CALCULUS (Global Governance) A.Y. 2025/26

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Extraordinary Final Exam: 21st May 2026

Last Name:	**First Name:**	**Student's ID:**

Instructions:

- You have 3 hours to answer the following questions. Students who successfully passed the midterms and are exempt from taking Part I have 1.5 hours (If they do not turn in the exam after 1.5 hours, it will be considered a forfeiture of their midterm score and they will be required to complete the entire final exam).
- Report your answers in the table (for Part I) and in the empty space (for Part II) provided. Any work done on draft sheets will not be considered.
- No notes, books, or any other reference materials is allowed during the test. Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

Part I (Multiple choice questions)

Answers

1	2	3	4	5	6	7	8	9	10

1. Compute the following limit

$$\lim_{x\to+\infty} \frac{8x^9 - 4x^8 - 8x + 16}{36x^8 - 6x^7 + 9} =$$

(a) $\frac{16}{9}$

(b) $-\frac{1}{9}$

► (c) $+\infty$

(d) $\frac{2}{9}$

(e) 0

2. At $x = 3$, the function $g(x) = \log(12 - x^2)$ is:

► (a) decreasing and concave

(b) increasing and convex

(c) increasing and concave

(d) neither increasing nor decrasing (i.e. $x = -3$ is a stationary point)

(e) decreasing and convex

3. Determine the solution to the following linear differential equation

$$y'(x) + \frac{1}{x}y(x) = 6,\ y(1) = 0$$

(a) $y(x) = 3\frac{x+1}{x} - 6$

(b) $y(x) = 3\frac{x^2+1}{x} - 6$

(c) $y(x) = 3\frac{x^3-1}{x}$

(d) $y(x) = 3\frac{x-1}{x}$

► (e) $y(x) = 3\frac{x^2-1}{x}$

4. Find the critical point of the function

$$f(x,y) = \exp(x^2 + xy + 3x + y)$$

specifying wheter it is a relative minimum, maximum or saddle point:

(a) $(1,0)$, local minimum

(b) $(1,1)$, local minimum

(c) $(1,-1)$, local maximum

► (d) $(-1,-1)$, saddle point

(e) $(-1,1)$, local maximum

5. Compute

$$\int_0^7 \frac{x^2+7x+2}{x+7}\,dx$$

(a) $\frac{49}{2} - 2\ln 14$

► (b) $\frac{49}{2} + 2\ln 2$

(c) $\frac{7}{2} + \frac{2}{7}\ln 7$

(d) $49 - 2\ln 2$

(e) $49 + 2\ln 4$

6. Which of the following best characterizes the domain of the function $f(x,y) = \frac{1}{\sqrt{4x^2-9y^2}}$?

► (a) $x > \frac{3}{2}|y|$ or $x < -\frac{3}{2}|y|$

(b) $|y| > \frac{2}{3}$

(c) $-\frac{3}{2}|y| < x < \frac{3}{2}|y|$

(d) $x > \frac{3}{2}|y|$ and $x < -\frac{3}{2}|y|$

(e) $-\frac{3}{2}|y| \le x \le \frac{3}{2}|y|$

7. Determine the solution for the following separable differential equation:

$$y'(x) = \frac{e^{-3x}}{\sqrt{y-5}}, \quad y(0) = 6$$

(a) $y(x) = 7 - (\frac{4-e^{-3x}}{3})^{\frac{3}{2}}$

(b) $y(x) = 7 - \frac{5\,e^{-3x}}{\sqrt{x+25}}$

(c) $y(x) = 3\sqrt{3 + (\frac{1+e^{-3x}}{2})^3}$

► (d) $y(x) = 5 + (\frac{3-e^{-3x}}{2})^{\frac{2}{3}}$

(e) $y(x) = \sqrt[3]{6}\,(\frac{13-e^{-3x}}{2})^{\frac{2}{3}}$

8. Compute the following limit

$$\lim_{x\to -1}\left(\frac{-e^{(2x^2+7x+5)}+1}{4x+4}\right) =$$

(a) It does not exist

► (b) $-\frac{3}{4}$

(c) $+\infty$

(d) $-\frac{1}{2}$

(e) 0

9. Find the second order partial derivative f_{yy} of the given function:

$$f(x,y) = \exp\left(3y + \sqrt[4]{x}\right)$$

(a) $(3y+\sqrt[4]{x})^2 \exp(3y+\sqrt[4]{x})$

(b) $3\exp(3y+\sqrt[4]{x})$

(c) $\frac{\exp(3y+\sqrt[4]{x})}{16}\left(x^{-\frac{3}{2}} - 3x^{-\frac{7}{4}}\right)$

► (d) $9\exp(3y+\sqrt[4]{x})$

(e) $\frac{3}{4}\frac{\exp(3y+\sqrt[4]{x})}{x^{\frac{3}{4}}}$

10. The level of radiation in a contaminated area decreases over time. After t hours, the radiation level $R(t)$ in in millisieverts (mSv) is given by $R(t) = 0.1\left(1 + 4\,e^{-0.04t}\right)$. How long (in hours) will it take for the radiation level to reach 0.3 mSv?

(a) $\frac{5}{2}\ln(4)$

(b) $\frac{25}{2}\ln(3)$

(c) $\frac{5}{2}\ln(30) - \frac{1}{4}$

► (d) $25\ln(2)$

(e) It will never reach this concentration.

Part II (Problems)

Problem 1. Study the following function and sketch its graph:

$$g(x) = \frac{1-x}{6-x-x^2}.$$

In particular, address the following aspects: domain, intersection with axes, continuity, positivity and negativity, asymptotes (vertical, horizontal or oblique), intervals of monotonicity (increase or decrease), local maxima and minima (the study of the concavity is optional).

Solution:

- **Domain and continuity**

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- **Intersection with axis and positivity/negativity**

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- **Asymptotes**

- **First derivative, interval of monotonicity, local maxima/minima**

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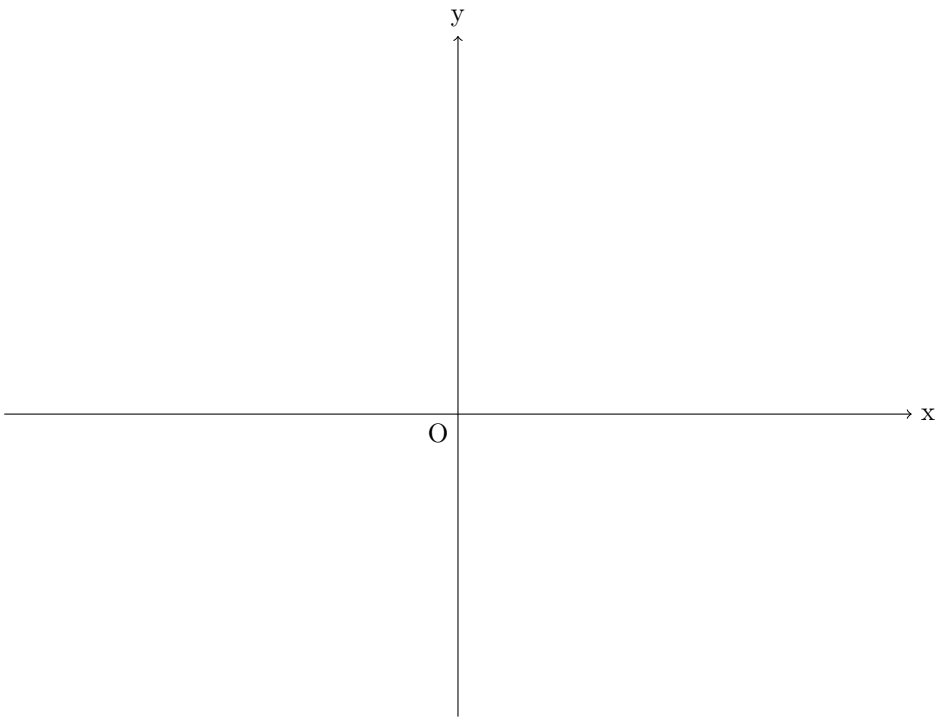
- **Graph**



Problem 2. A drug is delivered to a patient through an intravenous injection. The concentration of the drug in the patient's bloodstream (in mg/L) at time h hours following the injection follows the function:

$$F(h) := 2\left(\frac{h+2}{\sqrt{h^2+2}} - 1\right).$$

(a) After how many hours does the maximum concentration occur, and what is the value of this maximum?

(b) At what time is the concentration decreasing most rapidly (*i.e.*, when is its rate of change most negative)?

Solution