$$f(x) = x^2 e^{-x}$$

1) $D = \mathbb{R}$

2) ASINTOTI:

VERTICALI NESSUNO

ORIZZONTALI: $\lim_{x \to +\infty} x^2 e^{-x} = (+\infty) \cdot e^{-\infty} = (+\infty) \cdot 0 =$

$$= \lim_{x \to +\infty} \frac{x^2}{e^x} = \frac{+\infty}{+\infty} \overset{H}{=} \lim_{x \to +\infty} \frac{2x}{e^x} = \frac{+\infty}{+\infty} \overset{H}{=} \lim_{x \to +\infty} \frac{2}{e^x} = 0$$

$y=0$ È UN ASINTOTO ORIZZONTALE PER $x \to +\infty$

$$\lim_{x \to -\infty} x^2 e^{-x} = (-\infty)^2 e^{-(-\infty)} = (+\infty) \cdot e^{+\infty} = (+\infty) \cdot (+\infty) = +\infty$$

$f(-x) = (-x)^2 e^{-(-x)} = x^2 \cdot e^x$ NÉ PARI NÉ DISPARI

3) INTERSEZIONI CON GLI ASSI $y=0$ E $x=0$

$$x^2 \underbrace{e^{-x}}_{\neq 0} = 0 \iff x^2 = 0 \iff x = 0$$

QUINDI $f(x)$ INTERSECA $y=0$ NEL PUNTO $x=0$

$f(0) = 0$

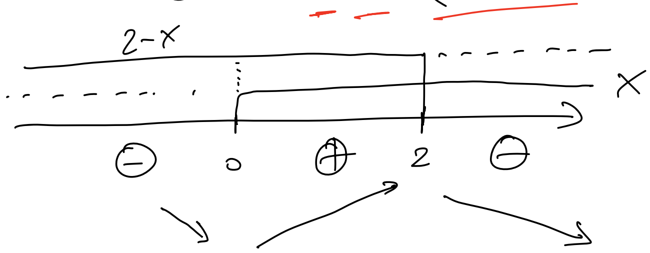
4) SEGNO DELLA FUNZIONE.

$f(x) \geq 0$ $\underbrace{x^2}_{\geq 0} \underbrace{e^{-x}}_{\geq 0} \geq 0 \quad \forall x \in \mathbb{R}$

5) MONOTONICITÀ.

$$f'(x) = (x^2 e^{-x})' = 2x e^{-x} - x^2 e^{-x} = x \overbrace{e^{-x}}^{\geq 0} (2-x)$$

STUDIO IL SEGNO $x(2-x)$



6) MASSIMI & MINIMI. $x=0$ È UN MINIMO LOCALE

$x=2$ È UN MASSIMO LOCALE

7) CONCAVITÀ / CONVESSITÀ.

$$f'(x) = \underbrace{e^{-x}} \underline{(2x-x^2)} \Rightarrow f''(x) = -e^{-x}(2x-x^2) + e^{-x}(2-2x)$$

$$= e^{-x}\left[-2x+x^2+2-2x\right]$$

$$= \underbrace{e^{-x}}_{\geq 0}\left[\underline{x^2-4x+2}\right]$$

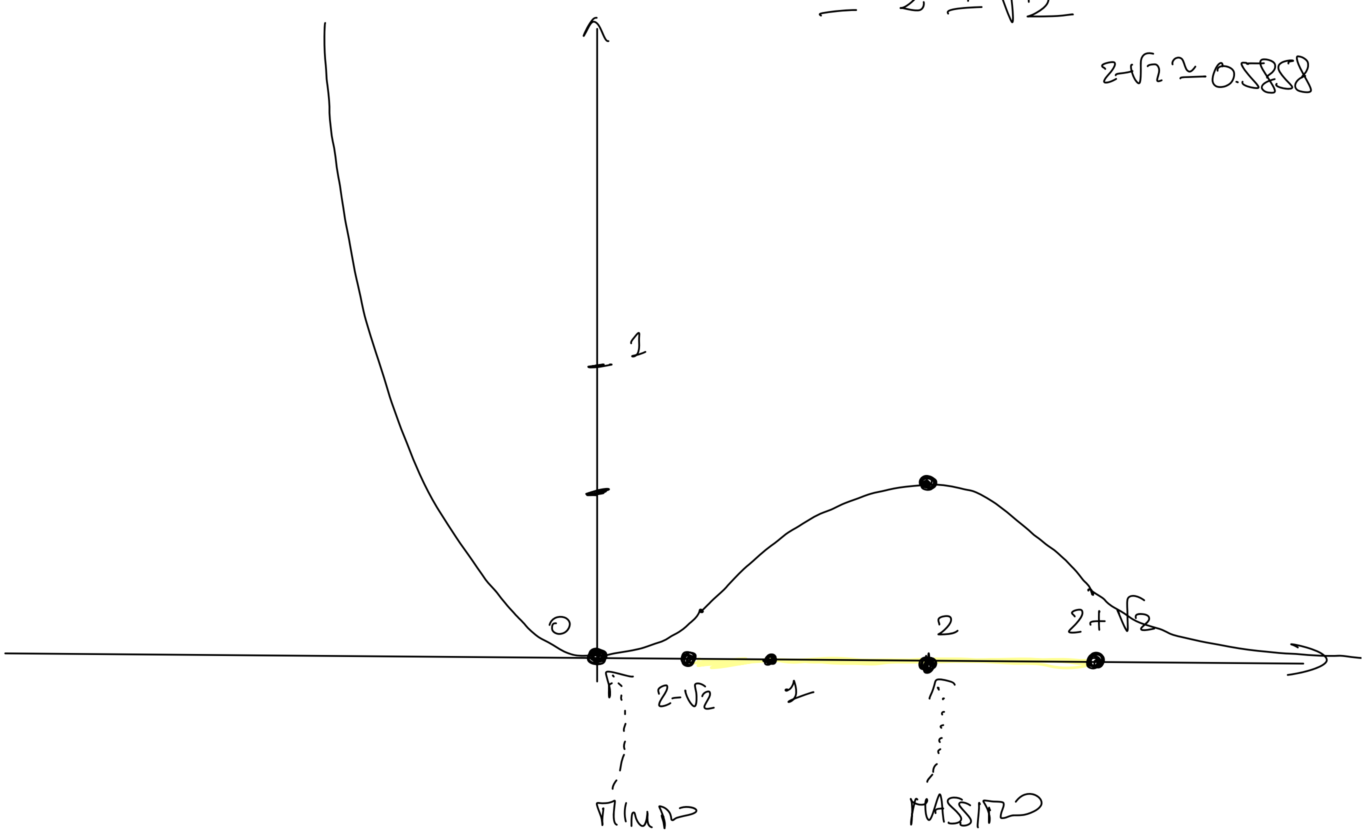
$$f''(x) \geq 0 \iff \underline{x^2-4x+2} \geq 0 \qquad x_{1,2} = \frac{4 \pm \sqrt{16-8}}{2}$$

$$f''(x) \geq 0 \iff x \leq 2-\sqrt{2}\,,\ x \geq 2+\sqrt{2} \qquad = \frac{4\pm\sqrt{8}}{2} = \frac{4\pm 2\sqrt{2}}{2}$$

CONVESSA

$$= 2 \pm \sqrt{2}$$

$2-\sqrt{2} \simeq 0.5858$



$f(0) = 0 \qquad f(2) = 4\bar{e}^{2} \simeq 0.5413$

$$f(x) = \frac{\sqrt{x}}{1+\log(x)}$$

1) $D = \begin{cases} \sqrt{x} \rightarrow x \geq 0 \\ \log(x) \rightarrow x > 0 \\ \frac{1}{1+\log(x)} \rightarrow 1+\log(x) \neq 0 \Rightarrow \log(x) \neq -1 \Rightarrow x \neq \frac{1}{e} = e^{-1} \end{cases}$

$D = \left(0, \frac{1}{e}\right) \cup \left(\frac{1}{e}, +\infty\right)$

2) VERTICALI: $\lim_{x \to 0^+} \frac{\sqrt{x}}{1+\log(x)} = \frac{0}{-\infty} = 0$

$\lim_{x \to \left(\frac{1}{e}\right)^-} \frac{\sqrt{x}}{1+\log(x)} = \frac{\sqrt{1/e}}{0^-} = -\infty$ | $\lim_{x \to \left(\frac{1}{e}\right)^+} \frac{\sqrt{x}}{1+\log(x)} = \frac{\sqrt{1/e}}{0^+} = +\infty$

QUINDI $x = \frac{1}{e}$ È UN ASINTOTO VERTICALE

ORIZZONTALI: $\lim_{x \to +\infty} \frac{\sqrt{x}}{1+\log(x)} = \frac{\infty}{\infty} \overset{H}{=} \lim_{x \to +\infty} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{x}} =$

$= \lim_{x \to +\infty} \frac{x}{2\sqrt{x}} = \lim_{x \to +\infty} \frac{\sqrt{x}}{2} = +\infty$

NESSUN ASINTOTO ORIZZONTALE.

3) INTERSEZIONI CON GLI ASSI.

$x = 0 \Rightarrow$ FUORI DAL DOMINIO

$y = 0 \Rightarrow \frac{\sqrt{x}}{1+\log(x)} = 0$ NESSUNA SOLUZIONE

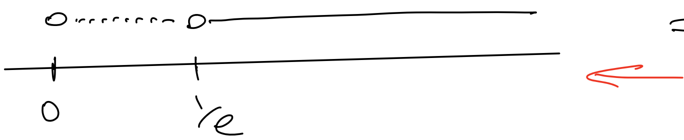
4) SEGNO DELLA FUNZIONE

$\sqrt{x} \geq 0 \quad \forall x \in D$

$$f(x) \geq 0 \Rightarrow \frac{\sqrt{x}}{1+\log(x)} \geq 0$$

$$1+\log(x) \geq 0 \Rightarrow \log(x) \geq -1$$

$$\Rightarrow x \geq e^{-1} = \frac{1}{e}$$



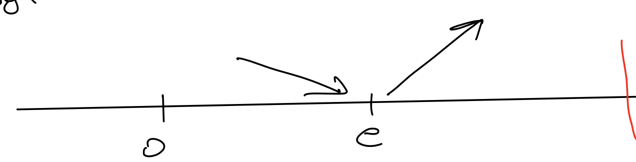
5) <u>MONOTONICITÀ</u>: $f(x) = \sqrt{x} \cdot \frac{1}{1+\log(x)}$

$$f'(x) = (\sqrt{x})' \cdot \frac{1}{1+\log(x)} + \sqrt{x} \cdot \left(\left(1+\log(x)\right)^{-1} \right)' = \frac{1}{2\sqrt{x}} \cdot \frac{1}{1+\log(x)} + \sqrt{x}(-1)(1+\log(x))^{-2} \frac{1}{x}$$

$$= \frac{1}{2\sqrt{x}(1+\log(x))} - \frac{\sqrt{x}}{x} \frac{1}{(1+\log(x))^2} = \frac{1}{2\sqrt{x}(1+\log(x))} - \frac{1}{\sqrt{x}} \frac{1}{(1+\log(x))^2}$$

$$= \frac{1+\log(x) - 2}{2\sqrt{x}(1+\log(x))^2} = \frac{\log(x) - 1}{2\sqrt{x}(1+\log(x))^2} = f'(x)$$

$$f'(x) \geq 0 \Leftrightarrow \log(x) - 1 \geq 0 \Leftrightarrow \log(x) \geq 1 \Leftrightarrow x \geq e$$



$\boxed{x = e \text{ è UN MINIMO LOCALE}}$

6) CONCAVITÀ / CONVESSITÀ:

$$f'(x) = (\log(x) - 1) \cdot \frac{1}{2\sqrt{x}(1+\log(x))^2} \Rightarrow$$

$$f''(x) = (\log(x) - 1)' \cdot \frac{1}{2\sqrt{x}(1+\log(x))^2} + \frac{\log(x) - 1}{2} \cdot \left(x^{-\frac{1}{2}} (1+\log(x))^{-2} \right)'$$

$$= \frac{1}{x} \frac{1}{2\sqrt{x}(1+\log(x))^2} + \frac{\log(x) - 1}{2} \left[\left(-\frac{1}{2}\right) x^{-\frac{3}{2}} \cdot (1+\log(x))^{-2} + x^{-\frac{1}{2}} (-2)(1+\log(x))^{-3} \frac{1}{x} \right]$$

$$= \frac{1}{2x^{3/2}(1+\log(x))^2} + \frac{\log(x)-1}{2} \cdot \left[-\frac{1}{2x^{3/2}(1+\log(x))^2} - \frac{2}{x^{3/2}(1+\log(x))^3} \right]$$

$$= \frac{1}{2x^{3/2}(1+\log(x))^2} - \frac{\log(x)-1}{4x^{3/2}(1+\log(x))^2} - \frac{\log(x)-1}{x^{3/2}(1+\log(x))^3} =$$

$$= \frac{2(1+\log(x)) - (\log(x)-1)(\log(x)+1) - (\log(x)-1)\,4}{4x^{3/2}(1+\log(x))^3}$$

$$= \frac{2(1+\log(x)) + (1-\log(x))(1+\log(x)) + (1-\log(x))\cdot 4}{4x^{3/2}(1+\log(x))^3}$$

$$= \frac{2(1+\log(x)) + (1-(\log(x))^2) + 4(1-\log(x))}{4x^{3/2}(1+\log(x))^3} =$$

$$= \frac{2 + 2\log(x) + 1 - (\log(x))^2 + 4 - 4\log(x)}{4x^{3/2}(1+\log(x))^3} =$$

$$= \frac{7 - 2\log(x) - (\log(x))^2}{4x^{3/2}(1+\log(x))^3} \qquad t = \log(x)$$

$$\boxed{7 - 2\log(x) - (\log(x))^2 \geq 0} \qquad 7 - 2t - t^2 \geq 0$$

$$\boxed{t^2 + 2t - 7 \leq 0}$$

$$t_{1,2} = \frac{-2 \pm \sqrt{4+7\cdot 4}}{2} = \frac{-2 \pm 2\sqrt{1+7}}{2} = -1 \pm \sqrt{8} =$$

$$= -1 \pm 2\sqrt{2}$$

$$-1-2\sqrt{2} \le t \le -1+2\sqrt{2}$$

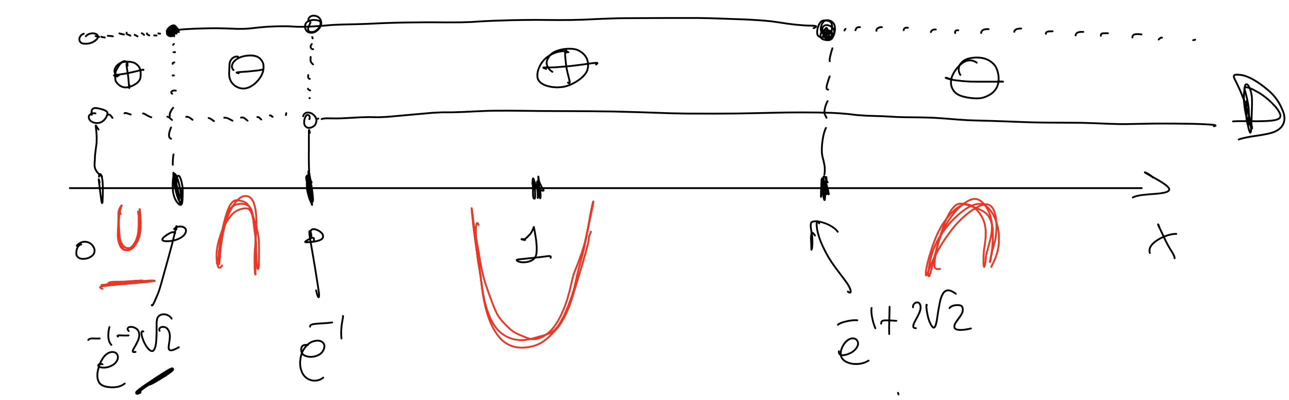
$$-1-2\sqrt{2} \le \log(x) \le -1+2\sqrt{2}$$

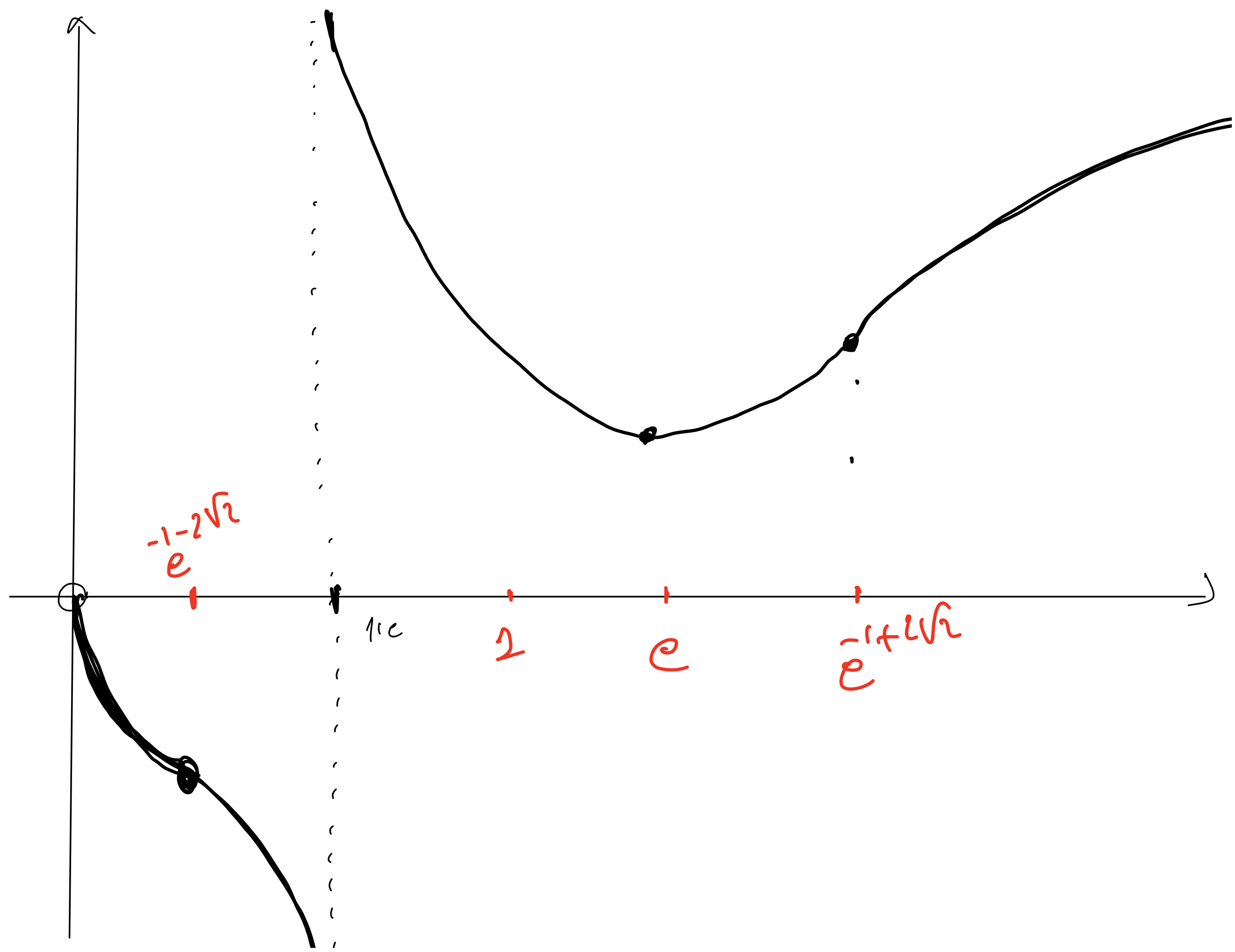
$$e^{-1-2\sqrt{2}} \le x \le e^{-1+2\sqrt{2}}$$

$e^{-1-2\sqrt{2}} \simeq 0,0217$

$e^{-1+2\sqrt{2}} \simeq 6,2241$

$1/e \simeq 0,3679$





$$f(e)=\frac{\sqrt{e}}{1+\log(e)}=\frac{\sqrt{e}}{2}\simeq 0{,}8$$

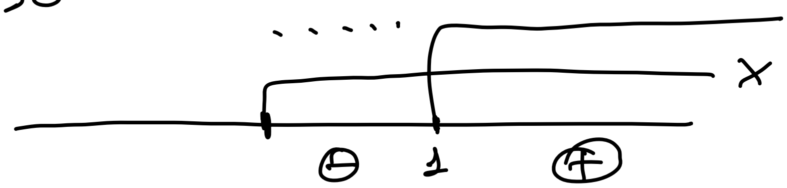
$$f(x)=\log\left(\frac{\log(x)}{x}\right)$$

$$\log(x)>0 \Rightarrow x>e^{0}=1$$

$$x\neq 0$$

$$\log(x) \Rightarrow \quad x>0$$

$$\frac{\log(x)}{x}>0$$



$$\Rightarrow \boxed{x>1} \qquad D=(1,+\infty)$$

$$\lim_{x\to 1^{+}} \log\left(\frac{\log(x)}{x}\right)=\log\left(\frac{0^{+}}{1}\right)=-\infty$$

$$\lim_{x\to +\infty} \log\left(\frac{\log(x)}{x}\right)$$

$$\lim_{x\to +\infty} \frac{\log(x)}{x}=\frac{\infty}{\infty}\overset{H}{=}\lim_{x\to\infty}\frac{\frac{1}{x}}{1}=0$$

$$\lim_{x\to +\infty} \log\left(\frac{\log(x)}{x}\right)=\log(0^{+})=-\infty$$