Statistics, Fall 2025 - Practice Session 5
Asymptotics & Hypothesis Testing

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Problem 1

Consider a random sample $\{X_i\}_{i=1}^n$ where X_i follows a distribution with density

$$f(x;a) = (1+a)x^a \mathbb{1}_{[0,1]}(x)$$

Given some significance level α, find the best critical region for testing the hypotheses

$$\mathcal{H}_0 : a = 1 \ , \quad \mathcal{H}_1 : a = 2$$

using the Likelihood Ratio method.

Problem 2

Consider two independent random samples $\{X_i\}_{i=1}^n$ and $\{Y_j\}_{j=1}^m$, where

$$X_i \sim N(a,c) \ , \quad Y_j \sim N(b,c)$$

where all three parameters are unknown. For a significance level α, construct the rejection region for the null hypothesis $\mathcal{H}_0 : a = b$.

Notes

- If $X \sim N$ and $Y \sim \chi^2_k$, and the two random variables are independent, then

$$T = \frac{X}{\sqrt{Y/k}} \sim t_k$$