CALCULUS (Global Governance) A.Y. 2025/26

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FIRST FINAL EXAM: 19th JANUARY 2026

Last Name:	**First Name:**	**Student's ID:**

Instructions:

- You have 3 hours to answer the following questions. Students who successfully passed the midterms and are exempt from taking Part I have 1.5 hours (If they do not turn in the exam after 1.5 hours, it will be considered a forfeiture of their midterm score and they will be required to complete the entire final exam).
- Report your answers in the table (for Part I) and in the empty space (for Part II) provided. Any work done on draft sheets will not be considered.
- No notes, books, or any other reference materials is allowed during the test. Electronic devices, including computers, tablets, and cell phones, are strictly prohibited. You are only permitted to use a non-scientific and non-graphing calculator.

Part I (Multiple choice questions)

Answers

1	2	3	4	5	6	7	8	9	10
D	A	B	D	A	A	A	B	D	B

1. Find the second order partial derivative f_{xy} of the given function:

$$f(x,y) = y\left(x^6 + \cos y\right)^2$$

(a) $12x^{11} + 12x^5 \sin y + 12x^5 y \cos y$

(b) $-12x^5 \sin y$

(c) $12y^{11} + 12y^5 \cos x + 12xy^5 \cos x$

► (d) $12x^{11} + 12x^5 \cos y - 12x^5 y \sin y$

(e) $12x^5 \cos y$

2. Compute the following limit:

$$\lim_{x\to+\infty} e^{-(x^2+4)}\left(\frac{4x^3+x+4}{x^3+1}\right) =$$

► (a) 0

(b) 4

(c) $4e^{-4}$

(d) $+\infty$

(e) None of the above

3. Compute the following limit

$$\lim_{x\to 2}\left(\frac{-e^{(-3x^2+15x-18)}+1}{3x-6}\right) =$$

(a) 0

► (b) -1

(c) $-\frac{2}{3}$

(d) $+\infty$

(e) It does not exist

4. Compute the following definite integral:

$$\int_1^2 6x\ln(4x)\,dx =$$

(a) $\frac{9}{2} - 3\ln 16 + 3\ln 4$

(b) $6\ln 16 - \frac{9}{2}\ln 4$

(c) $3\ln 16 - 3\ln 4$

► (d) $-\frac{9}{2} + 3\ln 16 - 3\ln 4$

(e) $-9 + 9\ln 4$

5. Determine the solution to the following linear differential equation

$$y'(x) + \frac{1}{x}y(x) = 6, \; y(1) = 0$$

▶ (a) $y(x) = 3\frac{x^2-1}{x}$

(b) $y(x) = 3\frac{x^3-1}{x}$

(c) $y(x) = 3\frac{x^2+1}{x} - 6$

(d) $y(x) = 3\frac{x-1}{x}$

(e) $y(x) = 3\frac{x+1}{x} - 6$

6. A substance is released into a solution, and the concentration of the substance t seconds later, is given by $S(t)$ in grams per cubic centimeter (g/cm^3), where $S(t) = 0.1\left(1 + 2\,e^{-0.03t}\right)$. How much time (in seconds) will it take for the concentration to reach 0.15 g/cm^3 ?

▶ (a) $\frac{100}{3}\ln(4)$

(b) $5\ln(15) - \frac{1}{2}$

(c) $\frac{50}{3}\ln(5)$

(d) $\frac{10}{3}\ln(3)$

(e) It will never reach this concentration.

7. Which of the following best characterizes the domain of the function $\log(x^2 - y)$?

▶ (a) $x^2 > y$

(b) $y < 0$ or $x > 0$

(c) $y < 0$ and $x \neq 0$

(d) $x^2 \geq y$

(e) $y \leq 0$

8. At $x = -1$, the function $g(x) = x^3 + 6x^2 + 10x + 4$ is:

(a) increasing and concave (concavity down)

▶ (b) increasing and convex (concavity up)

(c) neither increasing nor decreasing (i.e. $x = -2$ is a stationary point)

(d) decreasing and concave (concavity down)

(e) decreasing and convex (concavity up)

9. Determine the solution to the following separable differential equation:

$$y'(x) = \frac{y-5}{x+25}, \quad y(5) = 0$$

(a) $y(x) = \frac{1}{2} - \frac{x}{10}$

(b) $y(x) = 1 - \frac{x}{5}$

(c) $y(x) = \frac{x}{6} - \frac{5}{6}$

▶ (d) $y(x) = \frac{5}{6} - \frac{x}{6}$

(e) $y(x) = -1 + \frac{x}{5}$

10. Find the critical point of the function

$$f(x, y) = \exp(x^2 + y^2 - 2y)$$

specifying wheter it is a relative minimum, maximum or saddle point:

(a) $(1, 0)$, local minimum

▶ (b) $(0, 1)$, local minimum

(c) $(0, 1)$, local maximum

(d) $(1, 0)$, local maximum

(e) $(1, 0)$, saddle point

Part II (Problems)

Problem 1. Study the following function and sketch its graph:

$$f(x) = \frac{6x^2 + 10x}{x^2 - 5}.$$

In particular, address the following aspects: domain, intersection with axes, continuity, positivity and negativity, asymptotes (vertical, horizontal or oblique), intervals of monotonicity (increase or decrease), local maxima and minima (the study of the concavity is optional).

Solution:

- **Domain and continuity**

DOMAIN: $x^2-5 \neq 0 \iff x \neq \pm\sqrt{5}$ $\qquad \mathbb{D} := \mathbb{R} \setminus \{\pm\sqrt{5}\}$

THE FUNCTION IS CONTINUOUS AT EVERY POINT IN ITS DOMAIN (IT IS A RATIONAL FUNCTION)

- **Intersection with axis and positivity/negativity**

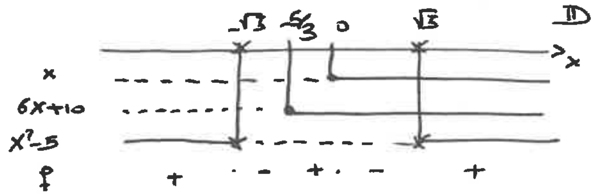
INTERSECTIONS: X-AXIS: $f(x)=0 \iff 6x^2+10x=0 \iff x(6x+10)=0 \iff x=0,\ -\frac{5}{3}$

$A=(-\frac{5}{3},0) \quad B=(0,0)$

Y-AXIS: $f(0)=0 \implies B=(0,0)$

STUDY OF THE SIGN:

N. $6x^2+10x = x(6x+10)$



$f>0$ IN $(-\infty,-\sqrt{5}) \cup (-\frac{5}{3},0) \cup (\sqrt{5},\infty)$

$f<0$ IN $(-\sqrt{3}, -\frac{5}{3}) \cup (0, \sqrt{5})$

- **Asymptotes**

VERTICAL: $x = \pm\sqrt{5}$ $\qquad \lim_{x\to\sqrt{5}^\pm} f(x) = \pm\infty \qquad \lim_{x\to-\sqrt{5}^\pm} f(x) = \mp\infty$

HORIZONTAL: $\lim_{x\to\pm\infty} f(x) = 6 \implies y=6$ HORIZONTAL ASYMP. AT $\pm\infty$

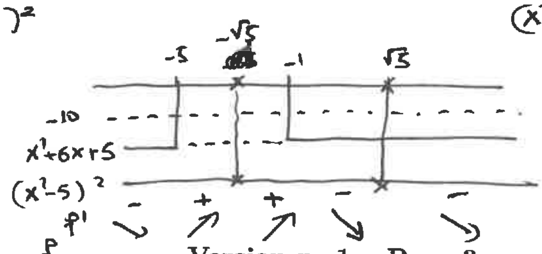
- **First derivative, interval of monotonicity, local maxima/minima**

$$f'(x) = \frac{(12x+10)(x^2-5) - (6x^2+10x)2x}{(x^2-5)^2} = -10\,\frac{(x^2+6x+5)}{(x^2-5)^2}.$$

SIGN OF $f'(x)$:

$x^2+6x+5=0$

$\iff x = \frac{-6\pm\sqrt{36-20}}{2} = \frac{-6\pm 4}{2}$ $\quad -5,\ -1$



$f' > 0 \quad (-5, -\sqrt{5}) \cup (-\sqrt{5}, -1) \quad f$ increasing

$f' < 0 \quad (-\infty, -5) \cup (-1, +\sqrt{5}) \cup (+\sqrt{5}, +\infty) \quad f$ decreasing

$x = -5$ LOCAL MINIMUM $\quad f(-5) = 5 \quad C = (-5, 5)$

$x = -1$ LOCAL MAXIMUM $\quad f(-1) = 1 \quad D = (-1, 1)$

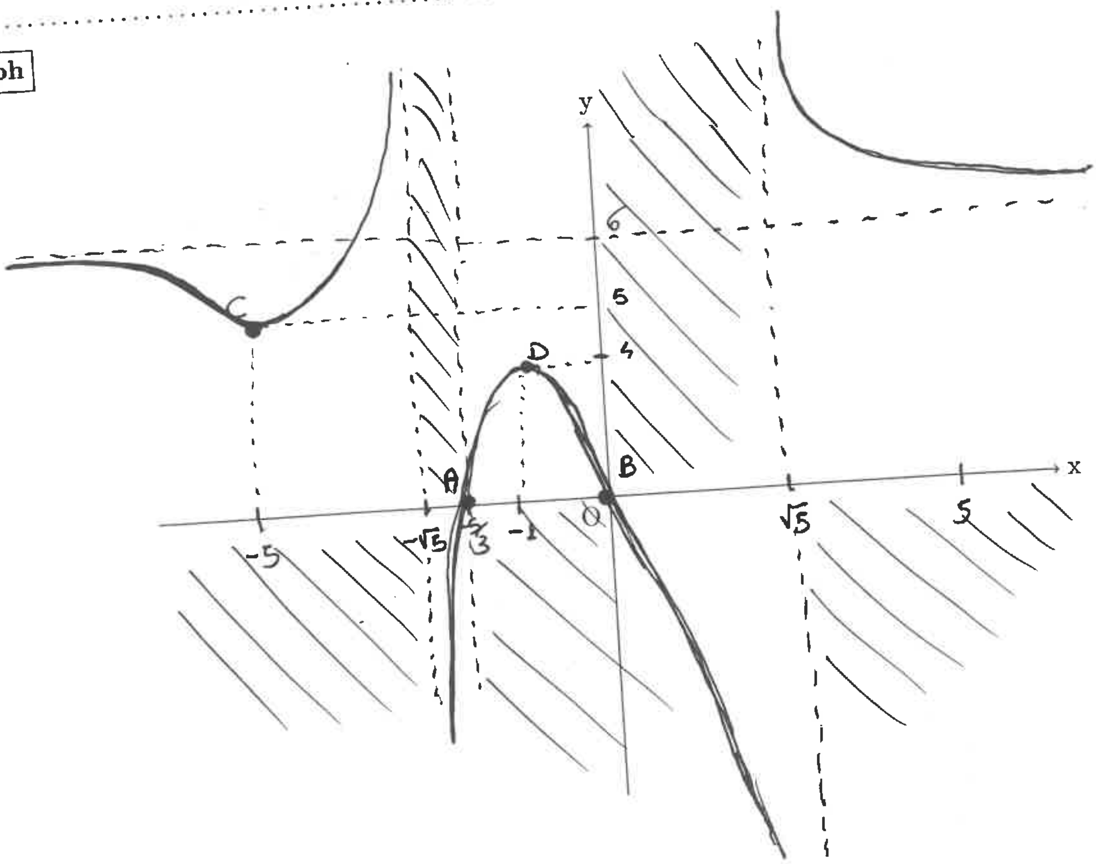
- **Second derivative and concavity**

$$f''(x) = -10 \left[\frac{(2x+6)(x^2-5)^2 - (x^2+6x+5)\, 2 \cdot (x^2-5)\, 2x}{(x^2-5)^4} \right]$$

$$= 20 \left[\frac{x^3 + 9x^2 + 15x + 15}{(x^2-5)^3} \right]$$

(We do not study its sign)

- **Graph**



Problem 2. After consuming an alcoholic drink, a person's blood alcohol concentration (in mg/dL) is approximated by

$$A(h) := \frac{4+2h}{\sqrt{h^2+2}} - 2.$$

where h is in hours since ingestion.

(a) After how many hours does the blood alcohol concentration reach its peak, and what is the value of this maximum value?

(b) After how many hours is the blood alcohol concentration decreasing most rapidly (*i.e.*, when is its rate of change most negative)?

Solution $h \geq 0$

(a) $$A'(h) = \frac{2\sqrt{h^2+2} - (4+2h)\frac{\cancel{2}h}{\cancel{2}\sqrt{h^2+2}}}{(h^2+2)} = \frac{2h^2+4-4h-2h^2}{(h^2+2)^{3/2}} = \frac{4-4h}{(h^2+2)^{3/2}}$$

$A'(h)=0 \iff h=1$

h	0		1		
A':		$+$		$-$	
A:		$\nearrow$	$\bullet$	$\searrow$	

$h=1$ IS A GLOBAL MAXIMUM (unique critical point in $(0,+\infty)$)

$\boxed{\text{PEAK AT } h=1 \text{ hour}}$

$A(1) = \frac{6}{\sqrt{3}} - 2 = 2\sqrt{3}-2$ (mg/dL) $\leftarrow$ $\boxed{\text{MAXIMAL CONCENTRATION}}$

(b) $$A''(h) = \frac{-4(h^2+2)^{3/2} - (4-4h)\frac{3}{\cancel{2}}(h^2+2)^{1/2}\cancel{2}h}{(h^2+2)^3} = \frac{-4(h^2+2)-3h(4-4h)}{(h^2+2)^{5/2}}$$

$$= \frac{-4h^2-8-12h+12h^2}{(h^2+2)^{5/2}} = \frac{8h^2-12h-8}{(h^2+2)^{5/2}} = \frac{4(2h^2-3h-2)}{(h^2+2)^{5/2}}$$

$A''(h)=0 \iff 2h^2-3h-2=0 \iff h = \frac{3\pm\sqrt{9+16}}{4} = \frac{3\pm 5}{4}$; $\frac{8}{4}=2>0$; $\frac{-2}{4} = -\frac{1}{2} < 0$

Sign of A'':

h	0		2		
A''					
A'		$\searrow$		$\nearrow$	

$\nwarrow$ global min pt of A'

$A'(2) = \frac{4-8}{(8)^{3/2}} = \underbrace{-\frac{4}{8^{3/2}}}_{=} < 0$ $\leftarrow$ $\boxed{h=2 \text{ point of most rapid decrease.}}$

$\left[-2^{2-\frac{9}{2}} = -(2^{-5/2}) = \frac{-1}{4\sqrt{2}}\right]$

$\boxed{\text{maximal rate of decrease: } A'(2) = -\frac{1}{4\sqrt{2}}}$