Labor Demand 1

Outline

- One factor of production.
- Two factors of production: isoquants and isocosts.
- Comparative statics: substitution and scale effects.
- The elasticity of substitution.
- Many factors of production: capital-skill complementarity.
- Application: Technology and wage inequality.

Labor Demand

- Main decision that employers (firms) have to make: how many inputs to hire, as a function of costs.
- Employers are price takers in both input and output markets.
- Firm's objective: maximize profits
  - abstracts from: conflicting managerial incentives, social behavior, etc.
- Criterion for maximum profits: hire inputs up to the point where

$$\text{marginal revenue} = \text{marginal cost}$$

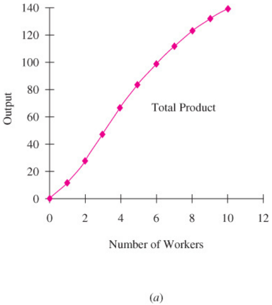
  - any other allocation would not be profit-maximizing.

One factor of production

- The firm's production technology is described by a *production function*: describes how inputs are transformed into outputs.
- In general:
$$Q = f\left(L, K, land, ...\right)$$
- One factor of production:
$$Q = f\left(L\right)$$
- We can think of it as:
  - short-run production function, when other inputs are fixed.
  - production function when costs of capital and other inputs is negligible.

- Properties of the production function:
  - $f'(L) > 0$: increasing in L – more labor, more input.
  - $f''(L) < 0$: diminishing marginal product: with fixed amount of capital, the additional output produced by the second worker is smaller than that produced by the first worker.

FIGURE 3-1 **The Total Product, the Marginal Product, and the Average Product Curves**
(*a*) The total product curve gives the relationship between output and the number of workers hired by the firm (holding capital fixed). (*b*) The marginal product curve gives the output produced by each additional worker, and the average product curve gives the output per worker.



- Firm's optimization problem:

$$\max_{L} \Pi = pQ - wL = pf(L) - wL.$$

- First order conditions:

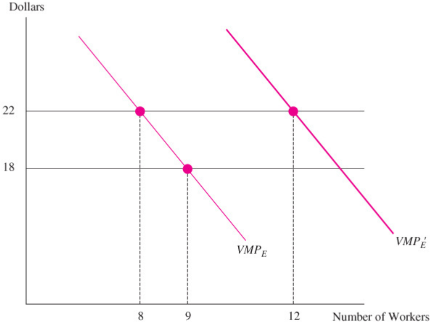
$$pf'(L) - w = 0$$

- $pf'(l)$ is the *value of the marginal product* of labor (VMPL), or *marginal revenue product* of labor (MRPL).

- In words: hire labor up to the point where the MRPL is equal to the wage.

FIGURE 3-3 **The Short-Run Demand Curve for Labor**

Because marginal product eventually declines, the short-run demand curve for labor is downward sloping. A drop in the wage from \$22 to \$18 increases the firm's employment. An increase in the price of the output shifts the value of marginal product curve upward and increases employment.



- Slope of the demand curve determined by technology.
- The more concave is the production function, the steeper (less elastic) is the labor demand curve.
    - Intuitively right, if workers become quickly less productive, firm is less likely to adjust demand when the wage changes.
- Elasticity of labor demand:

$$\eta = \frac{\%\Delta L^D}{\%\Delta w} = \frac{dL/L}{dw/w}$$

Many Inputs

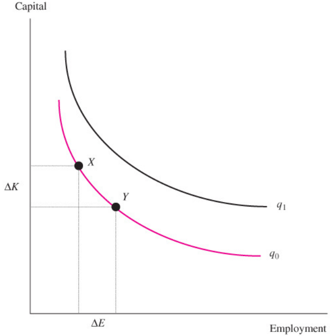
- Start with two inputs, labor (L) and capital (K).
- The production function is

$$Q = f(L, K).$$

- Properties of the production function:
  - $f_L, f_K > 0$: positive marginal productivities.
  - $f_{LL}, f_{KK} < 0$: diminishing marginal productivities.
  - quasi-concavity: *isoquants* are convex.

- Convenient to show the properties of the production function using isoquants, drawn in K, L space.
- An isoquant describes all combinations of capital and labor that can be used to produce a given level of output.

FIGURE 3-6 **Isoquant Curves**
All capital-labor combinations that lie along a single isoquant produce the same level of output. The input combinations at points X and Y produce q_0 units of output. Input combinations that lie on higher isoquants produce more output.



Properties of isoquants

1. Downward sloping: reflects *substitution* between inputs. Can produce the same quantity using either a lot of labor and little capital, or, a lot of capital and little labor.

2. Convex: the slope of the isoquant becomes smaller (the isoquant is flatter) when we move from left to right.

- Why convex? Diminishing marginal returns.
    - when firm uses large K and little L, L has high marginal productivity, K low marginal productivity.
    - firm can hire just a little more L, get rid of a lot of K, produce same amount.
    - when firm uses little K, lots of L, getting rid of (highly productive) last unit of K reduces output significantly, must hire a lot more (not so productive) workers to produce the same amount.

The Marginal Rate of Technical Substitution

- The slope of the isoquant is called **MRTS:** *Marginal Rate of Technical Substitution.*

- $\text{MRTS} = \frac{f_L}{f_K}$: ratio of marginal productivities.

- Why?

$$Q = f\left(L, K\right)$$

total differential, holding Q constant:

$$\begin{aligned} 0 &= f_L dL + f_K dK \\ &\Rightarrow -\frac{dL}{dK}\Big|_{Q=\bar{Q}} = \frac{f_L}{f_K}. \end{aligned}$$

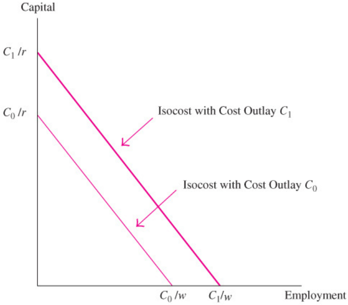
- MRTS tells us how easy it is to substitute between L and K.

Isocost functions

- Firms face market prices for inputs, w and r.
- Firm's cost of production when hiring L units of labor and K units of capital:
$$C = wL + rK$$
- Isocost curves (lines): all combinations of K and L that cost the firm the same amount.
- Slope of the isocost line: w/r.

FIGURE 3-7 **Isocost Lines**

All capital-labor combinations that lie along a single isocost curve are equally costly. Capital-labor combinations that lie on a higher isocost curve are more costly. The slope of an isoquant equals the ratio of input prices ($-w/r$).



Firm's optimization problem

- In the product market:

$$\max_{Q} \Pi = pQ - C(Q)$$

- First order condition (usual stopping rule):

$$p = MC(Q) \Rightarrow Q^{*}$$

- From here we identify the optimal amount of Q^{*} (as $MC(Q)$ is defined from technology $f(L, K)$)
- What is the optimal choice of inputs conditional on the firm producing exactly Q^{*} units of output?

- Firm's cost minimization problem:

$$\min_{L,K} wL + rK$$
$$s.t. \quad : \quad f\left(L,K\right) \geq Q^*$$

- Lagrangian and first order conditions:

$$\mathcal{L} = -wL - rK + \lambda\left[f\left(L,K\right) - Q^*\right]$$

- First order conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial L} &= -w + \lambda f_L = 0 \\
\frac{\partial \mathcal{L}}{\partial K} &= -r + \lambda f_K = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= f\left(L,K\right) - Q^* = 0
\end{aligned}$$

- Rearranging the first two conditions and simplifying, we obtain the key result:

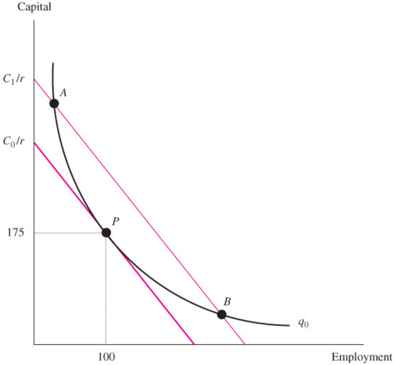
$$MRTS = \frac{f_L}{f_K} = \frac{w}{r}.$$

- Optimal allocation of inputs is achieved at tangency point between isoquant and isocost curves.

- Any other allocation would not be cost minimizing. Easy also to see it graphically.

FIGURE 3-8 The Firm's Optimal Combination of Inputs

A firm minimizes the costs of producing q_0 units of output by using the capital-labor combination at point P, where the isoquant is tangent to the isocost. All other capital-labor combinations (such as those given by points A and B) lie on a higher isocost curve.



Comparative Statics

- What happens if price p changes?

1. Firm will want to produce more output (see profit maximization), demand for both K and L will increase.
2. If production function is *homothetic* will increase proportionally so that K/L stays constant.
    - Homothetic production function

$$f(\lambda L, \lambda K) = \lambda f(L, K).$$

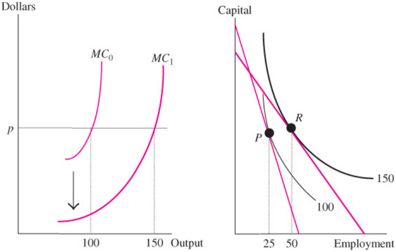
- More interesting, what if factor prices change? For example, decrease in w?

  1. Substitution effect: *holding output constant*, firm will want to use relatively less of the input that has become more expensive, relatively more of the input that has become less expensive.

     But output will not stay constant: lower factor prices imply that marginal costs curve will also be lower. From product market, lower MC implies higher Q^*

  2. Scale effect: produce more, use more of both inputs.

FIGURE 3-10 **The Impact of a Wage Reduction on the Output and Employment of a Profit-Maximizing Firm**
(*a*) A wage cut reduces the marginal cost of production and encourages the firm to expand (from producing 100 to 150 units). (*b*) The firm moves from point *P* to point *R*, increasing the number of workers hired from 25 to 50.



(*a*) Firm's Output Decision (*b*) Firm's Hiring Decision

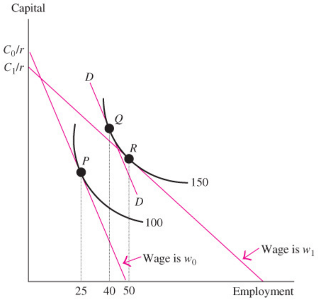
- Note that the effect on total costs is ambiguous:

$$C(Q^*) = wL(w, r, Q^*) + rK(w, r, Q^*).$$

  - w goes down
  - L goes up (positive substitution and scale effect)
  - K uncertain (negative substitution and positive scale effect)

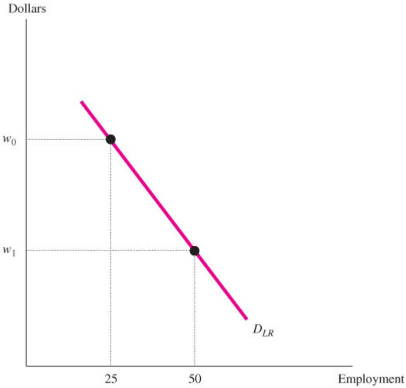
FIGURE 3-12 **Substitution and Scale Effects**

A wage cut generates substitution and scale effects. The scale effect (the move from point P to point Q) encourages the firm to expand, increasing the firm's employment. The substitution effect (from Q to R) encourages the firm to use a more labor-intensive method of production, further increasing employment.



- To sum up, a decrease in w:
  - Substitution effect: hire more L, relatively less expensive.
  - Scale effect: hire more L, because firm wants to produce more.
- Both effects operate in the same direction: a decrease in w unambiguously raises the demand for L. The labor demand curve is unambiguously downward sloping.

FIGURE 3-11 **The Long-Run Demand Curve for Labor**
The long-run demand curve for labor gives the firm's employment at a given wage and is downward sloping.



Two special cases

- Factors of production are **perfect substitutes.**

- One unit of output can always be produced with **either** a_L units of labor **or** a_K units of capital, or any linear combination of the two (e.g., $a_L/2$ units of labor **and** $a_K/2$ units of capital).

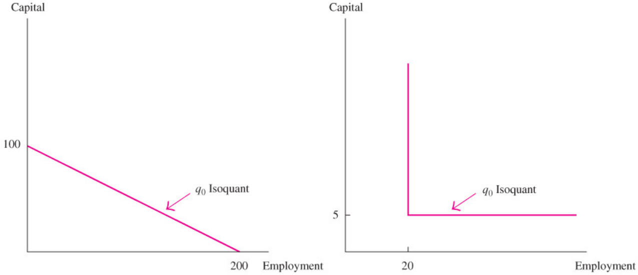
- The production function is

$$Q = \frac{L}{a_L} + \frac{K}{a_K}.$$

- Isoquants are straight lines with slope a_K/a_L.

- When w/r is high, produce using only K. When w/r is low, produce using only L. How does the labor demand function look like?

FIGURE 3-14 **Isoquants When Inputs Are Either Perfect Substitutes or Perfect Complements**
Capital and labor are perfect substitutes if the isoquant is linear (so that two workers can always be substituted for one machine). The two inputs are perfect complements if the isoquant is right-angled. The firm then gets the same output when it hires 5 machines and 20 workers as when it hires 5 machines and 25 workers.



(*a*) Perfect Substitutes (*b*) Perfect Complements

[7] Note that our definition of perfect substitution does *not* imply that the two inputs have to be exchanged on a one-to-one basis; that is, one machine hired for each worker laid off. Our definition implies only that the rate at which capital can be replaced for labor is constant.

- Factors of production are **perfect complements.**

- One unit of output can be produced using **both** b_L units of labor **and** b_K units of capital.

- The production function is

$$Q = \min\left(\frac{L}{b_L}, \frac{K}{b_K}\right).$$

- Isoquants are L-shaped. The ray from the origin connecting the vertexes of the L's has slope b_K / b_L.

- No substitution effect. But when the wage changes there is a scale effect, so that the labor demand function is downward sloping.

- Perfect substitutes and perfect complements are polar cases. In general the degree of substitution between capital and labor is somewhere in between.
- The parameter that characterizes how easy it is to substitute between factors of production is the *elasticity of substitution.*

$$\sigma = \frac{d \ln (K / L)}{d \ln (f_L / f_K)}$$

- Elasticity of substitution tells how "flat" the belly of the isoquants is.
- The higher σ, the flatter the isoquant, the easier it is to substitute between capital and labor along an isoquant.

Cross-price elasticities

- What happens to labor demand function when r (cost of capital) increases?
- Substitution effect: labor has become relatively less expensive, use more of it. L^d increases.
- Scale effect: firm will want to reduce the scale of production because marginal costs have gone up. Use *less* labor. L^d decreases.
- Two effects operate in opposite directions.
  - If the **substitution effect** dominates: $\partial L^d / \partial r > 0$, factors are said to be **gross substitutes.**
  - If the scale effect dominates: $\partial L^d / \partial r < 0$, factors are said to be **gross complements.**

Many Factors of Production

- Production function may depend on more than two factors.
- Case that is often very relevant for labor economics: skilled and unskilled labor.
$$Q = f\left(K, L_S, L_U\right)$$
- There is evidence for the notion of **capital-skill complementarity**. The elasticity of substitution between capital and unskilled labor is greater than the elasticity of substitution between capital and skilled labor.

Capital-skill complementarity

- An example: snow-removal firm.
- Snow-removal firm can produce one unit of output using two (perfectly substitutable) technologies.
  - a snow plow (K), operated by a driver (L_S).
  - unskilled workers (L_U) removing snow with shovels.
- In this case, K and L_S are perfect complements (you need one driver for every snowplow). K and L_U are perfect substitutes.
- Analogous example: bank tellers, ATM machines and engineers.

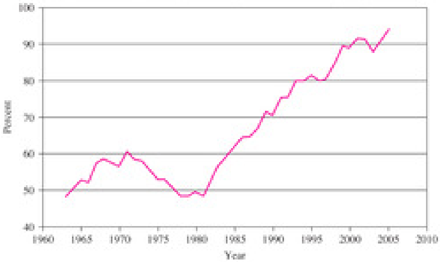
- How does the demand for L_S and L_U change when the price of K changes ?
- A decrease in the price of K:
  - substitution effect: use less L_U, more K and more L_S
  - scale effect: use more of all three factors.
- But overall, the demand for L_S is likely to increase more than the demand for L_U, because of capital-skill complementarity.

- Implications for wage inequality.
- Introduction of new technologies since 1980s (personal computer, information technology, automatization of production processes, robotization, etc.) can be thought of as a decrease in the price of capital.
- With capital-skill complementarity (or technology-skill complementarity), the relative demand for high-skill workers will have increased more than the demand for low-skilled workers.
- Relative increase in the wage premium paid to high skill workers. This is exactly what has happened since the early 1980s.

Figure 1.—Change in Log Real Weekly Wage by Percentile, Full-Time Workers, 1963–2005

FIGURE 7-5 **Wage Differential between College Graduates and High School Graduates, 1963–2005**

Source: David H. Autor, Lawrence F. Katz, and Melissa S. Kearney, "Trends in U.S. Wage Inequality: Revising the Revisionists," *Review of Economics and Statistics* 90 (May 2008): 300–23. The percent wage differentials give the differences in weekly earnings for full-time, full-year workers who are 18 to 65 years old.



? The college premium falls in the 1970s, but increases very rapidly from the 1980s onwards.

? Also (not shown), large increases in the returns to experience in the 1980s.