Mathematics
Program of the course
Teacher: P. Gibilisco

Linear Algebra

Systems of linear equations. Matrix Algebra. Algebra of square matrices. Transpose and its properties. Determinant. Vector spaces. Linear independence and basis. Dimension of vector spaces. Linear transformations. Kernels. Scalar products. Cauchy-Schwartz inequality. Eigenvalues, eigenvectors and the characteristic polynomial of a square matrix. Basic properties of eigenspaces. Symmetric, and orthogonal matrices. Positive definite matrices. Projection operators. Cholesky decomposition. Diagonalizable matrices. The spectral theorem.

Calculus

Series. The complex numbers. Complex series and the complex exponential. The Euler formula. Differentiability for functions of several variables: examples and counterexamples. The gradient. The Jacobian matrix. The chain rule for differentials. Mixed partial derivative. The Schwartz (Young) theorem. Integration in n dimension. The Fubini theorem. The change of variable formula. Integration using polar coordinates. Differentiation under the integral sign. Introduction to differential equations. The Cauchy problem. The L^2 scalar product on R^2, on $\mathcal{C}[0,1]$ and for random variables. Quasi-convex functions. The separating hyperplane theorem. Brouwer's fixed point theorem.

Optimization

The Taylor polynomial in n-dimensions. The Hessian matrix. Unconstrained optimization: necessary and sufficient conditions for maxima and minima. Constrained optimization. Lagrangian function and Lagrange multiplier. Introduction to Kuhn-Tucker. The Envelope Theorem.

Probability

Elements of a probability space. Algebras of events and information about random experiments. Introduction to combinatorial calculus. Finite probability spaces, probability measures, introduction to Kolmogorov theory. Conditional probability, total probability formula, Bayes formula. Independent events. Random variables and their properties. Probability distribution, distribution function and densities function of a random variable. Expectation and variance of a random variable and their properties.

Expectation and variance for the main kinds of random variables. Covariance and scale-invariance of the correlation coefficient. Random vectors and their properties. Probability distribution, distribution functions and densities functions of a random vector. Independent random variables, covariance and correlation. Conditional expectation of a random variable and its properties. Conditional expectation as best estimator. Geometric approach to the conditional expectation. Sequences of random variables. Convergence in probability and in law. The (weak) law of large numbers. The characteristic function. Central limit theorem. Multivariate Gaussian distribution. Conditional expectation for the bivariate gaussian.