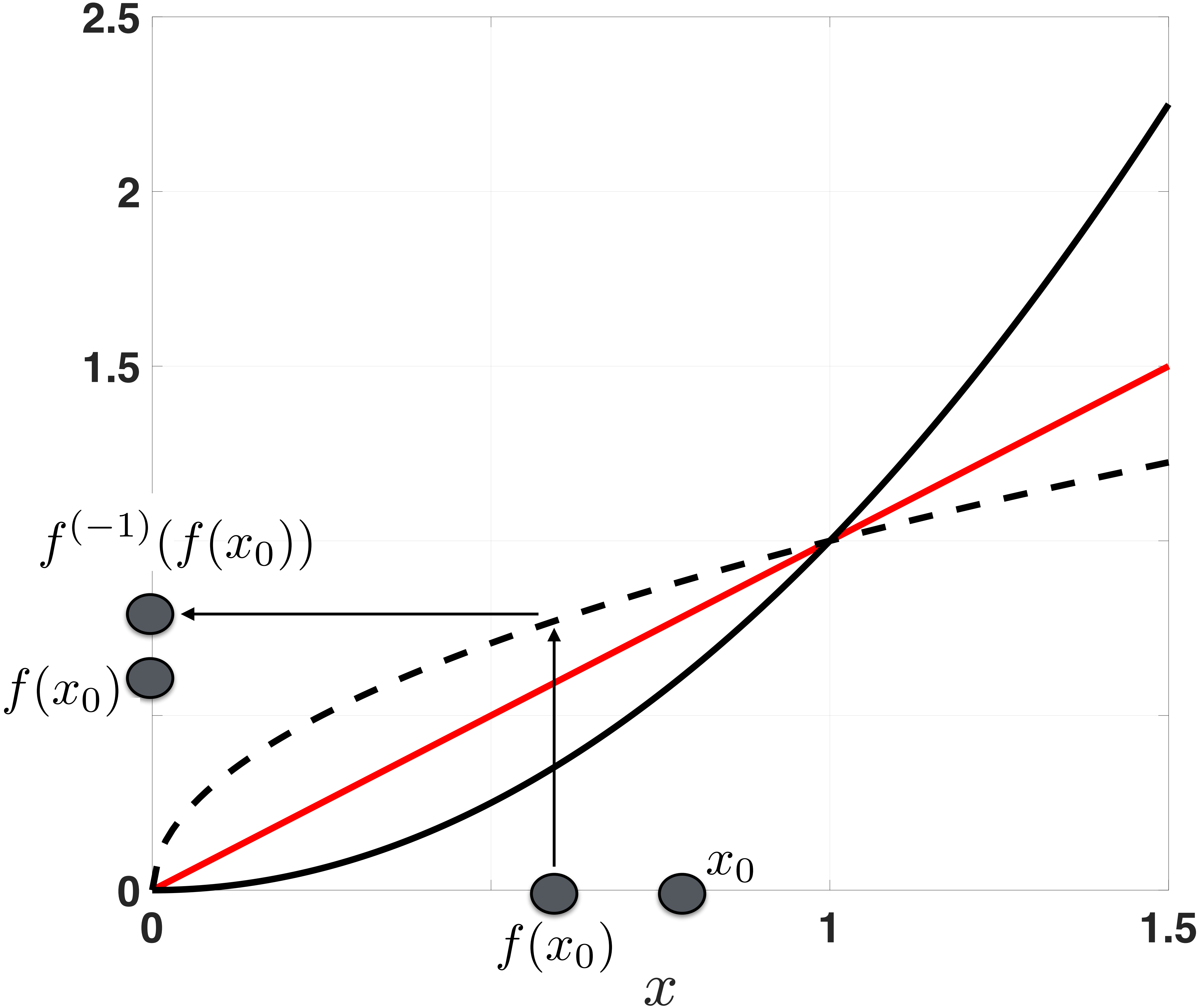
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
x
$f(x_0)$
x_0



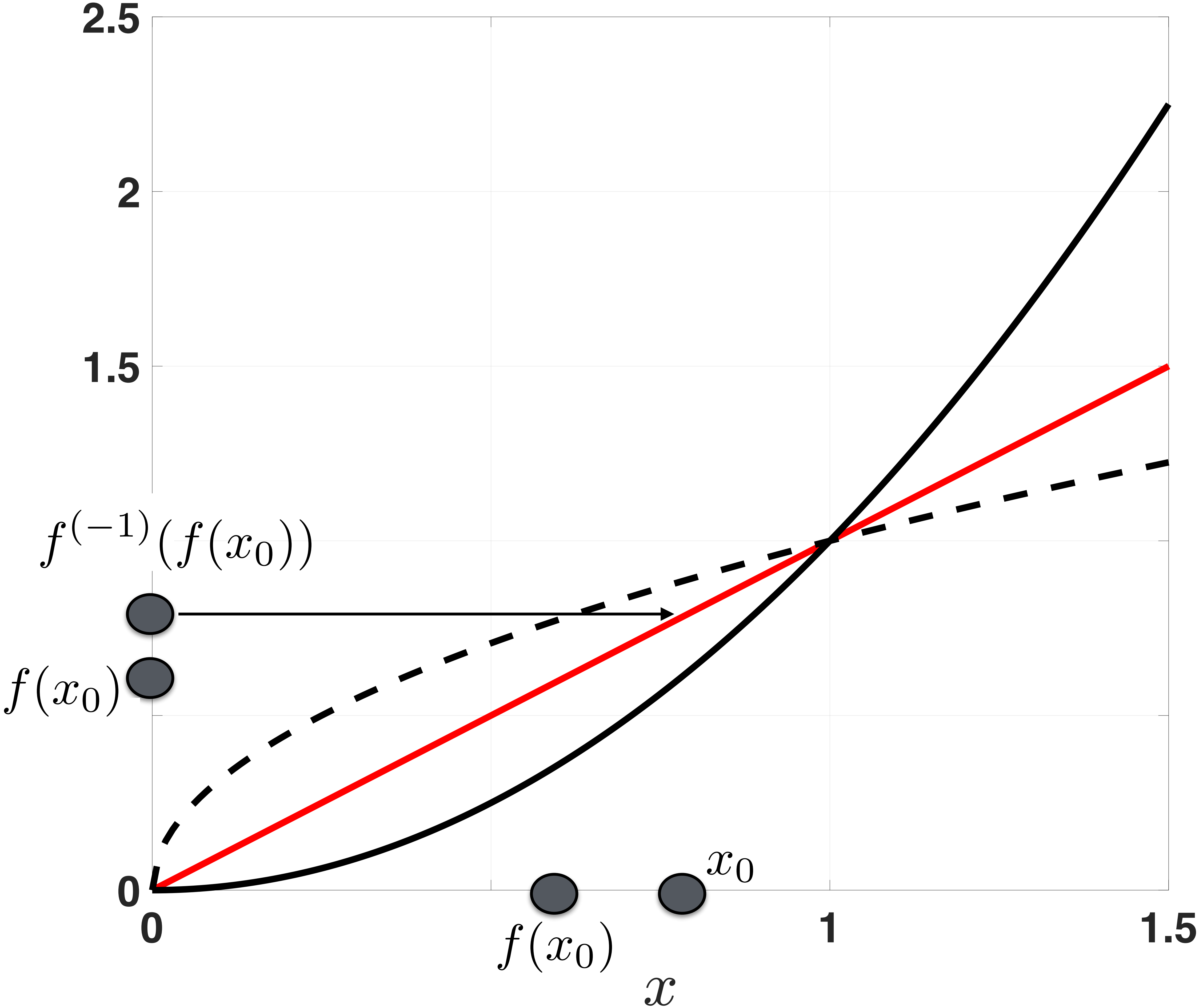
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
x
$f(x_0)$
x_0


2.5
2
1.5
1
0.5
0
0
1
1.5
$f(x_0)$
$f(x_0)$
x_0
x

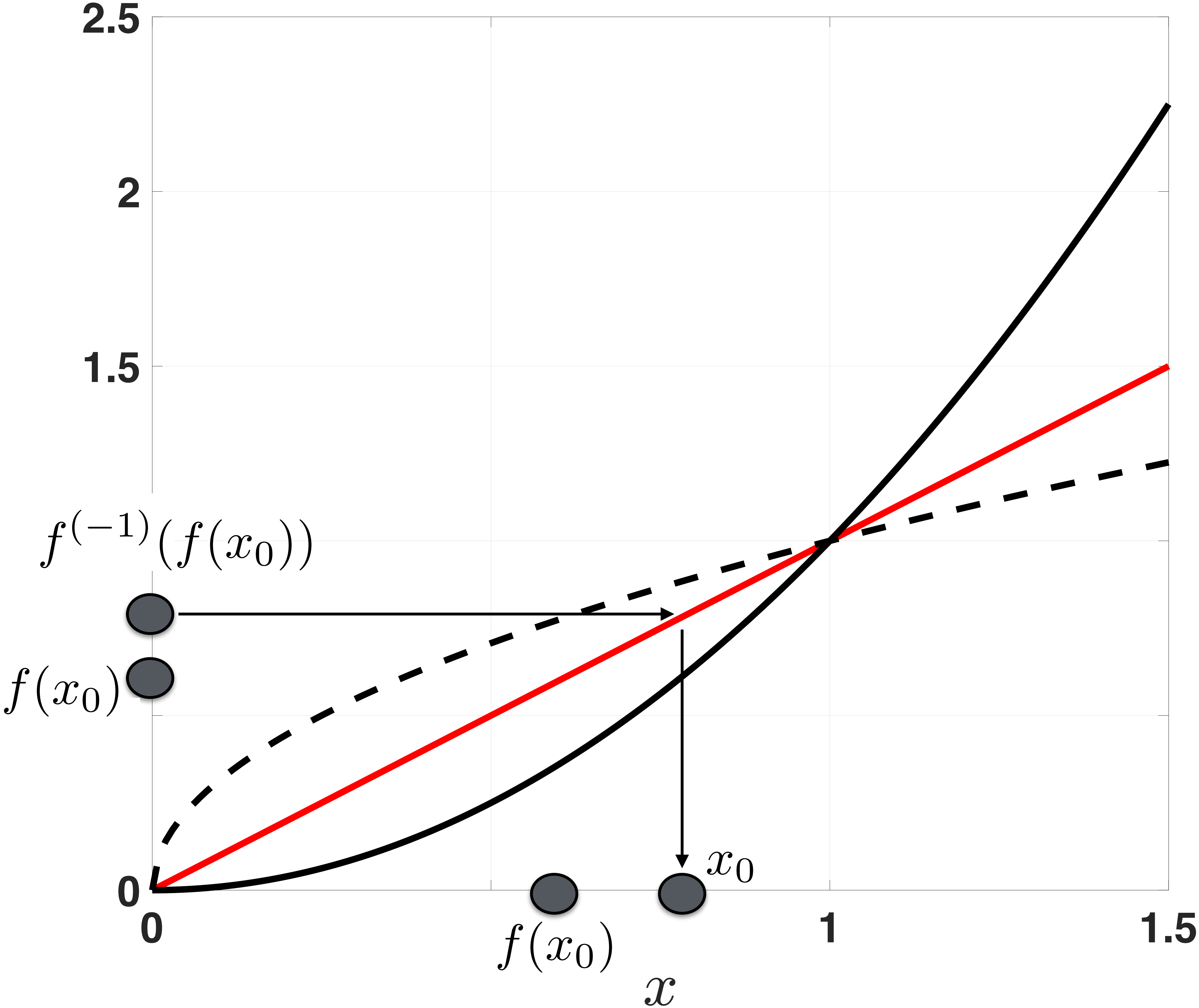

2.5
2
1.5
1
0
0
1
1.5
$f(x_0)$
x_0
$f(x_0)$
x



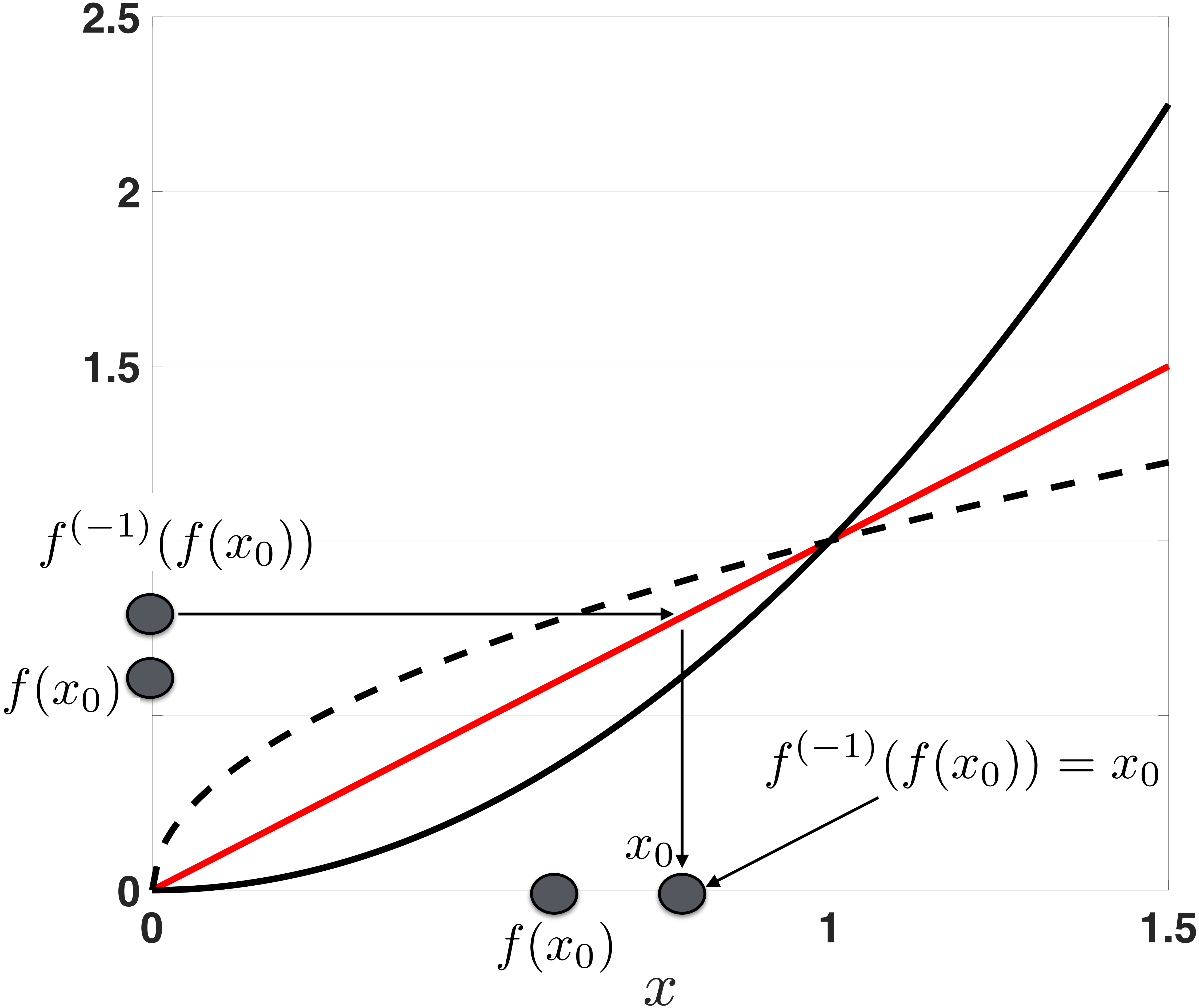
2.5
2
1.5
0
0
1
1.5
$f^{(-1)}(f(x_0))$
$f(x_0)$
x_0
$f(x_0)$
x

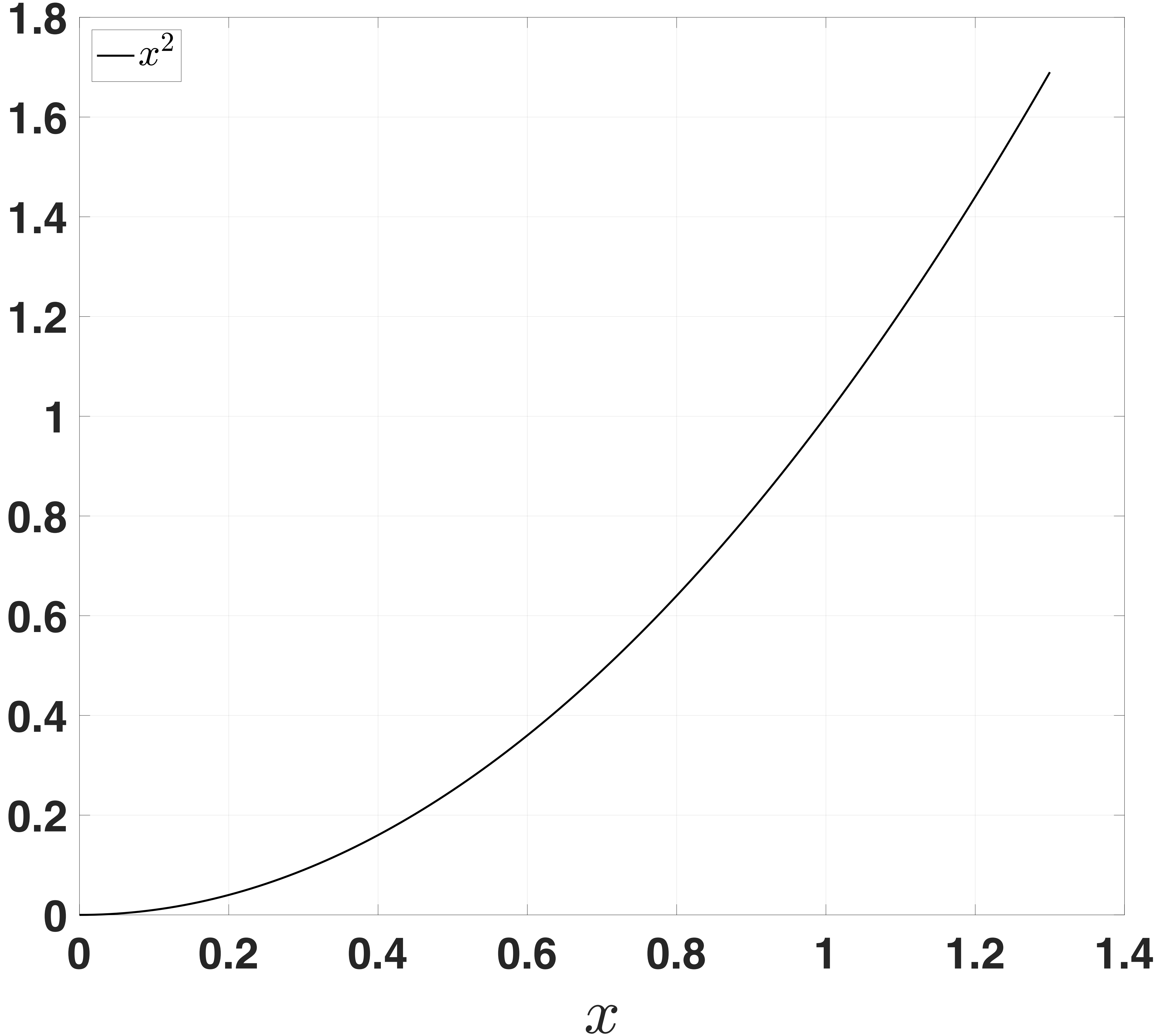


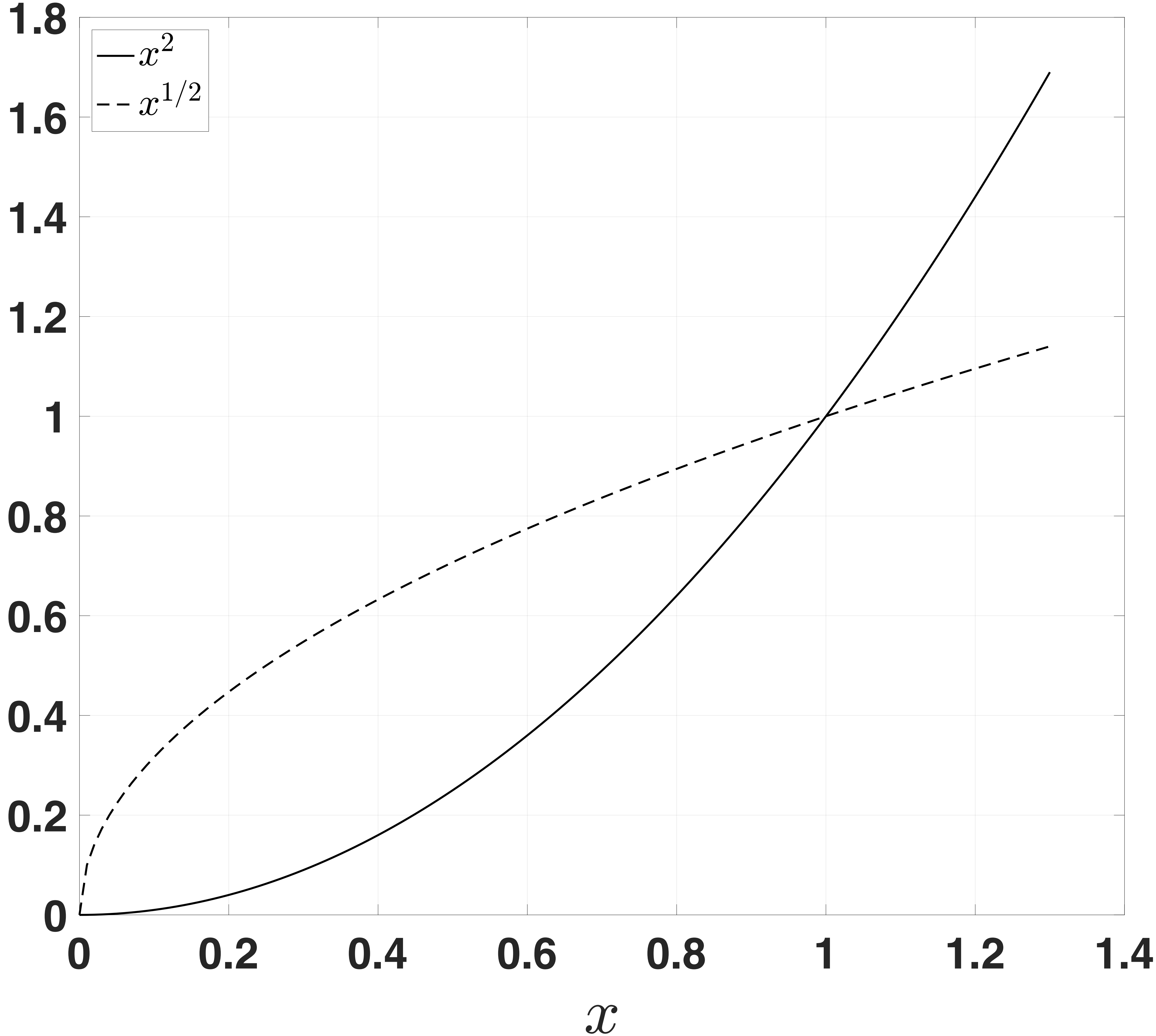
2.5
2
1.5
0
$f^{(-1)}(f(x_0))$
$f(x_0)$
x_0
$f(x_0)$
0
1
1.5
x

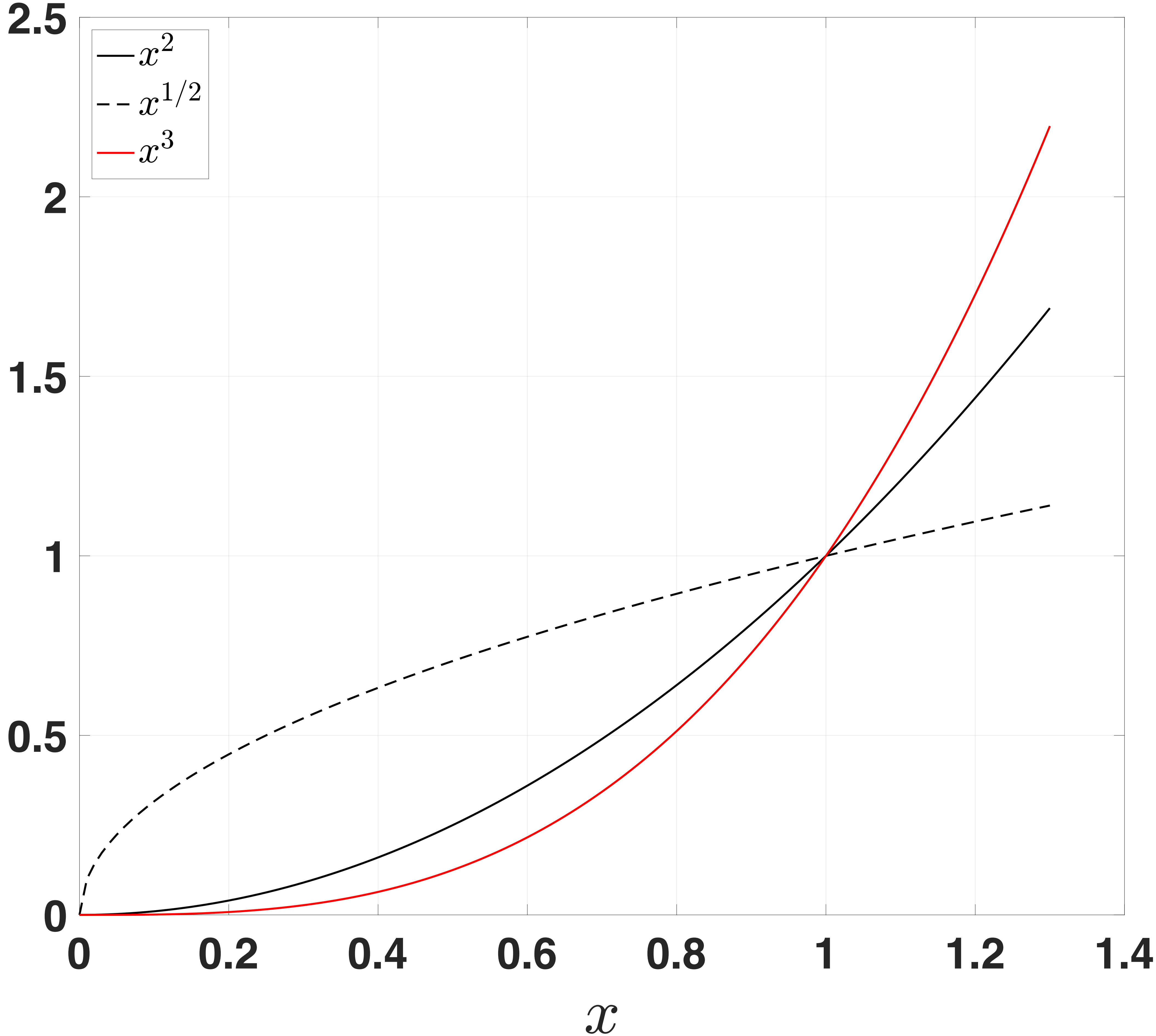


2.5
2
1.5
0
0
1
1.5
$f^{(-1)}(f(x_0))$
$f(x_0)$
x_0
$f(x_0)$
x

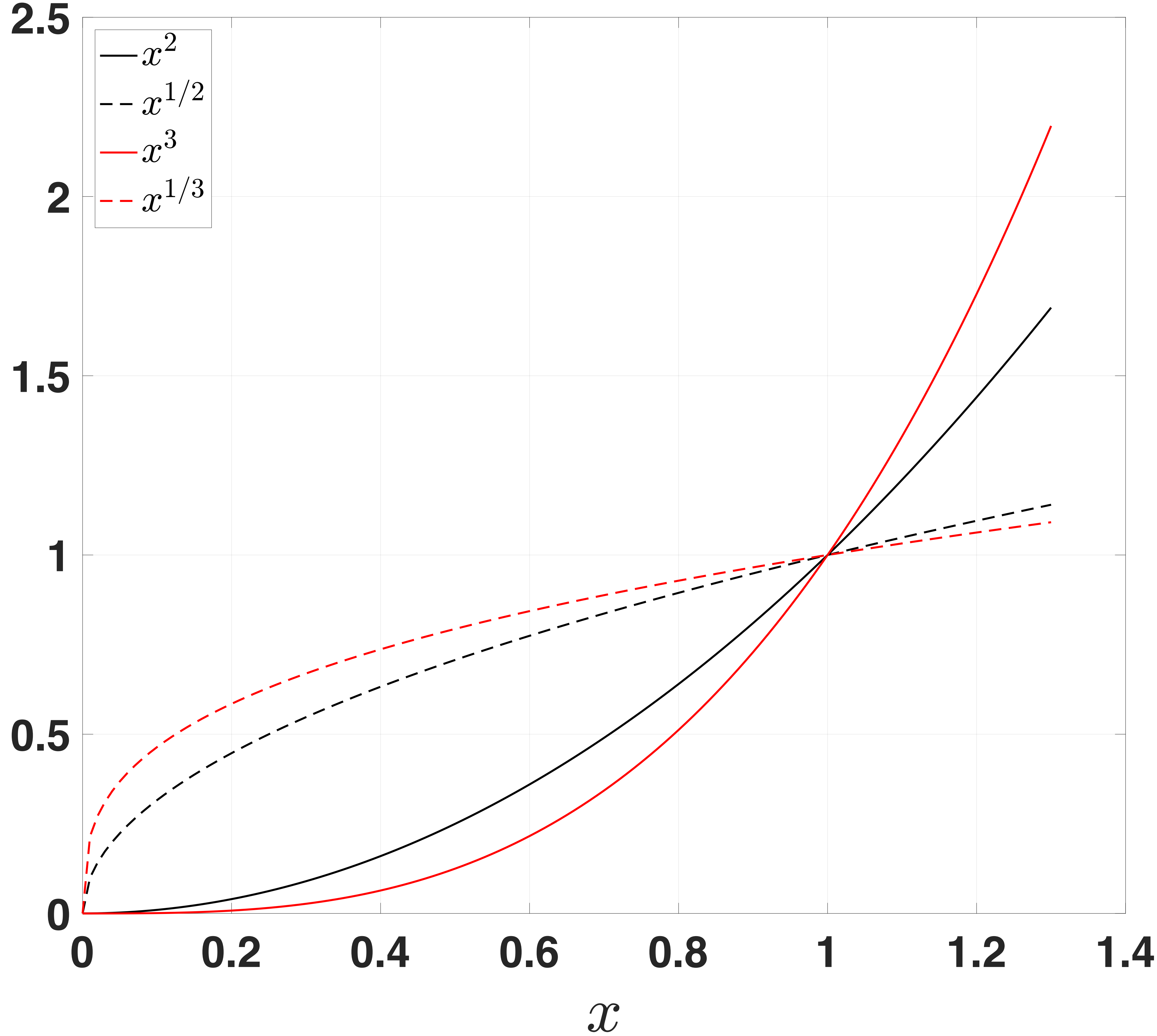

2.5
2
1.5
0
0
1
1.5
$f^{(-1)}(f(x_0))$
$f(x_0)$
$f^{(-1)}(f(x_0)) = x_0$
x_0
$f(x_0)$
x


x^2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x

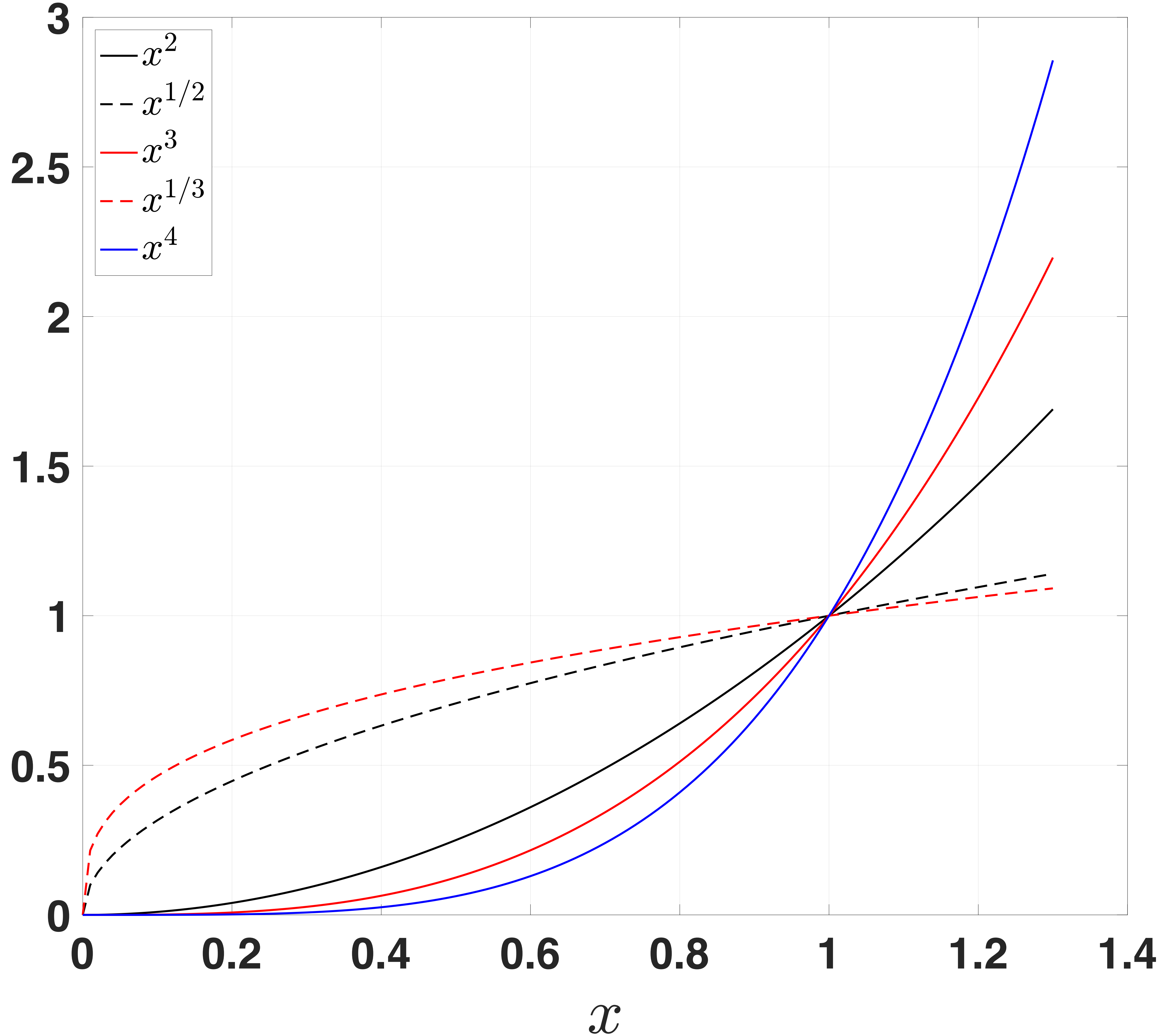

1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x
x^2
$x^{1/2}$



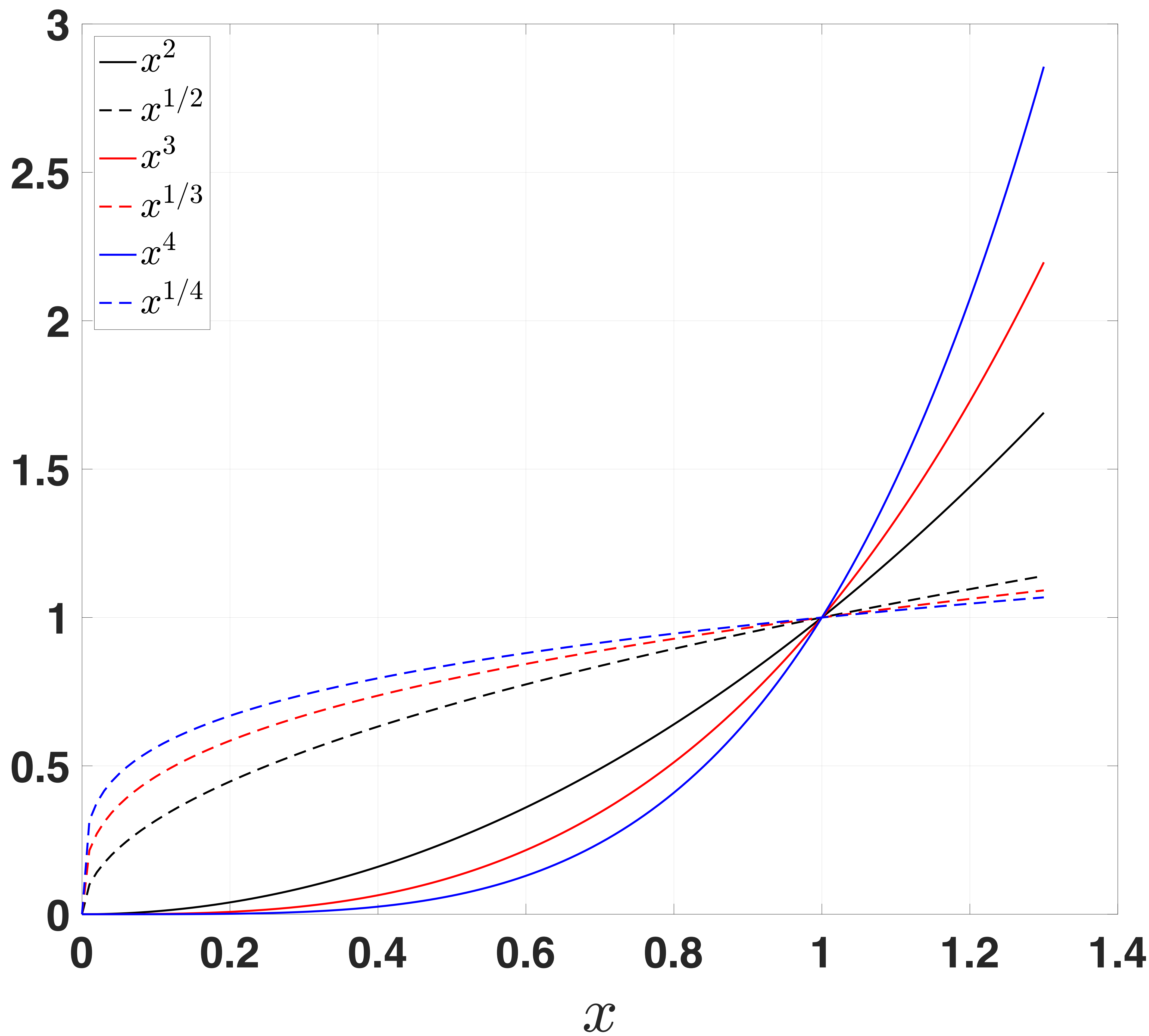
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x
x^2
$x^{1/2}$
x^3



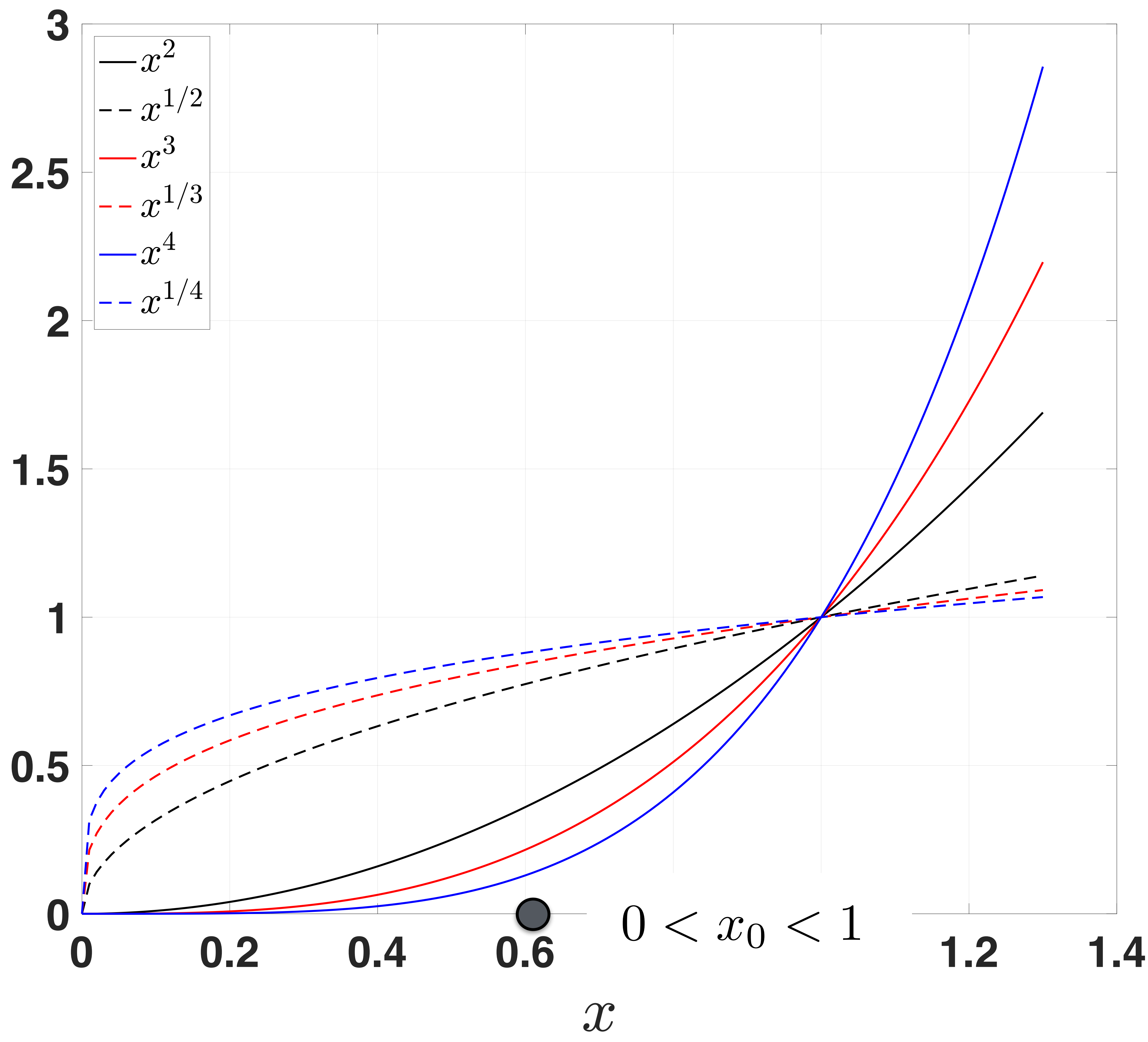
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x
x^2
$x^{1/2}$
x^3
$x^{1/3}$



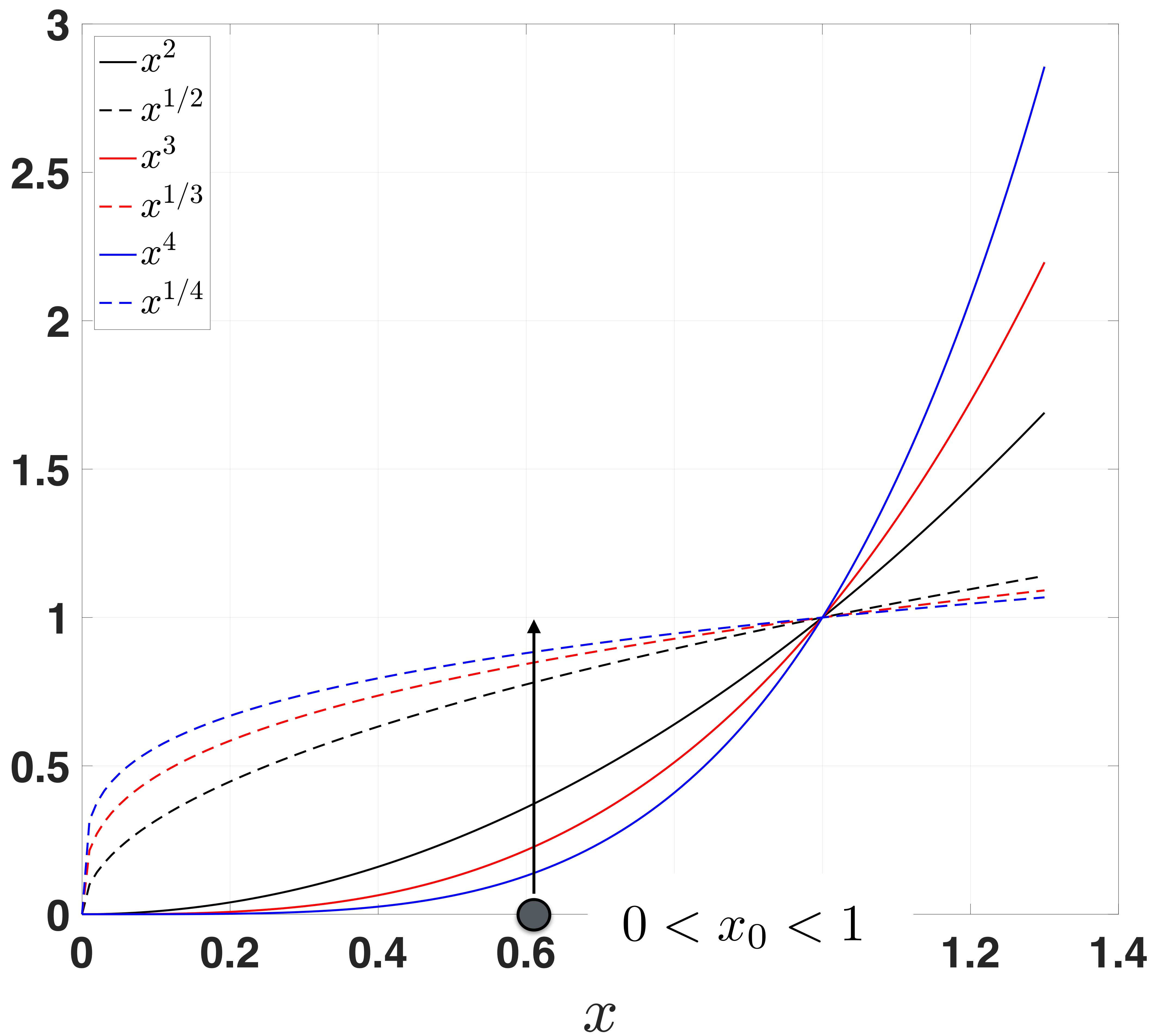
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x
x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4



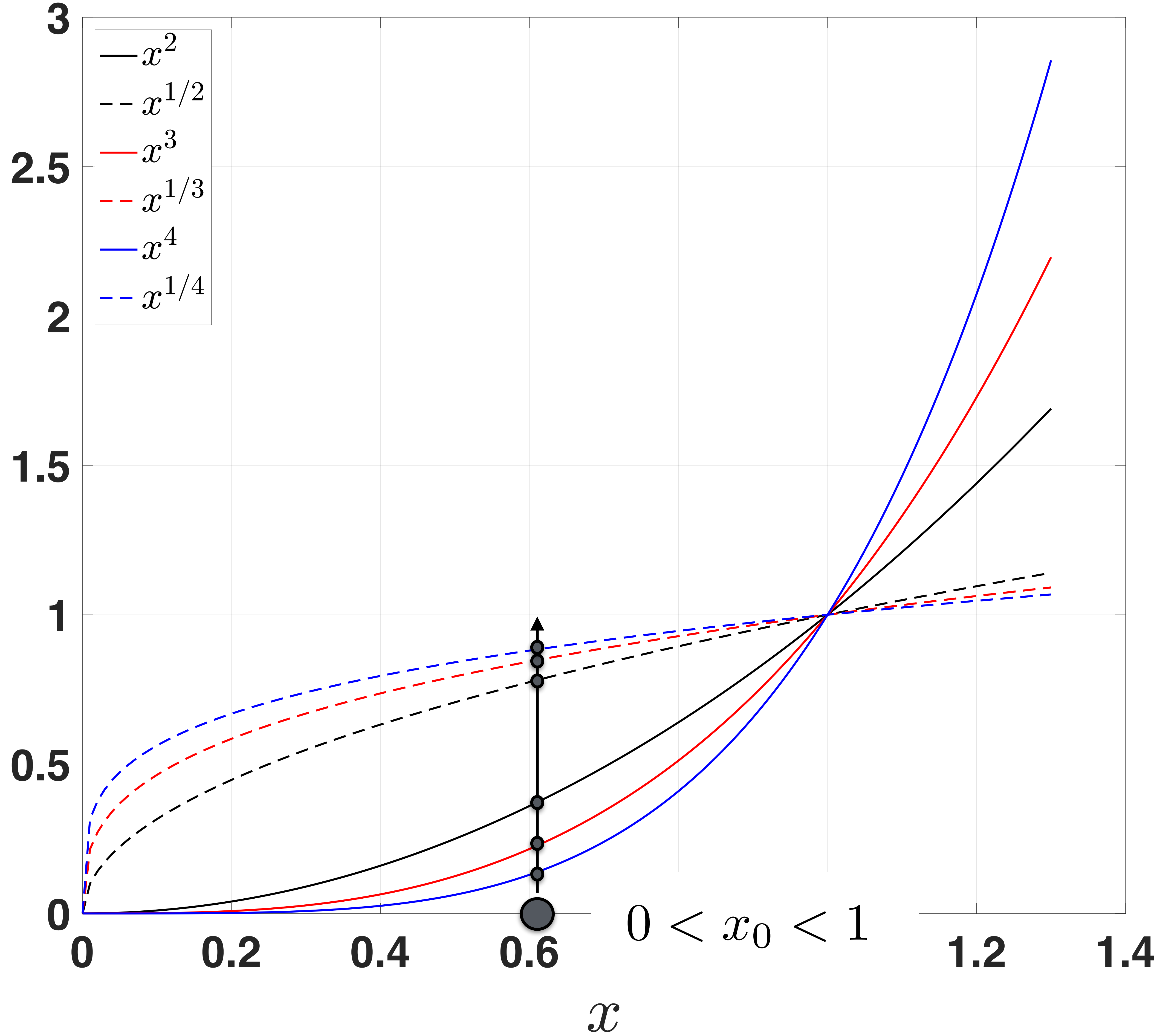
x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4
$x^{1/4}$
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x



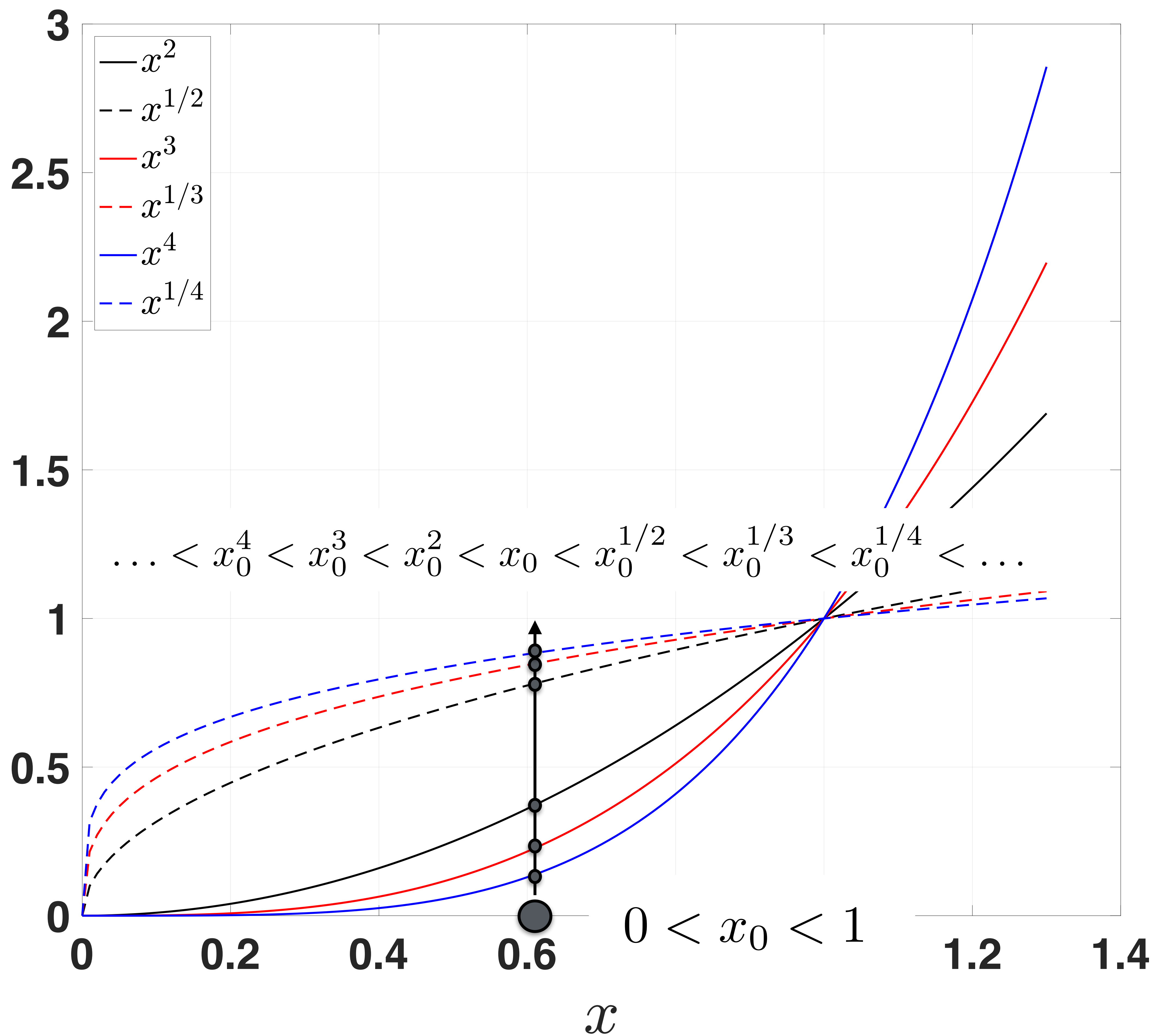
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
1.2
1.4
x
x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4
$x^{1/4}$
$0 < x_0 < 1$



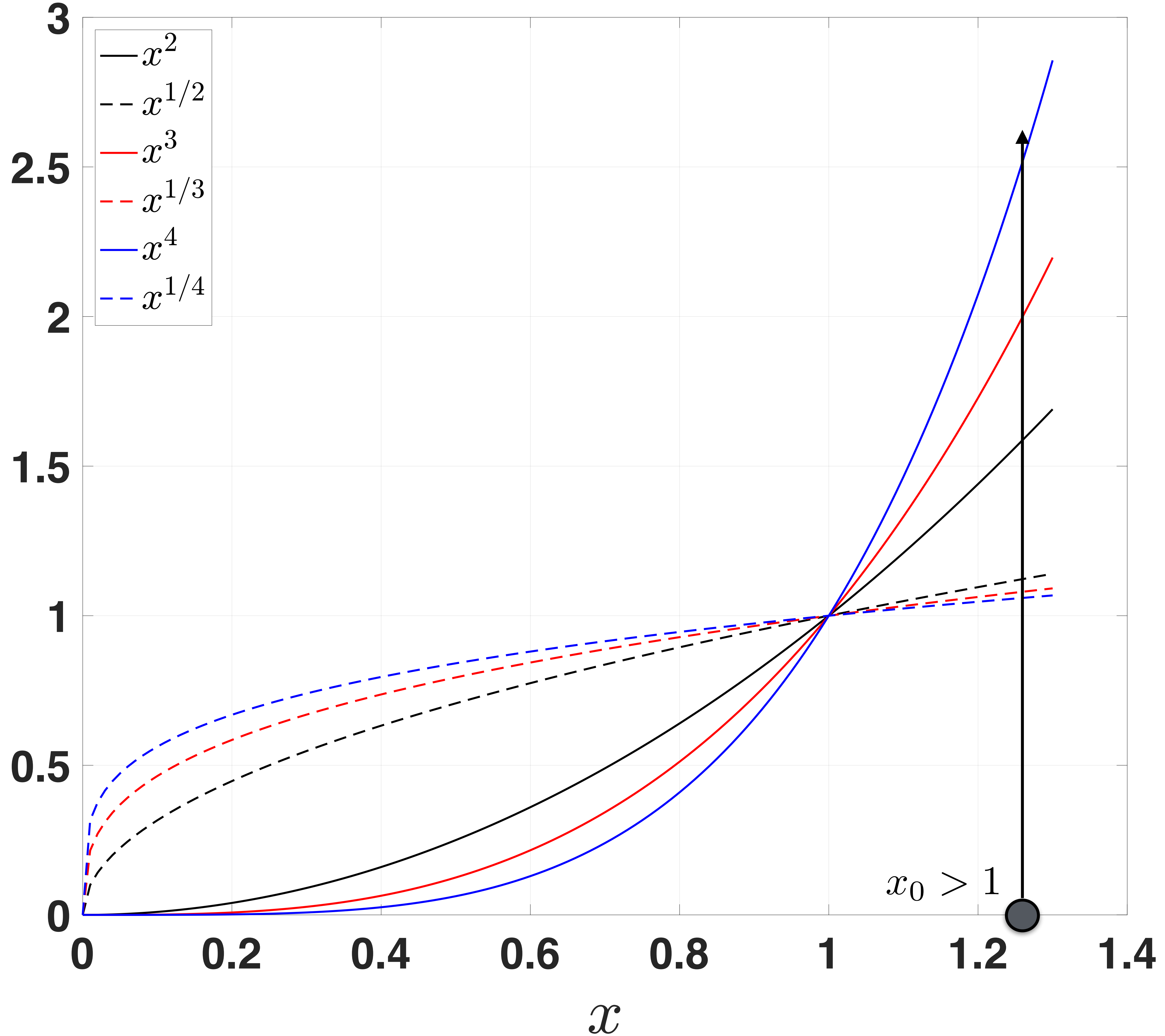
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
1.2
1.4
x
x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4
$x^{1/4}$
$0 < x_0 < 1$



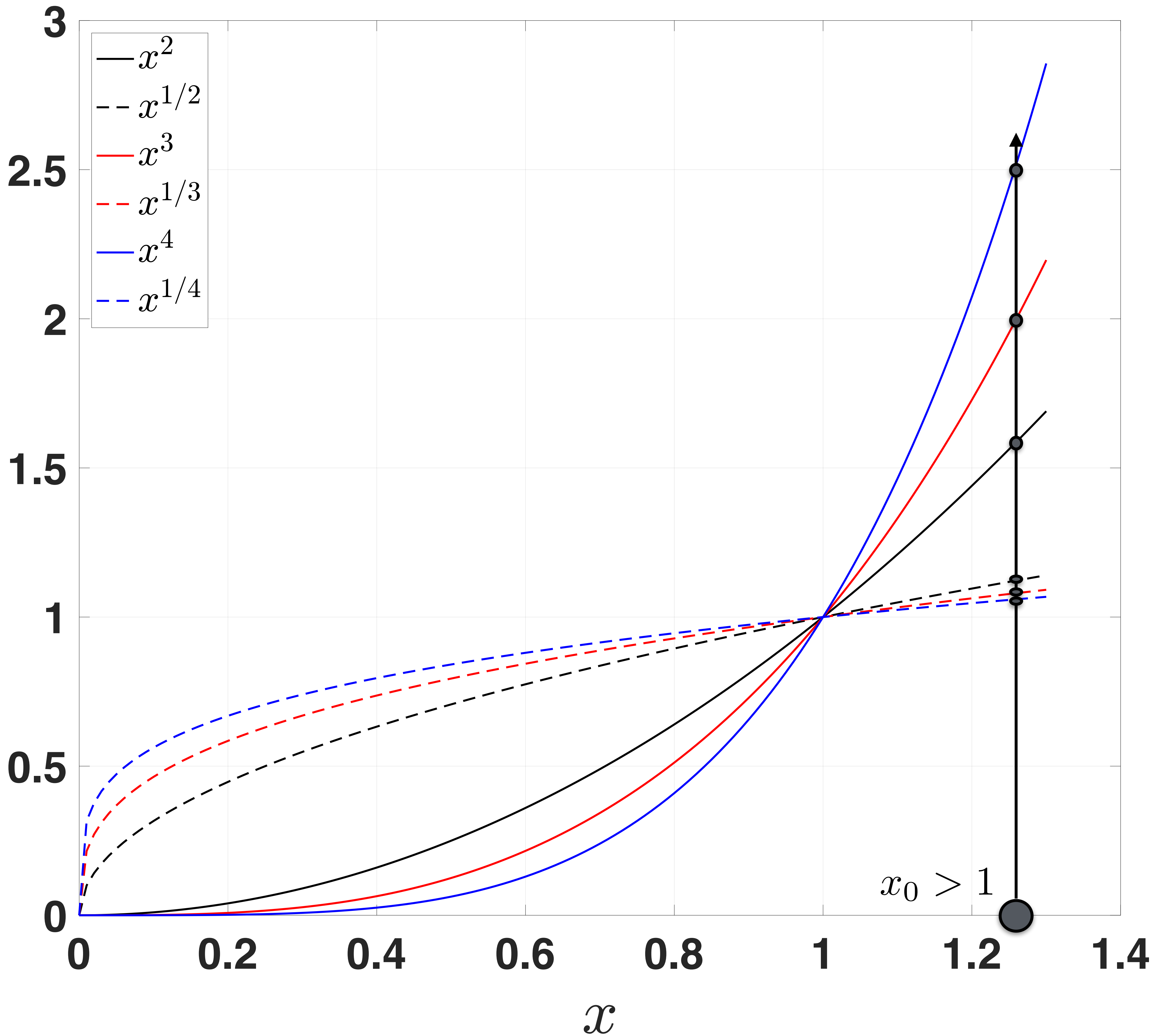
3
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
1.2
1.4
x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4
$x^{1/4}$
$0 < x_0 < 1$
x



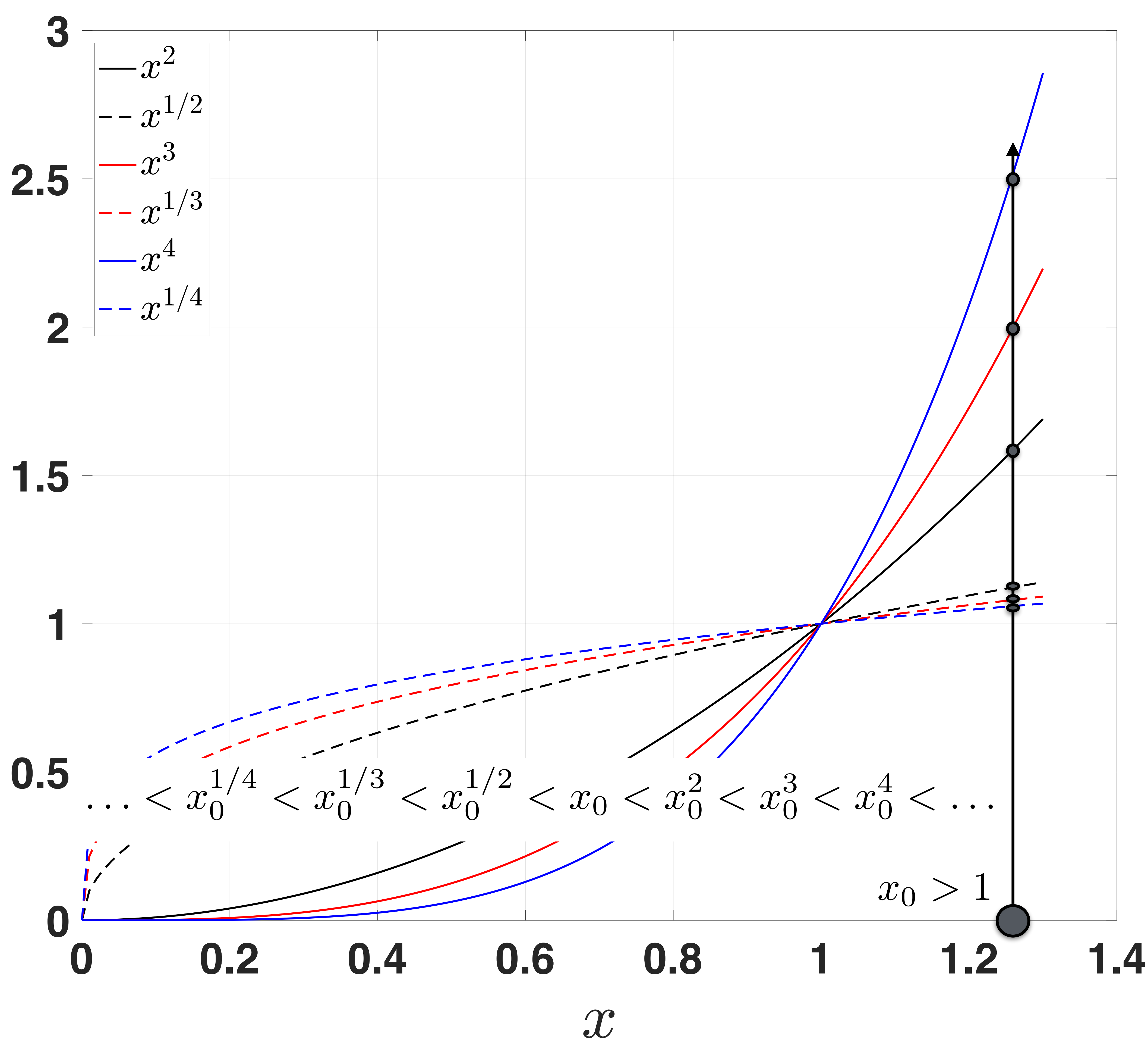
3
2.5
2
1.5
1
0.5
0
x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4
$x^{1/4}$
$\ldots < x_0^4 < x_0^3 < x_0^2 < x_0 < x_0^{1/2} < x_0^{1/3} < x_0^{1/4} < \ldots$
$0 < x_0 < 1$
0
0.2
0.4
0.6
1.2
1.4
x



x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4
$x^{1/4}$
$x_0 > 1$
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x

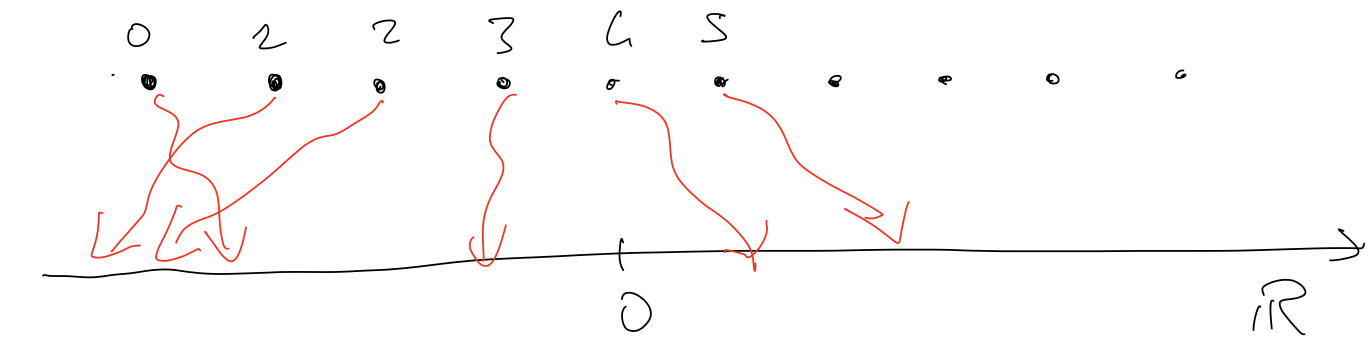


3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x
x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4
$x^{1/4}$
$x_0 > 1$



x^2
$x^{1/2}$
x^3
$x^{1/3}$
x^4
$x^{1/4}$
$\ldots < x_0^{1/4} < x_0^{1/3} < x_0^{1/2} < x_0 < x_0^2 < x_0^3 < x_0^4 < \ldots$
$x_0 > 1$
0
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x

DEF: UNA APPLICAZIONE $s: \mathbb{N} \longrightarrow \mathbb{R}$ SI CHIAMA SUCCESSIONE.



$s(n) = s_n$

$$\boxed{s_n = \frac{1}{n}}$$

$n=1 \longrightarrow 1$

$n=2 \longrightarrow 1/2$

$n=3 \longrightarrow 1/3$

$n=4 \longrightarrow 1/4$

DEF: SIA s_n UNA SUCCESSIONE DI NUMERI REALI

SIA $l \in \mathbb{R}$.

SI DICE CHE $\lim\limits_{n \to +\infty} s_n = l$

SE

$$\forall \varepsilon > 0 \quad \exists\, n_\varepsilon \in \mathbb{N} : \forall n \geq n_\varepsilon \Rightarrow |s_n - l| < \varepsilon$$

LA DISTANZA DI s_n DA l È ARBITRARIAMENTE PICCOLA POSTO CHE n SIA SUFFICIENTEMENTE GRANDE

$s_n = \frac{1}{n}$ L'IDEA È CHE $\ell = 0$

APPLICO LA DEFINIZIONE. È VERO CHE

$$\forall \varepsilon > 0 \ \exists n_\varepsilon \in \mathbb{N} : \forall n \geq n_\varepsilon \Rightarrow \left| \frac{1}{n} - 0 \right| < \varepsilon$$

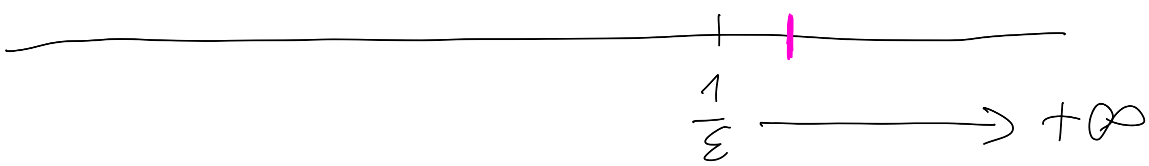
$$\forall \varepsilon > 0 \ \exists n_\varepsilon \in \mathbb{N} : \forall n \geq n_\varepsilon \Rightarrow \left| \frac{1}{n} \right| < \varepsilon$$

$$\left| \frac{1}{n} \right| = \frac{1}{n}$$

OK

$$\frac{1}{n} < \varepsilon \iff n > \frac{1}{\varepsilon}$$

SIA $\forall \varepsilon > 0$ $\quad n_\varepsilon = \min \left\{ n \in \mathbb{N} \;\middle|\; n > \frac{1}{\varepsilon} \right\}$



SE $n \geq n_\varepsilon > \frac{1}{\varepsilon} \Rightarrow \frac{1}{n} < \frac{1}{\varepsilon}$

$$\boxed{\lim_{n \to +\infty} \frac{1}{n} = 0}$$

$$S_n = \frac{n}{n+1}$$

$$S_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$S_2 = \frac{2}{2+1} = \frac{2}{3}$$

$$S_3 = \frac{3}{3+1} = \frac{3}{4}$$

$$\boxed{l = 1}$$

$$\forall \varepsilon > 0 \ \exists n_\varepsilon : \forall n \geq n_\varepsilon \Rightarrow \left| \underbrace{\frac{n}{n+1}}_{S_n} - \overset{l}{1} \right| < \varepsilon$$

$$\left| \frac{\cancel{n} - \cancel{n} - 1}{n+1} \right| < \varepsilon$$

$$\left| -\frac{1}{n+1} \right| < \varepsilon$$

$$\frac{1}{n+1} < \varepsilon$$

RISOLVO COME PER $\frac{1}{n}$

$$\lim_{n \to +\infty} \frac{n}{n+1} = 1$$

SUCCESSIONE DEFINITA PER RICORRENZA

$$\begin{cases} S_0 = 1 \\ S_{n+1} = \frac{S_n}{\ } + \frac{1}{\ } \end{cases}$$

$\{ S_{n+1} \qquad 2 \qquad S_n$

$$S_1 = \frac{S_0}{2} + \frac{1}{S_0} = \frac{1}{2} + 1 = \frac{3}{2} = 1.5 \in \mathbb{Q}$$

$$S_2 = \frac{S_1}{2} + \frac{1}{S_1} = \frac{3/2}{2} + \frac{1}{3/2} = \frac{3}{4} + \frac{2}{3} = \frac{9+8}{12} = \frac{17}{12}$$

$$= 1.416 \in \mathbb{Q}$$

$$S_3 = \frac{577}{408} = 1.414215 \in \mathbb{Q}$$

$$\boxed{\sqrt{2} = 1,4142135623 \ldots}$$

$\forall n \quad S_n \in \mathbb{Q}$

SI PUÒ DIMOSTRARE CHE $S_n \longrightarrow \sqrt{2}$

$$\lim_{n \to +\infty} S_n = \sqrt{2}$$

POICHÉ $S_n \in \mathbb{Q}$ $\forall n$ SI HA CHE

3^{S_n} È BEN DEFINITO $\forall n$

$$S_n = \frac{k}{q} \qquad 3^{S_n} = \left(3^{\frac{1}{q}}\right)^k = \left(3^k\right)^{1/q}$$

$$3^{\sqrt{2}} \equiv \lim_{n \to +\infty} 3^{s_n}$$

DEF: SIA $a \in \mathbb{R}$, $a > 0$, $a \neq 1$

SIA $x \in \mathbb{R}$.

SIA q_n UNA SUCCESSIONE DI NUMERI RAZIONALI TALI CHE

$$\lim_{n \to +\infty} q_n = x$$

SI DEFINISCE

$$a^x = \lim_{n \to +\infty} a^{q_n}$$

E TALE DEFINIZIONE NON DIPENDE DALLA PARTICOLARE SUCCESSIONE q_n PURCHÉ CHE

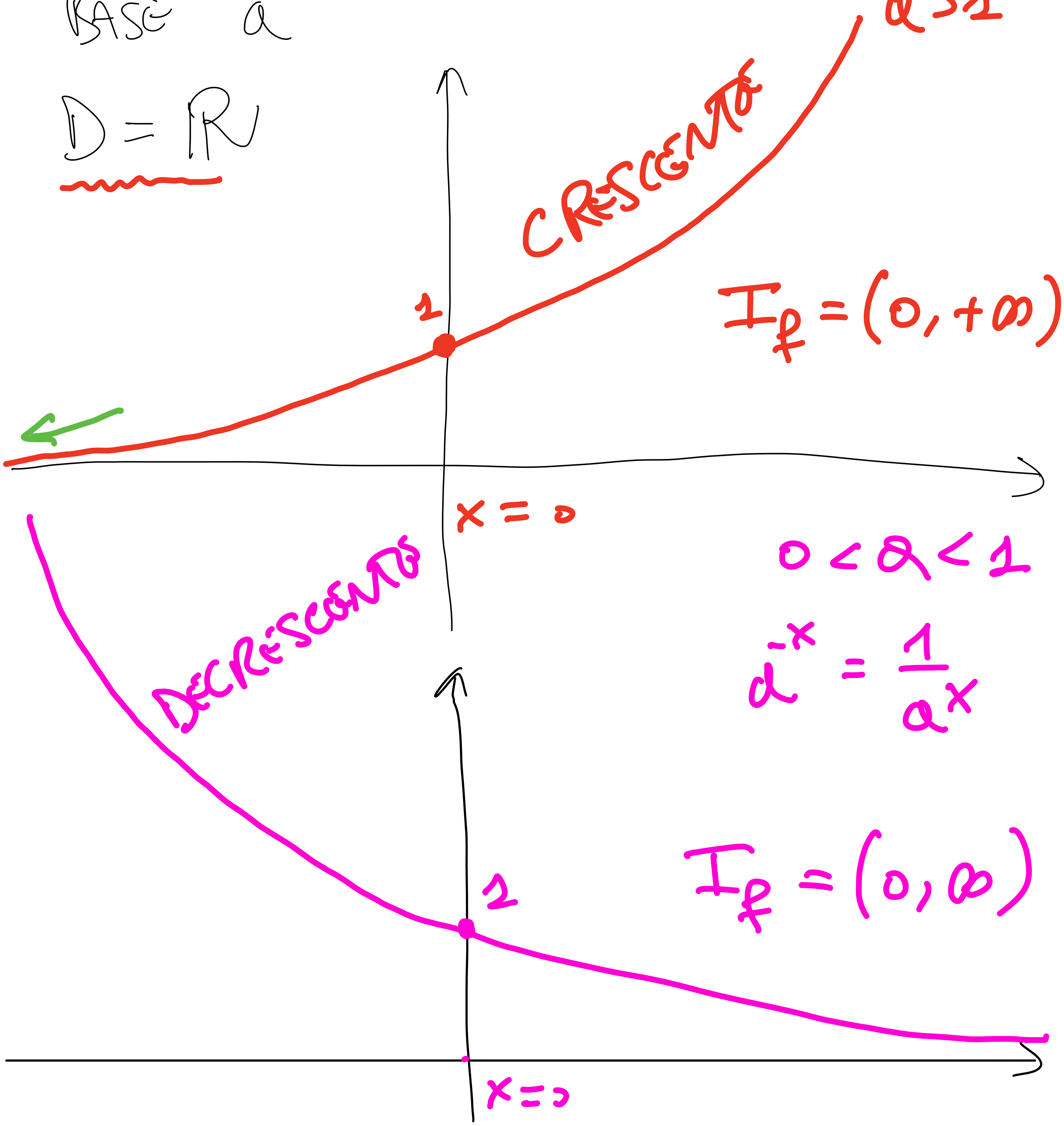
$$q_n \to x$$

DEF: SIA $a \in \mathbb{R}$, $a > 0$, $a \neq 1$

LA FUNZIONE $f(x) = a^x$ SI

CHIAMO FUNZIONE ESPONENZIALE CON BASE a

$D = \mathbb{R}$

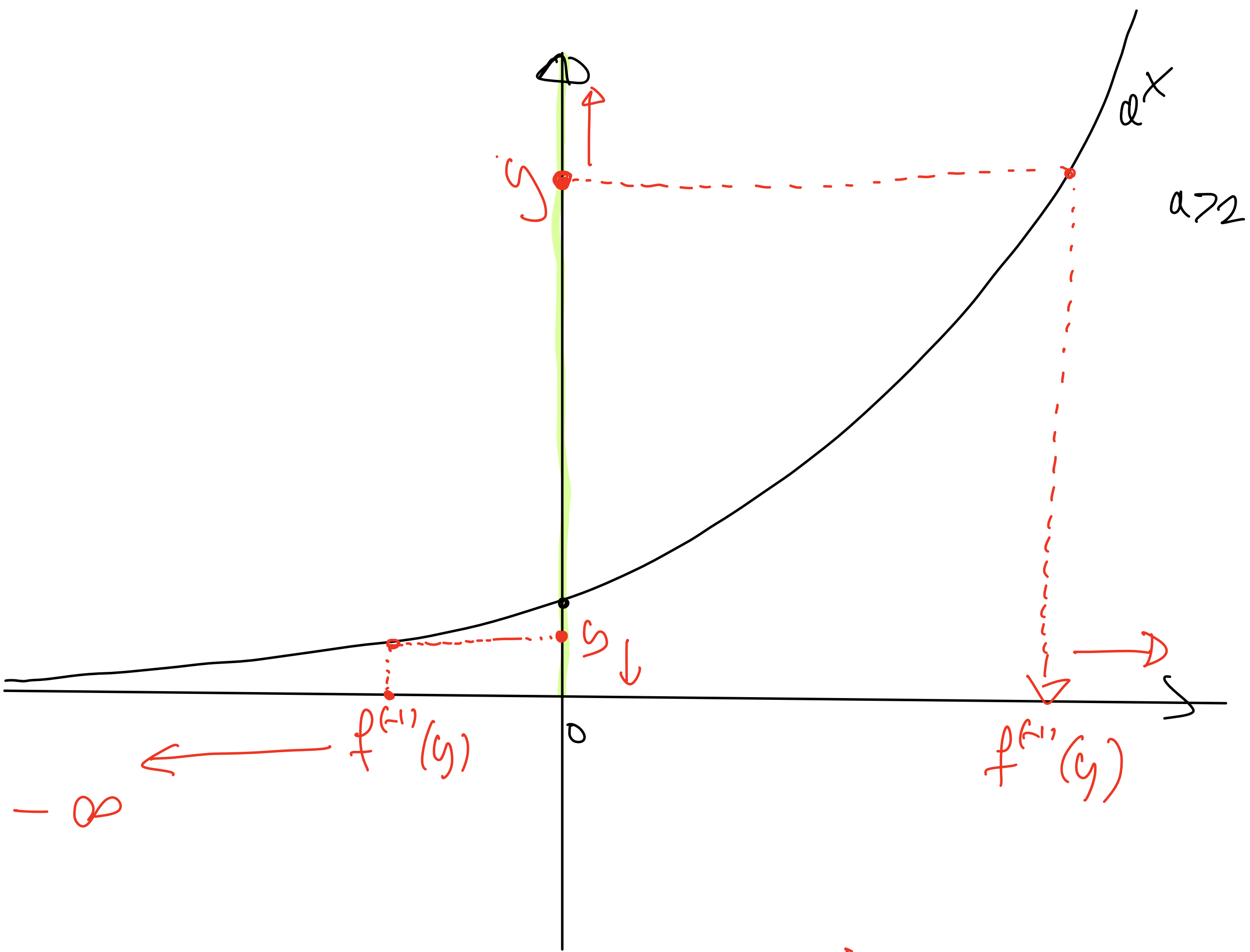


$\forall x, y \in \mathbb{R}$

1) $a^{x+y} = a^x \cdot a^y$

2) $(a^x)^y = (a^y)^x = a^{y \cdot x}$

3) $a^x > 0$



TEO: SIA $a > 0$, $a \neq 1$.

$\cancel{a^y = -2}$

$\forall x > 0 \quad \exists!\, y$ TALE CHE

$$\boxed{a^y} = x \qquad \cancel{a^y = 0}$$

E TALE y SI CHIAMA LOGARITMO DI BASE a DI x E SI INDICA COME

$$y = \log_a(x)$$

$$D = (0, +\infty) = \{x \in \mathbb{R} \mid x > 0\}$$

$$2^{\log_2(x)} = x$$

$$a^{\log_a(x)} = x$$

$$\log_a(a^x) = x$$

1) $a^{x+y} = a^x \cdot a^y$

$\log_a(x \cdot y) = \log_a(x) + \log_a(y)$

2) $(a^x)^y = (a^y)^x = a^{y \cdot x}$

$\log_a(x^b) = b \cdot \log_a(x)$

3) $a^0 = 1 \Rightarrow \log_a(1) = 0$

4) a^x CRESCENTE PER $a > 1$

$\log_a(x)$ CRESCENTE PER $a > 1$

a^x DE-CRESCENTE PER $0 < a < 1$

$\log_a(x)$ DE-CRESCENTE PER $0 < a < 1$