TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

Searching Algorithms

Algorithms, Data and Security
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Valeria Cardellini

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Science and Technology Major

Why searching and sorting algorithms?

- How do you find a contact in your phone or a particular email?
- If a deck of cards has less than 52 cards, how do you determine which card is missing?
- Searching and sorting are common tasks in our daily life!

Why searching and sorting algorithms?

- Searching and sorting are also common tasks in computer programs
- We have well-known algorithms for doing searching and sorting
- We'll look at two searching algorithms and four sorting algorithms
- Considering also their performance (i.e., running times)

Searching algorithms

- Given a collection of values, the goal of search is to find a target value in this collection or to recognize that the value does not exist in the collection
- We study two searching algorithms:
  - Sequential search
  - Binary search
- We assume that the values are stored in a list (or an array) and are numerical
  - Algorithms work also for non-numerical data
- To visualize how these algorithms work, see https://www.cs.usfca.edu/~galles/visualization/Search.html

Sequential search

- The simplest searching algorithm
  - Similar to scanning a book index for a specific topic
  - We look at each item in the list in turn, quitting once we find an item that matches the target value or once we reach the end of the list
  - Also known as linear search

```
algorithm SequentialSearch(item x, list L) → Boolean
    n = |L|                                   ▷ list L has n items
    for i = 1 to n do
        if L[i] = x then return found
        end if
    end for
    return not found
```


Sequential search: Running time

- Sequential search requires O(n) time (comparisons) in the worst case
  - Worst case means the most number of comparisons necessary to find the target value
  - In our case, it occurs when the target value is in the last slot in the list, or is not in the list. The number of comparisons is equal to the size of the list (say n)

- Can we do better?
- Yes, if the list L is sorted

Binary search

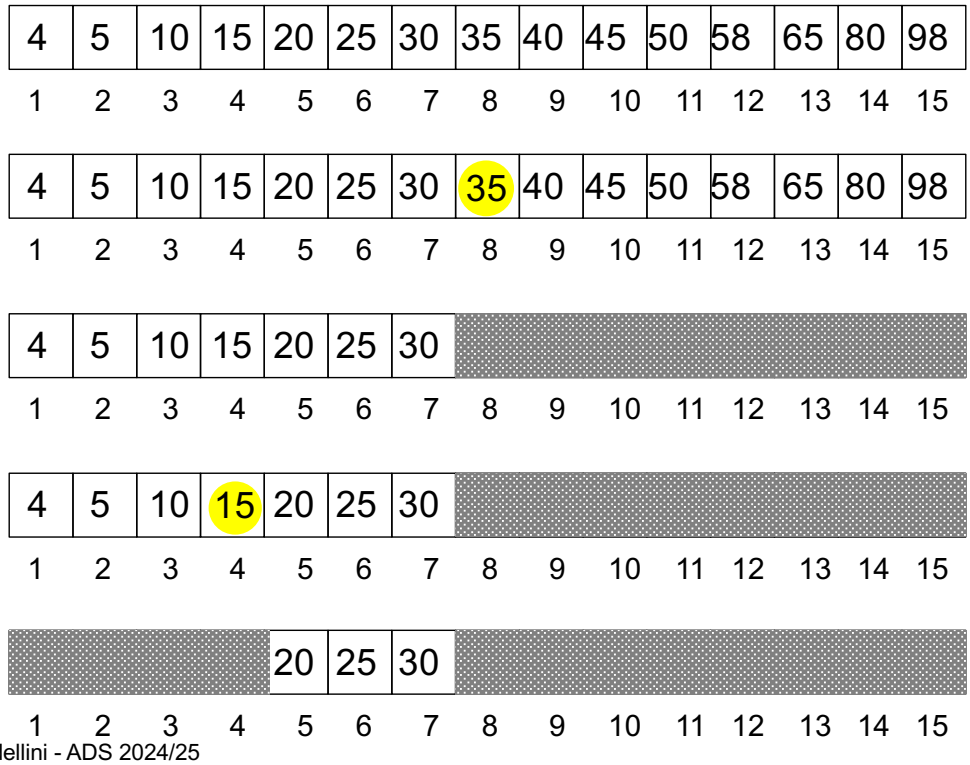
- Similar to finding a word in a dictionary
- Input: sorted list and target value
- Idea: each time we compare the target value to the middle element of the list, we eliminate half of the list and continue the search on the remaining half, until we either find the target value or determine that it is not in the list

Binary search

- The algorithm uses the middle element of list
- It works by repeatedly dividing the list in half until the target value is found or the list is empty
  - It compares the target value to the middle element
  - If they are not equal, the half in which the target cannot lie is eliminated and the search continues in the remaining half, again comparing the middle element to the target value, and repeating this until the target value is found or determined to not be in the list

Binary search: example

- Binary search for target value 20



Binary search: example

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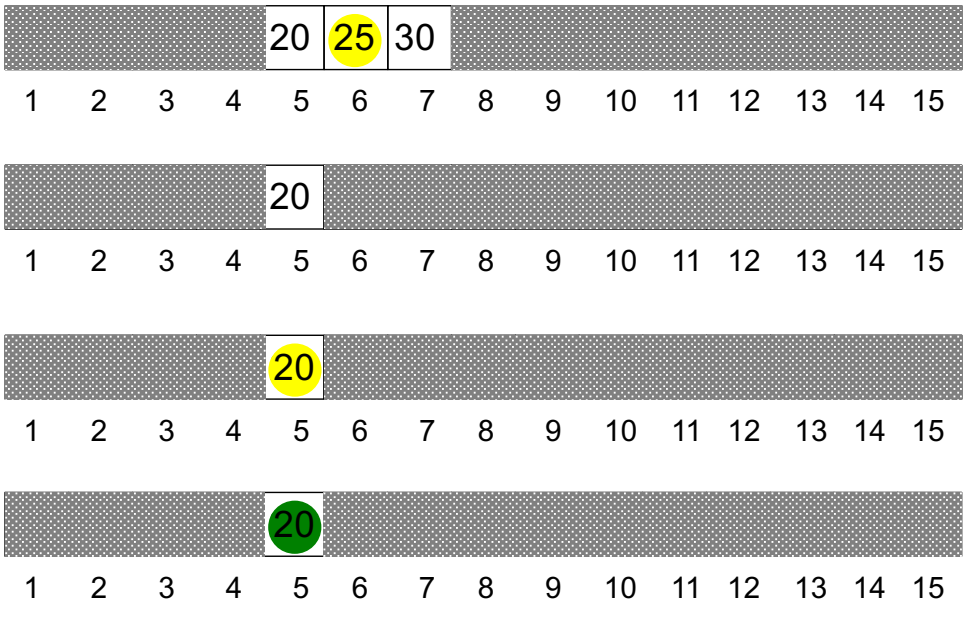

Binary search: Algorithm

- Given target value x and sorted list L as input, find if x is in the list or not

```
algorithm BinarySearch(item x, list L) → Boolean
    n = |L|
    if n = 0 then return not found
    end if
    mid = ⌈n/2⌉
    if L[mid] = x then
        return found
    else if L[mid] > x then
        return BinarySearch(x, L[1; mid − 1])
    else
        return BinarySearch(x, L[mid + 1; n])
    end if
```


Binary search: Algorithm

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    end if
```

List is sorted

length of list (number of elements in the list)

midpoint of list

recursive calls

left half of list

right half of list

Running time?

Binary search: Running time

- Let T(n) be number of comparisons done by BinarySearch to find an item x in a sorted list of size n

  $T(n) = 1 + T(n/2)$

  $T(1) = 1$
- T(1) = 1 (if the list has just one element, only 1 comparison is needed)
- It is a recurrence. How do we solve it?

Binary search: Running time

- We have to solve

  $T(n) = T(n/2) + 1$
- If this is true, then we can also say

  $T(n/2) = T(n/4) + 1$
- Putting this back into the recurrence, we obtain:

  $T(n) = T(n/2) + 1 = T(n/4) + 1 + 1$
- i.e.,

  $T(n) = T(n/4) + 2$

Binary search: Running time

- We have

  $T(n) = T(n/2) + 1 = T(n/4) + 2$
- We can also say

  $T(n/4) = T(n/8) + 1$
- Putting this back into the recurrence, we obtain:

  $T(n) = T(n/4) + 2 = T(n/8) + 1 + 2$
- i.e.,

  $T(n) = T(n/8) + 3$

Binary search: Running time

- We can go on like this:

  $T(n) = T(n/8) + 3 = T(n/16) + 4 = ...$
- In general, for k = 1, 2, 3,...

  $T(n) = T(n/2^k) + k$
- When do we stop? Can't go on forever…

Binary search: Running time

- We can stop when k is such that $n/2^k = 1$
  - i.e., $n = 2^k$, i.e., $k = \log_2 n$
- For this value of k:

  $T(n) = T(n/2^k) + k = T(1) + \log_2 n = 1 + \log_2 n$
- Namely:

  $T(n) = 1 + \log_2 n = O(\log n)$

Binary search: Running time

In summary:

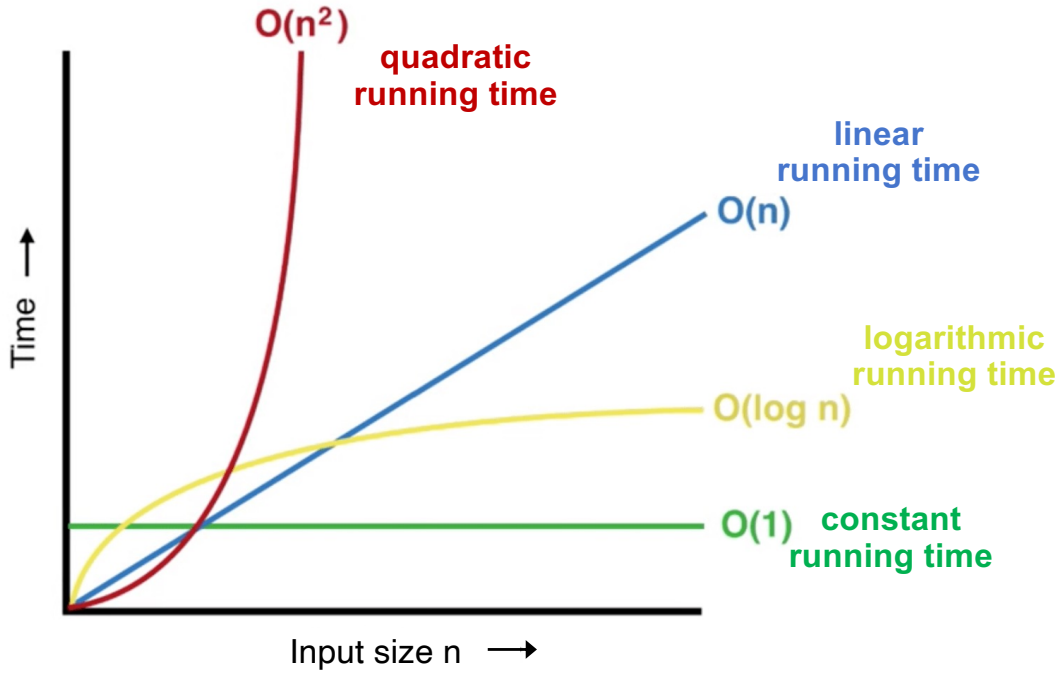
- The number of comparisons T(n) done by binary search to find an item in a sorted list of size n is described by the recurrence:

  $T(n) = T(n/2) + 1$
- The solution of this recurrence is

  $T(n) = O(\log n)$
- Binary search requires running time **O(log n)**
  - Better than sequential search, which is O(n)!

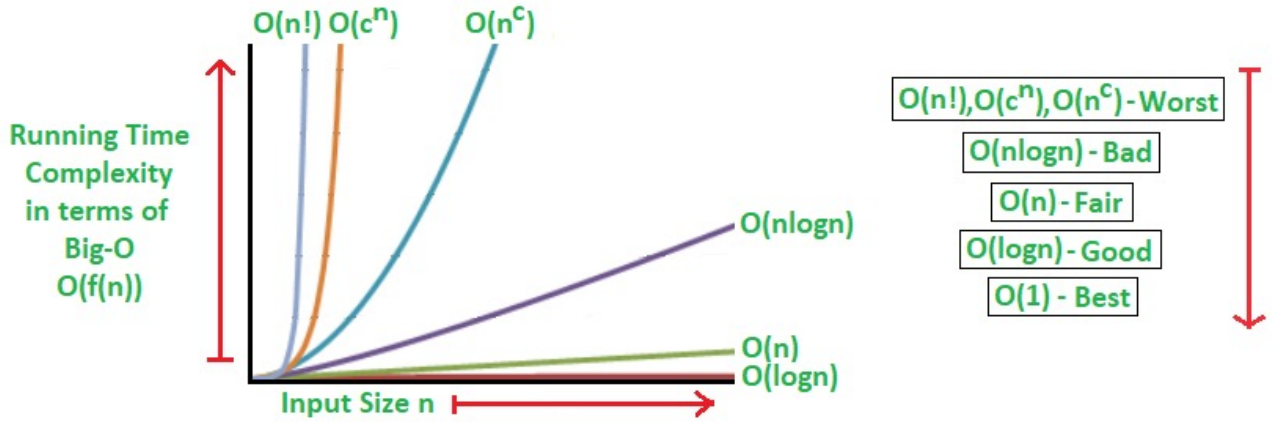
Big O notation: common cases

- Some common Big O notations:



Big O notation: common cases

- More Big O notations:



Big O notation: common cases

- Constant algorithm – O(1)
  - The fastest possible running time: the algorithm always takes the same amount of time to execute, regardless of the input size; ideal but rarely achievable
- Logarithmic algorithm – O(log n)
  - Running time grows logarithmically in proportion to n
  - E.g., binary search
- Linear algorithm – O(n)
  - Running time grows directly in proportion to n
  - E.g., sequential search, `fibonacci3`
- Superlinear algorithm – O(n log n)
  - Running time grows faster than n, still practical

Big O notation: common cases

- Polynomial algorithm – $O(n^c)$
  - Running time grows quicker than previous all
- Exponential algorithm – $O(c^n)$
  - Running time grows even faster than polynomial algorithm
  - E.g., `fibonacci2`
- Factorial algorithm – O(n!)
  - Running time grows the fastest and the algorithm becomes quickly unusable for even small values of n

Take-away

- We described two searching algorithms:
  - Sequential search: O(n)
  - Binary search (sorted input): O(log n)

n	10	100	1000	10^6	10^9
$\log_2 n$	~ 3.3	~ 6.65	~ 10	~ 20	~ 30

- If we have to search through trillion ($n=10^{12}$) of items, and each step takes 10 nanosec. (10^{-8} sec.):
  - n steps take: $10^{12} \times 10^{-8}$ sec $= 10^4$ sec ~ 3 hours!
  - $\log_2 n$ steps take: $\log_2(10^{12}) \times 10^{-8}$ sec =
    $= \log_2((10^3)^4) \times 10^{-8}$ sec ~ $\log_2((2^{10})^4) \times 10^{-8}$ sec =
    $= 40 \times 10^{-8}$ sec = 400 nanosec!

Exercise

- Consider the list [11, 6, 4, 21, 8] and search using sequential search for the target value 8

- Consider the sorted list [2, 4, 6, 8, 10, 12, 14, 16, 18, 20] and search using binary search for the target value 14
  - Write also the sequence of recursive calls BinarySearch(14, …)

References

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- C. Demetrescu, I. Finocchi, G. F. Italiano. Algoritmi e Strutture Dati, Mc-Graw Hill, 2008 (in Italian)
- Algorithms: Binary Search https://www.youtube.com/watch?v=P3YID7liBug