

Lecture II

a story from Kahneman

"As you consider the next question, please assume that Steve was selected at random from a representative sample: An individual has been described by a neighbor as follows: "Steve is very shy and withdrawn, invariably helpful but with little interest in people or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail." Is Steve more likely to be a librarian or a farmer?"

The resemblance of Steve's personality to that of a stereotypical librarian strikes everyone immediately, but equally relevant statistical considerations are almost always ignored. Did it occur to you that there are more than 20 male farmers for each male librarian in the United States? Because there are so many more farmers, it is almost certain that more "meek and tidy" souls will be found on tractors than at library information desks. However, we found that participants in our experiments ignored the relevant statistical facts and relied exclusively on resemblance. We proposed that they used resemblance as a simplifying heuristic (roughly, a rule of thumb) to make a difficult judgment. The reliance on the heuristic caused predictable biases (systematic errors) in their predictions."

Predictions: people make a forecast as extreme as the signal when they should use an average of the signal and the unconditional mean. Notice I said yesterday that people think a random walk is mean reverting. I guess this amounts, so far, to saying that people don't know what the experimenter knows (of course). Kahneman insists there is more going on – that people have a very firm effortless immediate sense if something is as extreme in one way as something else is in another – so we have a sense of someone who is as beautiful as Messi is able at calcio and things like that which make no sense.

We have a quick effortless sense of what is typical, ordinary, and average. This is a (very simple) figure from the book

Draw a line about the length of the average line segment in the figure.

No problem (and the lengths will be similar and not too far from the mathematical average)

Now draw a line the length of the sum of the lengths.

Huh ?!?

How many segments are there ? Not hard but *not* as effortless as estimating the average.

We definitely do not calculate the average as the sum divided by the number – not at all.

Approximate Calculations with real numbers can be easier than with integers. We effortlessly do things which require powerful computers and can't multiply two 4 digit numbers in our heads.

Herbert Simon on artificial intelligence "it is amazing how easy the hard things are and how hard the easy things are". We made artificial intelligence to do arithmetic for us (this is the history) because we did not evolve to do arithmetic.

This makes us unable to do the simplest calculations of probabilities and unwilling to even try. It means that we do not make choices when facing risks which maximize any expected value of a function of the final

outcome. This is Kahneman and Tversky's second pathbreaking article (still in the 70s). It has spawned a field of research (which has less to do with macroeconomics than the trouble forecasting time series).

It is called prospect theory. The key insight is due to Nobel Prizewinner Maurice Allais who contested Von Neuman and Morgenstern and argued that people do not maximize the expected value of a function of gambling winnings (so do not maximize expected utility for any reasonable utility function). This article is interesting for three reasons. OK first it is very important and pathbreaking. Second it is a reminder that, in theory, *Econometrica* is a bilingual journal which publishes in English and French (it is written in French and is the only article written in French that I have ever seen in *Econometrica*). Finally it shows a weak grasp of geography as he refers to Von Neuman and Morgenstern's theory as the "American" hypothesis when it is, in fact, Austro-Hungarian.

The point is that people put more weight on unlikely extreme outcomes than they would if they maximized the expected value of a function of winnings. To us the difference between 99% and 100% is much more important than the difference between 89% and 90%.

This means that people chose an 89% chance of a big prize over a 90% chance of a smaller prize but choose a 100% chance of the smaller prize over a 99% chance of the bigger prize. This is not consistent with maximizing a function of the money they take from the experiment. The reason is as follows. Let's say people are told they will have a 90% chance of good luck and they get to choose a lottery and a 10% chance of bad luck and they leave with 0. Then the lottery is a choice between 100% small prize and 99% big prize. They say they want the 100% chance. Or they are told they can participate in a lottery with an 89% chance of a big prize or one with a 90% chance of the small prize. Then they choose the 89% chance.

But the first is a choice between a $0.99 \times 0.9 = 0.891$ chance and a $1 \times 0.9 = 0.9$ chance. So 89% over 90% and (without thinking it through) 90% over 89.1%. Ooops. Part of the way Allais got so much attention is that he asked this question at conferences on choice under uncertainty and asked people who were about to present models on expected utility maximization (I don't know if he ever tricked Von Neuman but I very much doubt it as I know of no record of anyone outsmarting Von Neuman ever).

Kahneman and Tversky describe this as overweighting low probabilities. Here (for the first and last time) I criticize Kahneman. That's not quite right. People are not messed up by something like treating a 10% chance of a Euro as worth less than a 5% chance of 99 cents and 5% chance of 101 cents.

It is overweighting low probabilities of extreme outcomes. It is a statement about how we can't handle cumulative probability distributions (funzioni di partizione) not about how we can't handle probability densities. This was explained by John Quiggin (who is really smart) but it's still called prospect theory not cumulative prospect theory.

There is one possible explanation of the Allais paradox which definitely starts with a true statement about psychology (made by K and T). The true statement is called loss aversion. When gambling we care a lot about whether our winnings are positive or negative – that is if we win or lose overall. This is not rational – in this case 0 is not an especially important number. Clearly not rational behavior can be elicited by describing experiments with the exact same distribution of final outcomes differently.

Description 1 is you are paid 1 euro to participate and then have a choice between a high chance of a small 5 euros or a low chance of a large prize 9 Euros. Numbers chosen so most people choose the high chance.

Description 2 is you are paid to 10 euros to participate but then must choose between a high chance of having to pay back 5 euros or a smaller chance of having to pay back 9. then people chose the smaller chance.

This can't be maximizing any function of the amount of money they take away from the experiment. Their choices depend on how the different options are described (a "framing" effect) and not the distribution of final winnings (which are exactly the same with the two different "framings")

This can be seen in behavior when people who have lost like to bet double or nothing while people who have won do not.

OK back to Allais, the story could be that people think that if they get the good 90% chance they have won something of value and if they take the 99% probability they risk having something then losing it. This is painful. With the 89% vs 90% choice there is no risk of having then losing. Importantly, people make the same choice even if they are told they will not ever know if they had the good 90% probability outcome then the bad 1% or if they had the bad 10% outcome. People don't have to worry about knowing they had something but then they gambled it away.

I have 2 stories. One is the proof beyond reasonable doubt matters story. If someone says 100% then it doesn't happen we *know* for sure that his statement was false. If he says 99% we strongly suspect but won't be able to get him convicted because there is not proof beyond reasonable doubt. In the experiments people are dealing with honest psychologists who don't cheat them. In real life, we have to consider if someone might be a cheat who will get away with it. This is related to the next and best example of irrational choice under uncertainty (which is supported by massive massive real world data set involving prizes are economically significant value – also I have witnessed the experiment in real time).

There was this TV game show called "Let's Make a Deal" starring Monte Hall. Not the pinnacle of Western culture. The show had contestants answer quiz questions and Monte would then propose deals. He was slimy (untuoso) and the deals were not good deals (eg a mink (visone) turned out to be a mink hot water bottle). Monte in English is short for montebank. The alleged humor was that the guy was playing an obvious crook. This may be relevant to the real life experiment.

At the end of the show, the person with the most points had a chance to win a big prize (on the order of a new automobile). There were 3 large boxes on the stage one of which contained the big prize (the other two contained smaller prizes). The contestant indicated a box, let's say box 2. Monte Hall would have Carol Merrill open one of the other 2 boxes, let's say box 1. Then he would ask the contestant if the contestant wanted to stick to box 1 or switch and chose box 3. The contestant hardly ever switched (I never saw it happen). This demonstrates human failure to understand probability.

The contestants won the big prize about 1 time out of 3 (clearly the probability if one guesses out of 3 with no useful information). If they had switched they would have won 2 times out of 3. If you doubt this, I can prove it various ways (and even if you don't doubt it, I will prove it various ways). One simple way is to consider flipping a coin and changing if it comes up heads. Clearly the chance of winning if one does this is 0.5. It must be the average of the chance if one does not switch ($1/3$) and the chance if one does switch. Algebra now tells us that that chance is $2/3 > 1/3$.

The harder way is to use Bayes formula – people do not believe Bayes formula, do not understand it, consider it black magic, and it is the first thing one teaches about probability and statistics.

The correct calculation of a conditional probability is probability of x given signal s is proportional to the unconditional probability of x times the probability of s conditional on x .

So if the big prize were in box 3, then Monte Hall would definitely certainly have box 1 open (always a box with a small prize was opened – always). If the big prize were in box 2 the Monte Hall could have box 1 opened or box 3 opened. So the chance of box 1 getting open given that the big prize is in box 2 is $\frac{1}{2}$

so the probability that the big prize is in box 2 is $(\frac{1}{3})(\frac{1}{2}) / ((\frac{1}{3})(\frac{1}{2}) + (\frac{1}{3})) = \frac{1}{3}$

we know that.

The probability that the big prize is in box 3 is $(\frac{1}{3}) / ((\frac{1}{3})(\frac{1}{2}) + (\frac{1}{3})) = \frac{2}{3} > \frac{1}{3}$

The show was broadcast about every day for years. The number of big prize winners is known. The number who would have won if they switched is the total number of times it was done minus the number of winners. It is about twice the number of winners. This is very clear. A lot of wealth was left in the possession of the network which people could have taken.

This happened in front of my eyes *and* as I watched I thought it was unwise to switch. After I got my PhD my PhD supervisor asked me about it. I got the answer wrong. He said “oops I forgot to tell you there is one big prize”. He did forget, but I knew that, because I had watched the show (and I admit it). I didn’t explain to him that, no I messed up Bayes formula, because he is a smart guy who thought I was smart and I was embarrassed (and I admit that too – see how honest I am).

OK one thing going on is that Bayes formula is correct (I mean to apply it correctly you have to have probabilities and conditional probabilities but if they are correct, the formula can be tested as a hypothesis and is not rejected by the data). Human beings can not become comfortable with Bayes formula. It feels wrong and weird.

It is an explanation of probabilities which is not of the form, p is true, p causes q to be likely, so q is probably true. It is about selection after causation has already occurred (in which box was the big prize put).

There are 2 other things. One is loss aversion – it is very painful to have had the big prize then lost it. Another is cheater detection – this occurred after a whole show (half an hour an hour I forget) of Monte Hall tricking people. The lesson was do not accept his proposals.

I think we see many things. One is we are comfortable with stories about causation and probabilities based on chains of causation ending up with “and this causes the outcome which is therefore probable”. Bayes formula is counter intuitive. It remains counter intuitive no matter how many times we see it works (it remains weird to me). We have social intelligence – this guy seems honest or dishonest. It makes us respond to acting – even Monte Hall’s very clumsy acting.