# Microconomics I, 2023/2024

# Master of Science in Economics

# Problem Set 1

Lorenzo Bozzoli*

17/11/2023

**Question 1** *Suppose preferences are represented by the Cobb-Douglas utility function* $u(x_1, x_2) = Ax_1^{\alpha}x_2^{1-\alpha}$*, for* $A > 0$ *and* $\alpha \in (0, 1)$.

1. *Check that the Cobb-Douglas utility function is increasing in each commodity and that it is a strictly concave function. Once established concavity, what can you deduce on the properties of the Walrasian demand?*

2. *Considering an interior solution, solve for the Walrasian demand functions. Are the preferences that* $u(x)$ *represents homothetic?*

3. *Consider the logarithmic transformation* $v(x) = \ln u(x)$ *of the Cobb-Douglas utility function above and verify that the Walrasian demand functions are identical to those previously derived.*

4. *Suppose that the preferences of a consumer are represented by the utility function* $u(x)$*, and consider the transformation* $v(x) = f(u(x))$*. When is it that the transformation does not alter the original preference order? Is this the case of the previous questions?*

5. *Using the Walrasian demand functions, derive the consumer's indirect utility function. How does it vary with respect to wealth?*

**Question 2** *Consider a generalization of the Cobb-Douglas preferences that can be used to model the consumption of addictive goods, such as coffee. Let the utility function be:*

$$u(x_1, x_2; r_1) = (x_1 - r_1)^{\alpha}x_2^{\beta}$$

*with* $\alpha + \beta = 1$*,* $0 < \alpha < 1, 0 < \beta < 1$ *and* $r_1 > 0$*. Notice that the above utility is positive only for* $x_1 \geq r_1$ *and* $x_2 \geq 0$*. Assume that for* $x_1 < r_1$ *and for* $x_2 < 0$ *the utility is zero. Good* $x_1$ *is*

*e-mail: lor.bozzoli@gmail.com

*an addictive good (coffee) with addiction level $r_1$. One interpretation is that the more coffe the consumer has consumed in the past, the higher the addiction level ($r_1$).*

1. *Draw an approximate map of indifference curves.*
2. *How does the utility function change as $r_1$ changes? Interpret your findings.*
3. *Compute now the marginal utility with respect to $x_1$. How does this marginal utility change as $r_1$ changes? Explain your findings.*
4. *Write down the Utility Maximization problem (UMP) and solve for the Walrasian demands for $x_1$ and $x_2$.*
5. *Provide a definition of a normal good. Is good $x_1$ a normal good?*
6. *Compute the change in $x_1(p, w)$ as $r_1$ varies. Interpret you result: why does the consumer consume more of good 1 if she is more addicted to it?*
7. *Check whether the Walrasian demands for good 1 and 2 are homogeneous of degree zero in $(p, w)$ and satisfy Walras' law.*

**Question 3** *Consider the following utility function: $u(x_1, x_2) = \phi(x_1) + x_2$ with $\phi'(x_1) > 0$, and $\phi''(x_1) < 0$. At first, assume no particular functional form for $\phi(x)$. Consider $p = (p_1, p_2)$ strictly positive and $w > 0$.*

1. *Represent the indifference curves corresponding to this utility function and compute the Marginal Rate of Substitution ($MRS_{1,2}$) between commodity 1 and 2, explain its meaning.*
2. *Write down the Lagrange function for the UMP problem.*
3. *Write down the first order conditions for this problem. What do the first order conditions tell you regarding the value of $\lambda$? (Hint: Does the value of $\lambda$ depend on $p_1$ or $w$? Why is this the case?)*
4. *Solve for the Walrasian demand for commodity $x_1^*$.. Does it depend on wealth? Is good 1 a normal good, an inferior good or a neutral good, i.e. a commodity in which the wealth effect is null?*
5. *Compute $\frac{\partial x_1^*}{\partial p_1}$. You should find a negative sign, is this related to your findings in 3.?*
6. *Continue the exercise under the assumption $u(x_1, x_2) = \sqrt{x}_1 + x_2$. Provide an explicit solution for the Walrasian demands for the two commodities.*
7. *Under what conditions on $p_1, p_2$ and $w$ is the Walrasian demand for good 2 strictly positive?*
8. *Write down the budget lines for $(p_1 = 1, p_2 = 1, w = 1)$ and for $(p_1 = 1, p_2 = 1, w = 2)$. Find graphically the optimal consumption bundles that solve UMP. How do they look like? You should find that $x_1^*(1, 1, 1) = x_1^*(1, 1, 2)$? Is this consistent with your previous findings? Explain.*