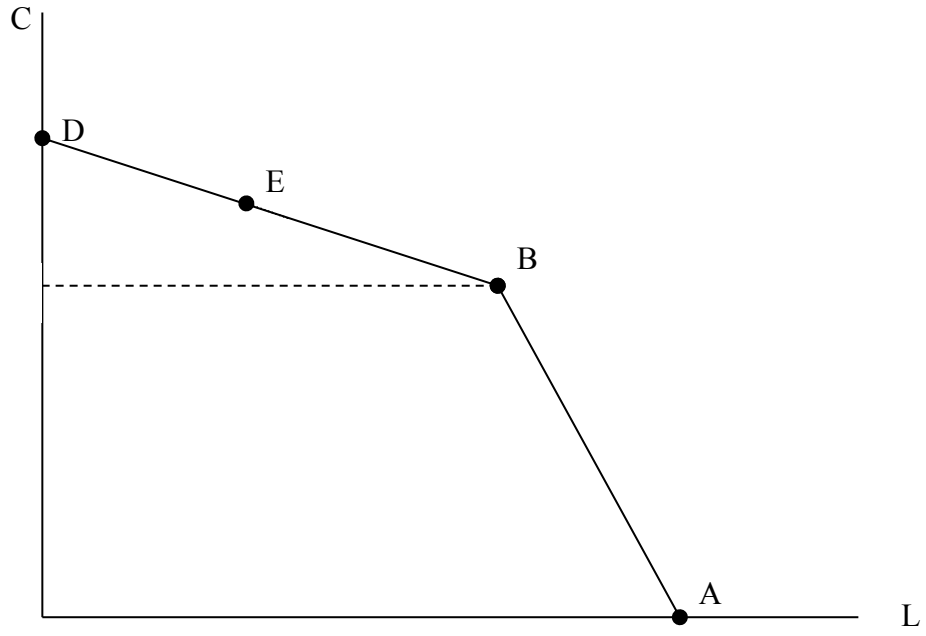
1. The graph below depicts the budget set for a worker who faces a progressive income tax schedule. The worker's wage rate is *w*. The marginal tax rate on the first $\bar{Y}$ dollars of income is equal to zero. The marginal tax rate on every additional dollar is t_0 %. The worker derives utility from consumption (C) and leisure (L). All the usual assumptions (downward sloping and convex indifference curves, C and L both normal goods, etc.) apply.



a) What is the slope of the budget line on the segment AB? What is the slope of the budget line on the segment BD?

The slope of the budget line is equal to the price of leisure: *w* on the segment AB, *w(1-t_0)* on the segment BD.

b) **For this part only**, assume that the worker's utility function is $U(C,L)=L^{\alpha}C^{1-\alpha}$. What conditions must the parameter α satisfy (in terms of the wage *w*, the endowment of leisure *T*, the tax rate t_0, and the level of tax-exempt income $\bar{Y}$) for the worker to choose exactly point B?

The worker will choose point B if the MRS at point B is lower than slope of the budget line on the segment AB, and higher than the slope of the budget line on the segment BD. That means that the following inequalities must hold:

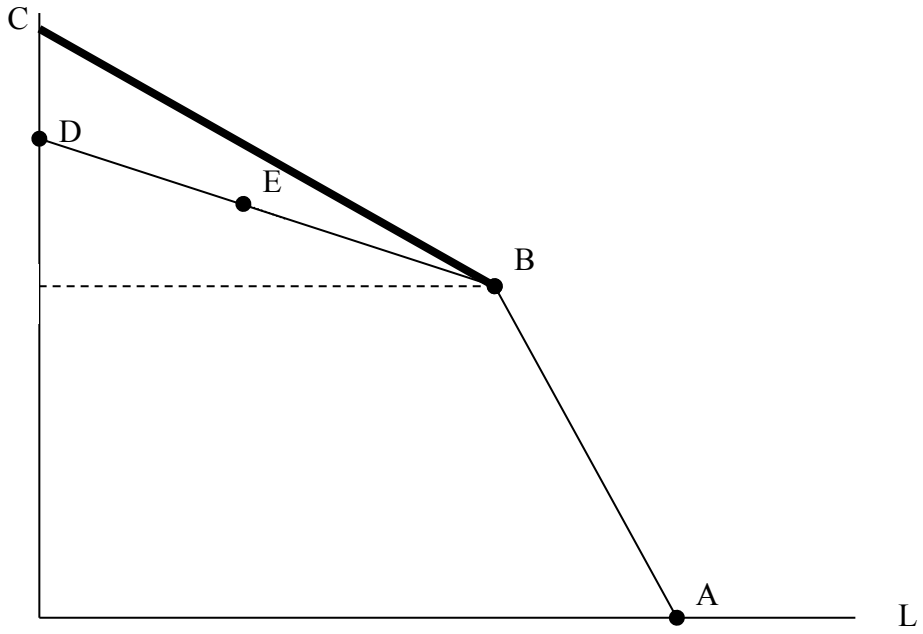
$$w(1-t_0)<\frac{\alpha}{(1-\alpha)}\frac{C}{L}<w$$

$$w(1-t_0)<\frac{\alpha}{(1-\alpha)}\frac{\bar{Y}}{(T-\bar{Y}/w)}<w$$

After some algebra, one can obtain:

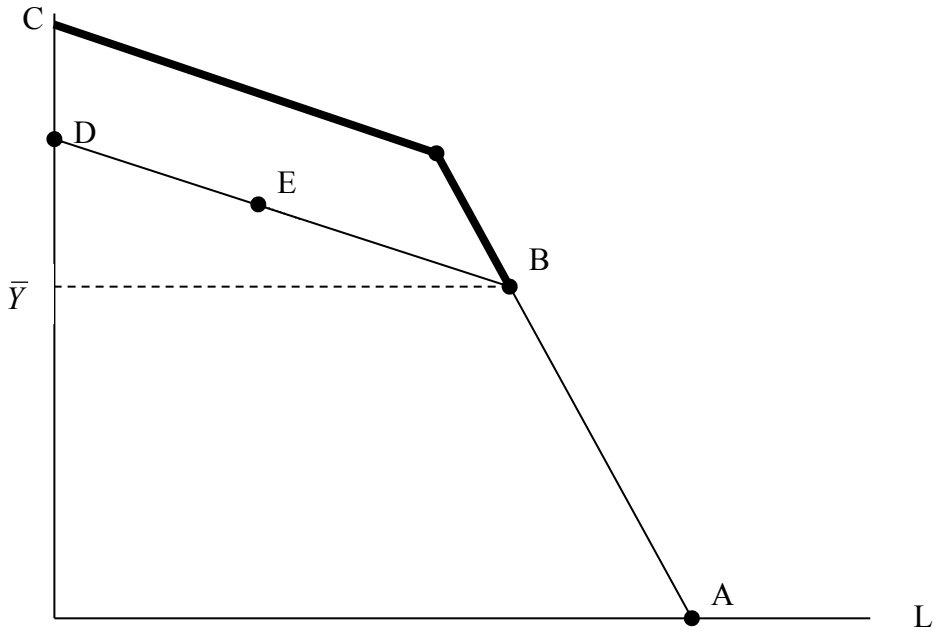
$$1-\frac{\bar{Y}}{wT-t_0(wT-\bar{Y})}<\alpha<1-\frac{\bar{Y}}{wT}$$

c) The government is considering a cut in the marginal income tax rate, from t_0 to $t_1 < t_0$. What effect will this policy have on the labor supply of a worker who prior to the tax cut chose point B? What effect will the policy have on the labor supply of a worker who prior to the tax cut chose point E?



The new budget line is the thick line. Somebody who originally chose B may either increase labor supply (there is only a positive substitution effect) or remain at point B. Somebody at point E will experience both (negative) income and (positive) substitution effects in opposite directions, so labor supply can either increase or decrease.

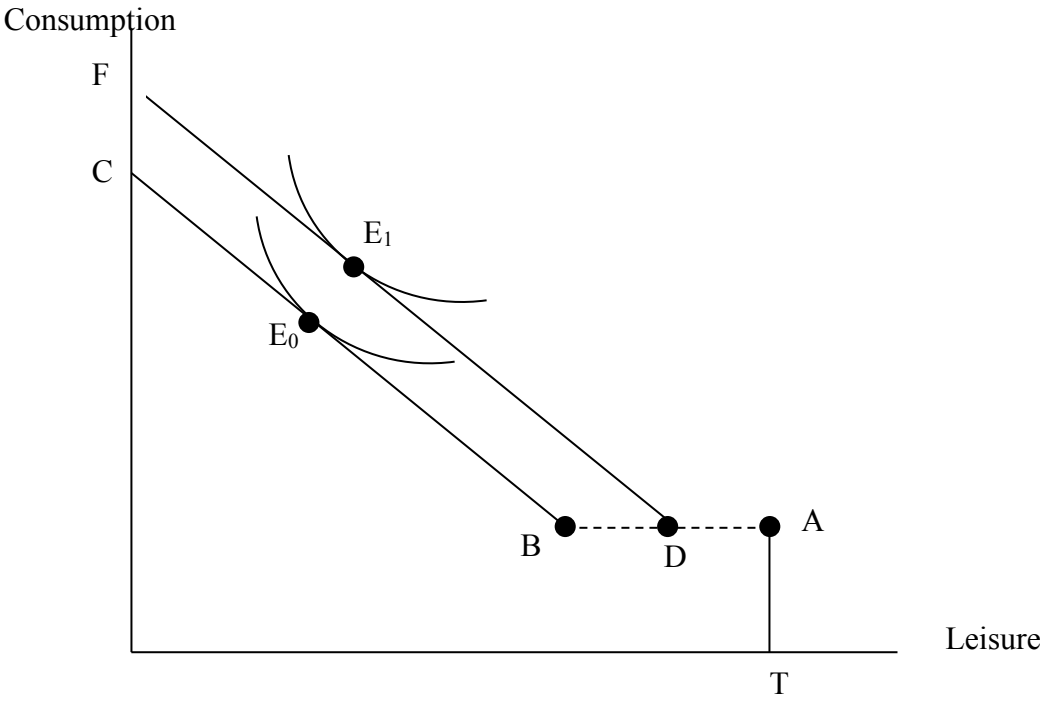
d) An alternative proposal is to maintain the tax rate unchanged, but to raise the level of income that is exempt from taxes from $\overline{Y}$ to $\overline{Y}' > \overline{Y}$. What effect will this policy have on the labor supply of a worker who prior to the tax cut chose point B? What effect will the policy have on the labor supply of a worker who prior to the tax cut chose point E?



The new budget line is the thick line. Those who prior to the change chose point B unambiguously increase their labor supply (they only experience a positive substitution effect). Those at point E experience only a (negative) income effect, and unambiguously decrease labor supply.

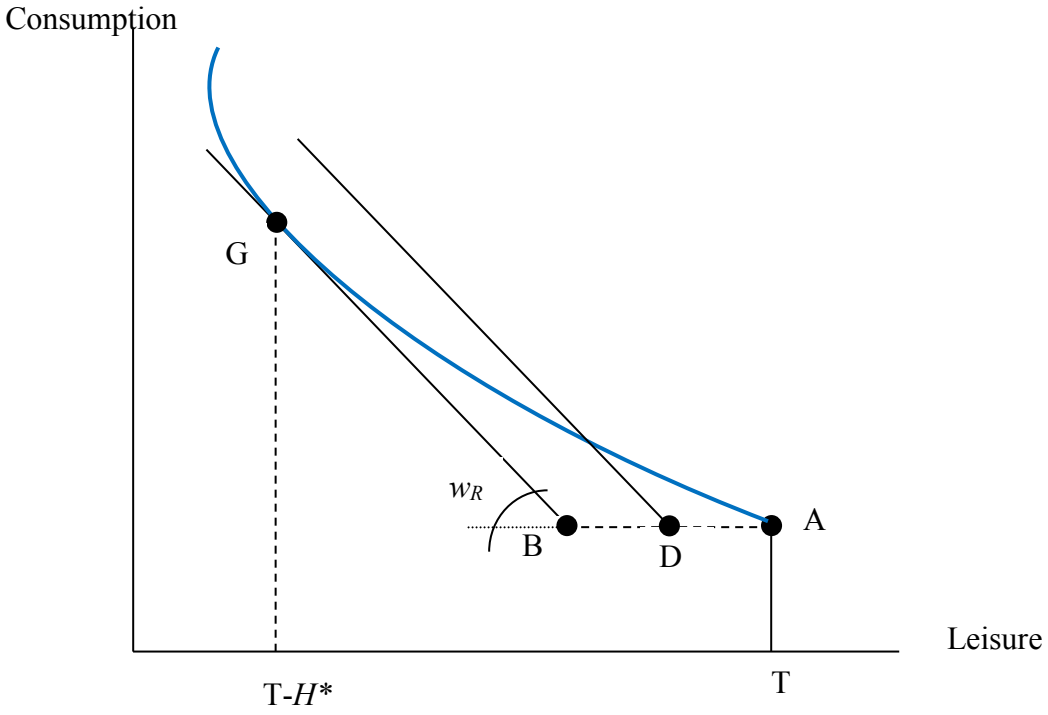
2. A worker draws utility from consumption and leisure. The worker allocates her time between leisure and work, but to reach her workplace she must commute C hours. The worker does not draw any utility from commuting. Assume that the monetary costs of commuting are negligible. For each one of the following statements, state the correct answer and explain in full.

a) A decrease in commuting time raises the number of hours of work. **True/False/Uncertain.**



TRUE. A decrease in commuting time shifts the budget constraint from TABC to TADF, and the optimal point moves from E_0 to E_1. Since consumption and leisure are both normal goods, this means that the increase in leisure is necessarily smaller than the increase in available hours (which is equal to the decrease in commuting time, BD). This implies that hours of work must increase as well.

b) A decrease in commuting time raises the worker's reservation wage. **True/False/Uncertain.**



FALSE. The reservation wage is given by the slope of the line going out of point B that is tangent to the indifference curve that goes through point A (i.e., the line connecting B and G). When the wage is w_R, the worker is indifferent between point G and point A. The worker will choose zero hours if the wage is below w_R, and a positive amount of hours if the wage is above w_R. A decrease in commuting costs implies that at w_R the worker strictly prefers working to not working. Therefore w_R is higher than the new reservation wage. In other words, a decrease in commuting time lowers the reservation wage.

3. Firm A is a competitive firm in both the product and input markets. It produces output (Y) using capital (K) and labor (L), using the following (CES) production function:

$$Y = \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

The firm can hire any amount of labor at the wage rate *w*, and any amount of capital at rental rate *r*.

a) Write down the first order conditions for the firm's cost minimization problem.

The Lagrangian is:

$$\mathcal{L} = wL + rK - \lambda \left\{ \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - Y_0 \right\}$$

And the first order conditions are:

$$w - \lambda(1-\alpha) \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} L^{-\frac{1}{\sigma}} = 0;$$

$$r - \lambda\alpha \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} K^{-\frac{1}{\sigma}} = 0$$

$$\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - Y_0 = 0$$

Rearranging the first two equations, one obtains:

$$\frac{w}{r} = \frac{1-\alpha}{\alpha} \left(\frac{L}{K} \right)^{-1/\sigma} \Rightarrow \frac{L}{K} = \left(\frac{\alpha}{1-\alpha} \right)^{-\sigma} \left(\frac{w}{r} \right)^{-\sigma}$$

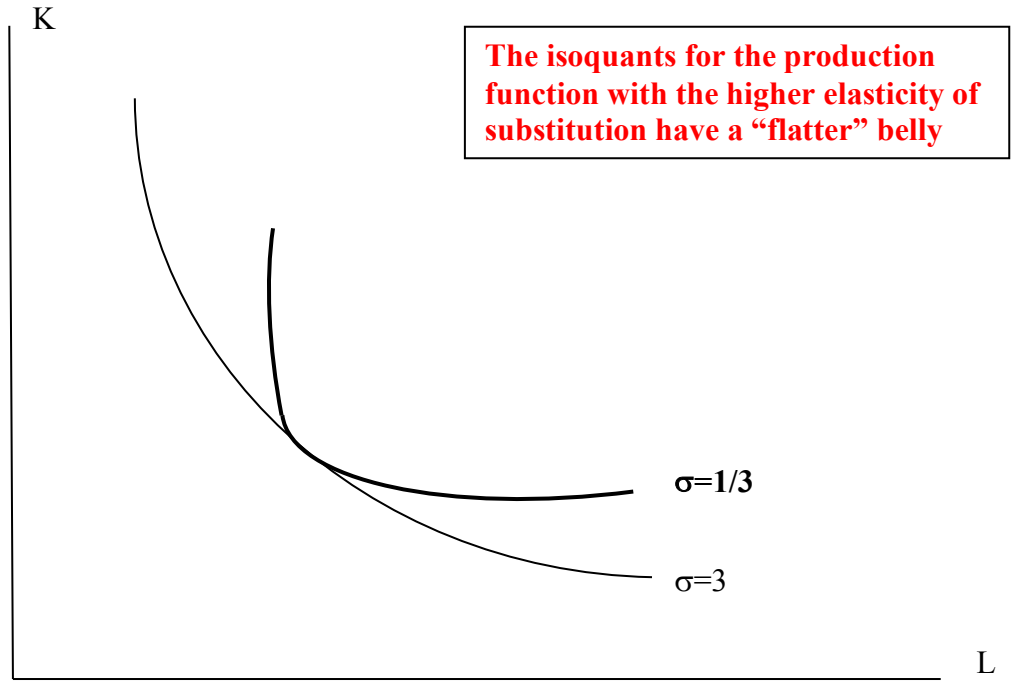
or:

$$\ln \frac{L}{K} = const. - \sigma \ln \frac{w}{r}$$

b) If the ratio of factor prices *w/r* increases by 1% and the firm wants to continue to produce the same amount of output, by how much will the ratio *L/K* change?

This is just a question about the elasticity of substitution. As the last equation shows, the elasticity of substitution is equal to σ, so if *w/r* increases by 1%, *L/K* will <u>decrease</u> by σ%.

c) Draw on the same graph a representative isoquant for Firm A when $\sigma = 3$ and when $\sigma = 1/3$. [You don't have to draw these exactly, just sketch what they look like relative to one another].


K
The isoquants for the production function with the higher elasticity of substitution have a “flatter” belly
σ=1/3
σ=3
L

4. A researcher is interested in estimating the effect of subsidized childcare on the labor supply of low-skill mothers. In 2005, State A passed a law that granted fully subsidized child care for children under the age of 2, but only for mothers with 10 or less years of schooling. The researcher collects data on the employment rates of mothers in State A and in neighboring State B, for different demographic groups.

   The data on average employment rates (for mothers with 10 or less years of schooling) is summarized in the following table:

State	Age of youngest child	Employment rate, 2003-2004	Employment rate, 2006-2007
A	Less than 2	25.0%	28.2%
A	3-4	38.9%	39.1%
B	Less than 2	24.6%	25.1%
B	3-4	37.7%	38.2%

a) Why is a simple before-after comparison of employment rates of mothers with children less than 2 *not* likely to yield an unbiased estimate of the effect of the law?

   It's likely that there were other macroeconomic shocks that affected labor supply over this period, so a simple before-after comparison cannot disentangle between the effects of the law and the effects of these macroeconomic shocks.

b) Assume that states A and B are identical in all respects, and faced the same macroeconomic conditions throughout the period, except for the fact that State B did not subsidize childcare. What strategy would you use to estimate the effect of the law? Explain clearly, and calculate the effect of subsidized childcare on mother's labor supply.

   To overcome the problem in part (a), we could set up a differences-in-differences estimator, comparing the change in employment rates of women with children ages 0-2 in State A, to the change in State B. This latter change is likely to pick up the effect of macroeconomic conditions, so the DD estimate will capture only the effect of the law. This DD estimate is (28.2 – 25.0) – (25.1 – 24.6) = 2.6%.

   Note that you could also set up a DD estimator by comparing the employment rates of women with children ages 0-2 in State A with the employment rates of women with children ages 3-4 in State A (unaffected by the law). A potential concern with this estimator is that the mothers of children 3-4 are likely to have different tastes for leisure, and therefore may not represent such a good control group.