Microeconomics I, 2024/2025
Master of Science in Economics
Problem Set 2

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Question 1 *Show the following:*

- *If $\succsim$ is strongly monotone, then it is monotone.*
- *If $\succsim$ is monotone, then it is locally non-satiated.*

Question 2 *Consider the following utility function $u(x_1, x_2) = x_1 - x_2$. Show that it represents a rational preference relation $\succsim$ that is locally non-satiated but not monotone.*

Question 3 *Which of the following v functions represents the same preferences of the corresponding u?*

1. $v = 2u - 13$;
2. $v = \frac{1}{u^2}$;
3. $v = ln(u)$;
4. $v = -e^{-u}$;

A monotonic transformation of a utility function does not affect the preference order of a consumer, because it does not change the marginal rate of substitution. Discuss.

Question 4 *Suppose preferences are represented by the Cobb-Douglas utility function $u(x_1, x_2) = Ax_1^{\alpha}x_2^{1-\alpha}$, for $A > 0$ and $\alpha \in (0, 1)$.*

1. *Assuming an interior solution, solve for the Walrasian demand functions.*
2. *Consider the logarithmic transformation $v(x) = \ln u(x)$ of the Cobb-Douglas utility function above and verify that the Walrasian demand functions are identical to those derived in the previous item. Explain.*

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3. *Show that, for every commodity, the Walrasian demand function is homogeneous of degree one in wealth.*

4. *Using the Walrasian demand functions you found before, derive the consumer's indirect utility function.*

5. *Show that the indirect utility function is strictly increasing in w. Argue that this is indeed a general property of any indirect utility function $v(p, w)$.*

Question 5 *Show that if $x(p, w)$ is homogeneous of degree one in w, i.e., $x(p, \alpha w) = \alpha x(p, w)$ for all $\alpha > 0$, and satisfies Walras law then, for every commodity l, the demand elasticity relative to wealth is equal to one, i.e., $\epsilon_{lw}(p, w) = 1$. What can you say about the wealth effects and the form of the Engel curves in this case?*

Question 6 *Let $(-\infty, +\infty) \cup \mathbb{R}_+$ denote the consumption set, and assume preferences are represented by the utility function $u(x_1, x_2) = x_1 + \phi(x_2)$ with $\phi(.)$ increasing and strictly convex.*

1. *Compute the Walrasian demand function for each commodity.*

2. *Show that the Walrasian demand function for commodity 2 is independent of wealth. What does this imply about the wealth effect of the demand of good 1?*

3. *Compute the indirect utility function for this problem, and show that it is indeed homogeneous of degree zero in prices and wealth. How can explain your finding?*