Advanced Microeconomics 2025/2026

Master of Science in Economics

Problem Set 4

28/11/2025

Question 1 Consider $u(x_1, x_2) = (x_1 - b_1)^{\alpha}(x_2 - b_2)^{1-\alpha}$ with $b_i > 0$ for $i = 1, 2$ and the utility being positive only when $x_1 > b_1$ and $x_2 > b_2$.

1. Derive the Hicksian demand and the expenditure functions.
2. Verify the homogeneity of degree one of $e(p, u)$ in p.
3. Verify the concavity of $e(p, u)$ in p and check that the compensated own-price effects are negative and that the compensated cross-price effects are symmetric for all commodities.

Question 2 Consider an economy with $L = 2$. Let the expenditure function for the price profile $(p_1, p_2) >> 0$ and the utility level $\overline{u} \geq 0$, be given by

$$e(p, \overline{u}) = p_1^{\alpha_1} p_2^{\alpha_2} \exp \overline{u},$$

in which $\alpha_1, \alpha_2 \geq 0$ and $\alpha_1 + \alpha_2 = 1$.

1. Find the Hickisian demand of every good.
2. Compute the associated indirect utility function. How does it vary with respect to w?
3. Which property/properties did you use to answer the previous questions?

Question 3 A consumer in an economy with three commodities, (x_1, x_2, x_3), with prices p_1, p_2 and p_3 respectively, and wealth $w > 0$, has demand functions for commodities 1 and 2 given by:

$$x_1 = 100 - 5\,\frac{p_1}{p_3} + \beta\,\frac{p_2}{p_3} + \delta\,\frac{w}{p_3}$$
$$x_2 = \alpha + \beta\,\frac{p_1}{p_3} + \gamma\,\frac{p_2}{p_3} + \delta\,\frac{w}{p_3}$$

where Greek letters are non-negative constants.

1. Indicate how to compute the demand for x_3, but do *not* do it.
2. Are the demands functions for x_1 and x_2 appropriately homogeneous?
3. Calculate the restrictions on the values of α, β, γ and δ implied by utility maximisation.
4. Given your results in $(3.)$, for fixed level of x_3 draw the consumer's indifference curves in the (x_1, x_2)-Cartesian space.
5. What does your answer to $(4.)$ imply about the form of the consumer's utility function $u(x_1, x_2, x_3)$?

Question 4 A function is of the Gorman form if $v(p, w) = a(p) + b(p)w$.

1. Which properties will the functions $a(p)$ and $b(p)$ have to satisfy for $v(p, w)$ to qualify as an indirect utility?
2. Verify that an indirect utility function of the Gorman form exhibits linear wealth expansion paths.