1. Consider the following version of the human capital model. The income generating function is of the form:

$$Y(S_i) = \exp\left(b_i S_i - \frac{S_i^2}{2}\right)$$

where $b_i > 0$, S_i is the schooling level chosen by individual i and Y is income. The costs of education are determined by the following function:

$$g(S_i) = c + r_i S_i$$

where $c > 0$ and $r_i > 0$. Individuals choose S to maximize utility: $U(S)=log[Y(S)] - g(S)$.

a) What is the optimal level of schooling S^* that individuals choose in this model?

The optimal level of schooling is given by maximizing the following:

$$\left(\boldsymbol{b_i S_i - \frac{S_i^2}{2}}\right) - \boldsymbol{c - r_i S_i}$$

The first order condition is

$$\boldsymbol{b_i - S_i - r_i = 0}$$

Which gives

$$\boldsymbol{S_i^* = b_i - r_i}$$

b) Given the optimal choice of S, what is the return to schooling $\beta^* = \frac{Y'(S)}{Y(S)}$?

$$\boldsymbol{\beta^* = \frac{Y'(S)}{Y(S)} = \frac{\exp\left(b_i S_i - \frac{S_i^2}{2}\right)(b_i - S_i)}{\exp\left(b_i S_i - \frac{S_i^2}{2}\right)} = b_i - S_i^* = b_i - (b_i - r_i) = r_i}$$

c) Now, interpret the variable S so that individual attends college if $S > 0$ and only gets the compulsory education if $S \leq 0$. Assume that parameters r and b are distributed in the population in the following way:

	b_i	
r_i	**0**	**1**
0	20%	30%
1	30%	20%

How large a fraction of the population attends college in this economy? Suppose that the government introduces a policy that reduces r_i to 1/2 for those whose r_i was previously 1. How large a fraction of the population will attend college as a result of this reform?

Prior to the policy change, S is positive for the part of the population that has b=1 and r=0, that is 30% of the population. After the policy change, S is also positive for the part of the population that has b=1 and r=1. Now, 30% + 20% = 50% of the population attends college.

2. Suppose the firm's production function is given by

$$q = 10\sqrt{E_w + E_b}\ ,$$

where E_w and E_b are the number of whites and blacks employed by the firm respectively.

Suppose the market wage for black workers is \$10, the market wage for whites is \$20, and the price of each unit of output is \$100.

(a) How many workers would a firm hire if it does not discriminate? How much profit does this non-discriminatory firm earn if there are no other costs?

The marginal product of labor is

$$MP_E = \frac{5}{\sqrt{E_w + E_b}}.$$

There are no complementarities between the types of labor as the quantity of labor enters the production function as a sum, E_w + E_b. Further, the market-determined wage of black labor is less than the market-determined wage of white labor. Thus, a profit-maximizing firm will not hire any white workers and will hire black workers up to the point where the black wage equals the value of their marginal product:

$$\mathbf{10 = w_b \text{ is set equal to }} VMP_B = p \times MP_E = \frac{100(5)}{\sqrt{E_b}}$$

which yields E_b = 2,500. The 2,500 black workers produce q = 10(sqrt(2,500)) = 500 units of output, and profits are:

$$\Pi = pq - w_b E_b = 100(500) - 10(2{,}500) = \$25{,}000.$$

(b) Consider a firm that discriminates against blacks with a discrimination coefficient of .25 (such that the adjusted cost of hiring a black worker is $w_b(1 + d)$). How many workers does this firm hire? How much profit does it earn?

The firm acts as if the black wage is $w_b(1 + d)$, where d is the discrimination coefficient. The employer's hiring decision, therefore, is based on a comparison of w_w and $w_b(1 + d)$. The employer will then hire whichever input has a lower utility-adjusted price. As d = 0.25, the employer is comparing a white wage of \$20 to a black (adjusted) wage of \$12.50. As \$12.50 < \$20, the firm will hire only blacks.
As before, the firm hires black workers up to the point where the utility-adjusted price of a black worker equals the value of marginal product, or

$$12.50 = \frac{100(5)}{\sqrt{E_b}}$$

so that E_b= 1,600 workers. The 1,600 workers produces 400 units of output, and profits are

$$\Pi = 100(400) - 10(1{,}600) = \$24{,}000.$$

(c) Finally, consider a firm that has a discrimination coefficient equal to 1.25. How many workers does this firm hire? How much profit does it earn?

As d = 1.25, the employer compares a white wage of \$20 against a black wage of \$22.50. Thus, the firm hires only whites. The firm hires white workers up to the point where the price of a white worker equals the value of marginal product:

$$20 = \frac{100(5)}{\sqrt{E_w}}$$

so the firm hires 625 whites, produces 250 units of output, and earns profits of

$$\mathbf{\Pi = 100(250) - 20(625) = \$12{,}500.}$$

3. Consider the following wage distribution and income and payroll tax schedule. Suppose 40 percent of households earn $30,000, 30 percent earn $60,000, 20 percent earn $120,000, and 10 percent earn $400,000. Marginal income tax rates are 0 percent up to $25,000, 15 percent on income earned from $25,000 to $50,000, 25 percent on income earned from $50,000 to $150,000, and 40 percent on income earned in excess of $150,000. There is also an 8% percent payroll tax on all income up to $90,000.

   a) What is the marginal tax rate and average tax rate for each of the four types of households? What percent of the total income tax is paid by each of the four types of households? What percent of the total payroll tax bill is paid by each of the four types of household?

Group	Share	Income	MTR	Income Tax	Payroll Tax	Average tax rate	Income tax share	Payroll tax share
A	0.4	30,000	23%	750	2,400	10.5%	0.48%	11.11%
B	0.3	60,000	33%	6,250	4,800	18.4%	3.98%	22.22%
C	0.2	120,000	25%	21,250	7,200	23.7%	13.54%	33.33%
D	0.1	400,000	40%	128,750	7,200	34.0%	82.01%	33.33%

   b) What is the Gini coefficient for the economy before taxes? What is the Gini coefficient after taxes? [Hint: assume there are 1,000 households in the economy].

   c) What happens to the after-tax Gini coefficient if all taxes were replaced by a single 20 percent flat tax on all incomes?

Group	Share	Pre-tax income	Cumulative share of pre-tax income	After-tax income (original tax schedule)	Cumulative share of after tax income	After-tax income new schedule	Cumulative share of after tax income
A	0.4	30,000	0.1277	26,850	0.1531	24,000	0.1277
B	0.7	60,000	0.3191	48,950	0.3625	48,000	0.3191
C	0.9	120,000	0.5745	91,550	0.6235	96,000	0.5755
D	1	400,000	1	264,050	1	320,000	1

The pre-tax Gini coefficient is equal to:

1-2*(0.4*0.1277/2 + 0.3*(.1277+(.3191-.1277)/2) + 0.2*(.3191+(.5755-.3191)/2) + 0.1*(.5745+(1-.5745)/2))=0.479

The after-tax Gini coefficient is:

1-2*(0.4*0.1531/2 + 0.3*(.1531+(.3625-.1531)/2) + 0.2*(.3625+(.6235-.3625)/2) + 0.1*(.6235+(1-.6235)/2))=0.425

If the current tax-schedule were replaced by a flat tax, the shares of income accruing to each group would be equal to the shares in pre-tax income, and hence the Gini coefficient would be 0.479.

d) How would your answer to part (c) change if you were to also take into account the changes in labor supply and earnings of the 4 different groups in response to the changes in the tax schedule? State clearly any assumptions you make to obtain your answer.

If we were to take into account the labor supply response to the change in tax-schedule, the total earnings of every income group would increase (relative to the original tax schedule) because the marginal tax rate decreases for every group. Without further information, it is impossible to say what will happen to the Gini coefficient. Presumably, though, since the biggest change in the marginal tax rate is experienced by the top income group, it is likely that they would have the largest labor supply response, and therefore the Gini coefficient would increase even further.

4. A researcher finds that earnings of college graduates are positively correlated with Grade Point Average (GPA) in college.

   Economist A: "These results are not surprising. Students with higher GPA have learned more material in school, and therefore are more productive workers. This is consistent with human capital theory."

   Economist B: "I don't think this is the explanation for the finding. The positive correlation is just due to signaling."

   a) Explain Economist B's claim.

      More able students have lower costs of "producing" a high GPA in college. Therefore, there can be a separating equilibrium in which high ability students put high effort in college, obtain high grades to signal that they are high ability, and are rewarded with higher wages, while low ability students put in low effort, have lower grades and lower wages.

   b) Assume that some students work in occupations that are highly compatible with their major field of study in college (e.g., journalist majors work as journalists, drama majors are involved in theater production, etc.), while others work in incompatible occupations (English majors as economic consultants). If the human capital theory (Economist A's theory) is correct, would you expect to observe a stronger correlation between earnings and GPA for workers in compatible or incompatible occupations?

      We would expect to observe a stronger correlation for workers in compatible occupations. A high GPA in classical studies is unlikely to have much of an effect on the productivity of an economic consultant.

   c) Assume that some students attend selective colleges, while others attend less selective colleges. If the signaling theory (Economist B's theory) is correct, would you expect to observe a stronger correlation between earnings and GPA for workers that attended more selective or less selective colleges?

      We would expect to observe a stronger correlation in less selective colleges. College selectivity is likely to convey some information about worker ability. Students attending highly selective colleges can already signaled to employers that they are high ability, the value of the high GPA is less strong.

   d) Assume that you have data for a sample of recent college graduates, on log earnings, a student's GPA in college, an index of compatibility between the student's major and his occupation, and a measure of the selectivity of the college. Write down a regression equation that would allow you test the theories of Economists A and B. Be specific about the hypotheses to be tested, and how you would implement the test.

      I would estimate the following regression equation:

$$\begin{aligned}\ln w_i = \beta_0 + \beta_1 GPA_i + \beta_2 CompatibilityIndex_i + \beta_3 Selectivity_i +\\ \beta_4 GPA_i * CompatibilityIndex_i + \beta_5 GPA_i * Selectivity_i + u_i\end{aligned}$$

The coefficients of interest are those on the interaction terms. Economist A's theory would be supported by a finding that β_4 is positive and significant; Economist B theory would be supported by a finding that β_5 is negative and significant.