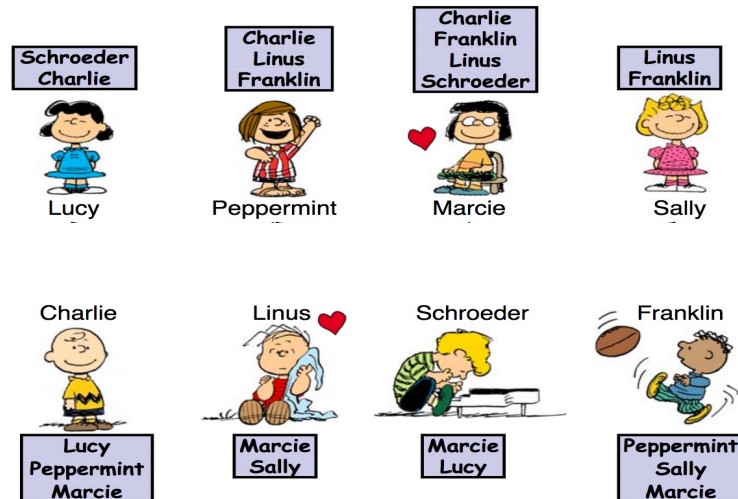


Exercise

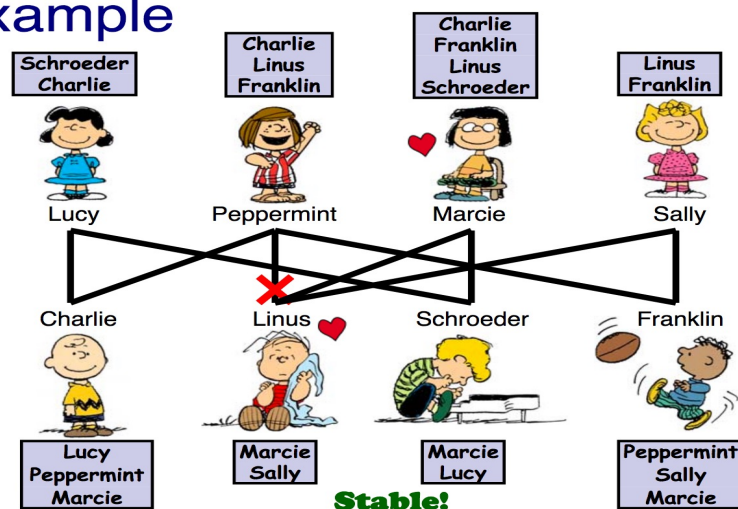
- Find a stable marriage for the Peanuts



Exercise

- Find a stable marriage for the Peanuts

Example



Gale-Shapley algorithm (women-optimal)

- Fix some ordering on the women
- Repeat until everyone is matched
 - Let X be the first unmatched woman in the ordering
 - Find man Y such that Y is the most desirable man in X's list such that Y is unmatched, or Y is currently matched to Z and X is more preferable to Y than Z
 - Match X and Y; possible this turns Z to be unmatched

Women-optimal solution

- Let's find a **women optimal stable marriage**
 - Women preference lists
 - Lucy: Schroeder, Charlie
 - Peppermint: Charlie, Linus, Franklin
 - Marcie: Charlie, Franklin, Linus, Schroeder
 - Sally: Linus, Franklin
 - Men preference lists
 - Charlie: Lucy, Peppermint, Marcie
 - Linus: Marcie, Sally
 - Schroeder: Marcie, Lucy
 - Franklin: Peppermint, Sally, Marcie
- Ordering on the women:
Lucy Peppermint Marcie Sally
- Lucy: Schroeder
 - Peppermint: Charlie
 - Marcie: Franklin
 - Sally: Linus
- Marcie prefers Charlie to Franklin, but Charlie prefers Peppermint over Marcie

Gale-Shapley algorithm (men-optimal)

- Fix some ordering on the men
- Repeat until everyone is matched
 - Let X be the first unmatched man in the ordering
 - Find woman Y such that Y is the most desirable woman in X's list such that Y is unmatched, or Y is currently matched to Z and X is more preferable to Y than Z
 - Match X and Y; possible this turns Z to be unmatched

Men-optimal solution

- Let's find a **men optimal stable marriage**

Ordering on the men:

Charlie Linus Schroeder Franklin

- Women preference lists

- Lucy: Schroeder, Charlie
- Peppermint: Charlie, Linus, Franklin
- Marcie: Charlie, Franklin, Linus, Schroeder
- Sally: Linus, Franklin

- Charlie: Lucy
- Linus: Marcie
- Schroeder: ~~Marcie~~ Lucy
- Franklin:

Schroeder prefers Marcie the most, but Marcie prefers Linus to Schroeder

- Men preference lists

- Charlie: Lucy, Peppermint, Marcie
- Linus: Marcie, Sally
- Schroeder: Marcie, Lucy
- Franklin: Peppermint, Sally, Marcie

- Charlie:
- Linus: Marcie
- Schroeder: Lucy
- Franklin:

Men-optimal solution

- Let's find a **men optimal stable marriage**
 - Women preference lists
 - Lucy: Schroeder, Charlie
 - Peppermint: Charlie, Linus, Franklin
 - Marcie: Charlie, Franklin, Linus, Schroeder
 - Sally: Linus, Franklin
 - Men preference lists
 - Charlie: Lucy, Peppermint, Marcie
 - Linus: Marcie, Sally
 - Schroeder: Marcie, Lucy
 - Franklin: Peppermint, Sally, Marcie
- Charlie: Peppermint
 - Linus: Marcie
 - Schroeder: Lucy
 - Franklin:

- Charlie: Peppermint
- Linus: Marcie
- Schroeder: Lucy
- Franklin: ~~Peppermint~~ Sally

Franklin prefers Peppermint the most, but Peppermint prefers Charlie over Franklin

Men-optimal solution

- What happens if at this point **we match first Franklin and then Charlie?**
 - Women preference lists
 - Lucy: Schroeder, Charlie
 - Peppermint: Charlie, Linus, Franklin
 - Marcie: Charlie, Franklin, Linus, Schroeder
 - Sally: Linus, Franklin
 - Men preference lists
 - Charlie: Lucy, Peppermint, Marcie
 - Linus: Marcie, Sally
 - Schroeder: Marcie, Lucy
 - Franklin: Peppermint, Sally, Marcie
- Charlie:
 - Linus: Marcie
 - Schroeder: Lucy
 - Franklin:
- Charlie:
 - Linus: Marcie
 - Schroeder: Lucy
 - Franklin: Peppermint

Men-optimal solution

- What happens if we match Franklin before Charlie?
 - Women preference lists
 - Lucy: Schroeder, Charlie
 - Peppermint: Charlie, Linus, Franklin
 - Marcie: Charlie, Franklin, Linus, Schroeder
 - Sally: Linus, Franklin
 - Men preference lists
 - Charlie: Lucy, Peppermint, Marcie
 - Linus: Marcie, Sally
 - Schroeder: Marcie, Lucy
 - Franklin: Peppermint, Sally, Marcie
- Charlie: Peppermint
 - Linus: Marcie
 - Schroeder: Lucy
 - Franklin: Peppermint
- Charlie: Peppermint
 - Linus: Marcie
 - Schroeder: Lucy
 - Franklin:
- Charlie: Peppermint
 - Linus: Marcie
 - Schroeder: Lucy
 - Franklin: Sally

Wrapping up

Women-optimal stable marriage	Men-optimal stable marriage
(Lucy, Schroeder)	(Charlie, Peppermint)
(Peppermint, Charlie)	(Linus, Marcie)
(Marcie, Franklin)	(Schroeder, Lucy)
(Sally, Linus)	(Franklin, Sally)

Women-optimal: Marcie prefers Franklin to Linus, and Sally prefers Linus to Franklin: this matching is the best for all women among all stable matchings, but the worst for all men

Men-optimal: Linus prefers Marcie to Sally, and Franklin prefers Sally to Marcie: this matching is the best for all men among all stable matchings, and the worst for all women