

CALCULUS
GLOBAL GOVERNANCE
A.Y. 2025/26

Recitation VIII

Exercise 1 (Critical points)

Find and classify the critical points of the following functions:

(a) $f(x, y) = 2x^2 + y^2 - 2xy - 2x + 1$, (b) $g(x, y) = e^{xy-x}$,

(c) $h(x, y) = y^3 + 3xy^2 + 3x^2y - 3y + 2$

Exercise 2 (Constrained optimization)

- (1) Find the maximum value of $f(x, y) = xy$ subject to the constraint $x + y = 1$.
- (2) Find the minimum value of $f(x, y) = x^2 + y^2$ subject to the constraint $xy = 1$.
- (3) Find the maximum and minimum values of $f(x, y) = xy$ subject to the constraint $x^2 + y^2 = 1$.

Exercise 3 (Application: optimizing an outcome)

Suppose that a combination of a carbon tax of T euros/ton and subsidies of S million euros are predicted to prevent

$$E(T, S) = 10T + 15S - 0.5T^2 - 0.3S^2 + 0.2TS$$

million tons of CO_2 per year.

What policy (that is, what level of subsidies and carbon tax) gives the largest reduction in emissions?

Exercise 4 (Application: minimizing cost for fixed production)

Suppose a certain company produces

$$P(K, L) = 10K^{2/5}L^{3/5}$$

tons of steel when it has K units of capital and L units of labor.

If capital costs €200 per unit and labor costs €100 per unit, how can the company make 1000 tons of steel at the lowest possible cost?