

Interval Estimation

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Outline

- 1 Pivotal Quantity
 - Gaussian Distribution
 - Uniform Distribution

Pivotal distribution for μ in $N(\mu, \sigma)$ σ^2 known

$$X \sim N(\mu, \sigma^2)$$

Sufficient Statistics

$$\bar{X}$$

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Distribution of $\bar{X}$?

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

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$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

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Pivotal Quantity?

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Pivotal Quantity?

- Distribution does not depend on μ

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$$

- Is it Invertible?

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$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Pivotal Quantity

$$P\left(-z_{1-\alpha/2} \leq \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z_{1-\alpha/2}\right) = 1 - \alpha$$

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$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

NO

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Pivotal Quantity?

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

NO

Pivotal Quantity?

$$\frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

YES!

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Is it Invertible?

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Pivotal Quantity

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$$X_{(n)} = \text{Max}(X_1, \dots, X_n)$$

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Distribution of $Y = X_{(n)}$?

$$f(y) = \frac{ny^{n-1}}{\theta^n} I_{[0, \theta]}(y)$$

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$$\left[X_{(n)}; \frac{X_{(n)}}{\alpha^{1/n}} \right]$$

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