Sampling

Statistics

What is a random sample?

$X = (X_1, X_2,..,X_n)$ n random variables i.i.d.

What is a statistics?

A statistics is a function of the sample that does not depend on any unknown parameter

Statistics

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$Max(X_1, \ldots X_n)$

$$S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n}$$

$Min(X_1, \ldots X_n)$

Statistics

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$\text{Max}(x_1,\ldots x_n)$

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

$\text{Min}(x_1,\ldots x_n)$

Distribution sample sum?

X~Bernoulli(π)

$\sum_{i=1}^{n} X_i$ ~?

X~Exp(λ)

$\bar{X}$ ~?

$$\sum_{i=1}^{n} X_i$$

X~N(μ,σ^2)

$\bar{X}$ ~?

X arbitraria

$\sum_{i=1}^{n} X_i$ ~?

Distribution sample sum/sample mean?

$X \sim \text{Bernoulli}(\pi)$

$\sum_{i=1}^{n} X_i \sim \text{Binomiale}(\pi, n)$

$X \sim \text{Exp}(\lambda)$

$\sum_{i=1}^{n} X_i \sim \text{Gamma}(n, 1/\lambda)$

$$\sum_{i=1}^{n} X_i$$

$X \sim N(\mu, \sigma^2)$

$\sum_{i=1}^{n} X_i \sim N(\mu, n\sigma^2)$

X arbitraria

$\sum_{i=1}^{n} X_i \sim N(\mu, n\sigma^2)$

Distribution of minimum e maximum?

$\mathrm{Min}(X_1, X_2, .., X_n)$ $\longrightarrow$ $$\boxed{f_{X_{(1)}}(x) = n\,[1 - F(X)]^{n-1}\, f(x)}$$

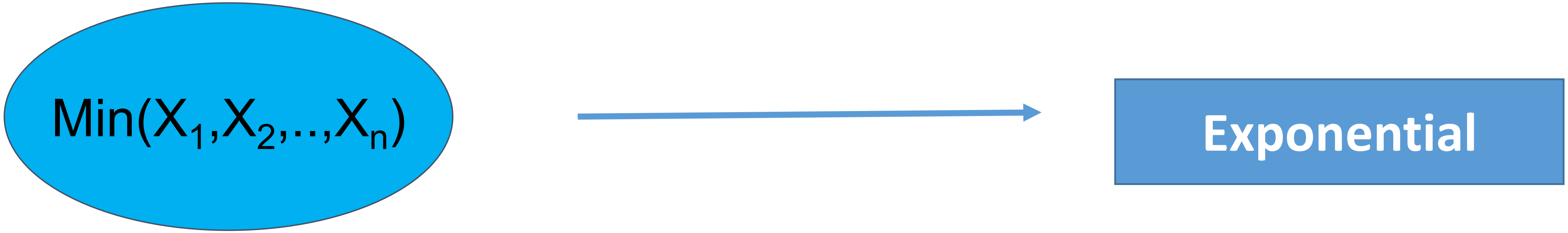
$\mathrm{Max}(X_1, X_2, .., X_n)$ $\longrightarrow$ $$\boxed{f_{X_{(n)}}(x) = n\,[F(X)]^{n-1}\, f(x)}$$

Distribution of minimum e maximum $X \sim U(a,b)$?

$\mathrm{Min}(X_1,X_2,..,X_n)$ $\rightarrow$ $$f_{X_{(1)}}(x) = \frac{n}{b-a}\left[\frac{b-x}{b-a}\right]^{n-1}$$

$\mathrm{Max}(X_1,X_2,..,X_n)$ $\rightarrow$ $$f_{X_{(n)}}(x) = \frac{n}{b-a}\left[\frac{x-a}{b-a}\right]^{n-1}$$

Distribution of minimum of X is Exponential?



SMALL Samples

	STATISTICS			
Distribution	$\sum_{i=1}^{n} X_i$	$\overline{X}$	$\max(X_i)$	$\min(X_i)$
$X \sim N(\mu,\sigma^2)$ σ^2 noto	$N(n\mu, n\sigma^2)$	$N(\mu, \sigma^2/n)$		
$X \sim$ Bernoulli(π)	Binom(n,π)	$\approx$Binom(n,π)	Bernoulli $(1-(1-\pi)^n)$	Bernoulli(π^n)
$X \sim$ Uniform$(0,\theta)$			$\theta \times$Beta$(n,1)$ $f_{X_{(n)}} = \frac{n}{\theta}\left(\frac{x}{\theta}\right)^{n-1} I_{[0,\theta]}(x)$	$\theta \times$Beta$(1,n)$ $f_{X_{(1)}} = \frac{n}{\theta}\left(1-\frac{x}{\theta}\right)^{n-1} I_{[0,\theta]}(x)$
$X \sim$ Exponential(λ)	$\Gamma(n,\lambda)$	$\Gamma(n,\lambda/n)$		Exponential(λ/n).

LARGE Samples

	STATISTICS			
Distribution	$\sum_{i=1}^{n} X_i$	$\overline{X}$	$\max(X_i)$	$\min(X_i)$
$X \sim N(\mu,\sigma^2)$ σ^2 noto	$N(n\mu, n\sigma^2)$	$N(\mu, \sigma^2/n)$		
$X \sim \text{Bernoulli}(\pi)$	$N(n\pi, n\pi(1-\pi))$	$N(\pi, \pi(1-\pi)/n)$		
$X \sim \text{Uniform}(0,\theta)$	$N(n\theta/2, n\theta^2/12)$	$N(\theta/2, \theta^2/(12n))$		
$X \sim \text{Exponential}(\lambda)$	$N(n\lambda, n\lambda^2)$	$N(\lambda, \lambda^2/n)$		