Microconomics I, 2024/2025
Master of Science in Economics
Problem Set 3

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Only the solutions to the Questions 1-3 will be graded, the remaining questions are given to practice new topics.

Question 1 Consider a consumer's preferences represented by the following utility function:

$$u(x_1, x_2) = min\{ax_1, bx_2\}$$

with $a, b > 0$.

- Compute the consumer's Walrasian demands for goods 1 and 2.
- Compute the indirect utility function.
- Check that the Walrasian demands satisfy homogeneity of degree zero in (p, w) and Walras' law.
- For the indirect utility, check that it is non-increasing in prices and increasing in wealth.

Question 2 Establish the following result: a continuous $\succeq$ is homothetic if and only if it admits a utility function $u(x)$ that is homogenous of degree one; i.e., $u(\alpha x) = \alpha u(x)$ for all $\alpha > 0$.

Question 3 Suppose that $u(x)$ is differentiable and strictly quasiconcave and that the Walrasian demand function $x(p, w)$ is differentiable. Show the following: if $u(x)$ is homogeneous of degree one, then the Walrasian demand function $x(p, w)$ and the indirect utility function $v(p, w)$ are homogeneous of degree one [and hence can be written in the form $x(p, w) = w\tilde{x}(p)$ and and $v(p, w) = w\tilde{v}(p)$] and the wealth expansion path is a straight line through the origin. What does this imply about the wealth elasticity of demand?

Question 4 Prove that a solution to the EMP exists if $p >> 0$ and there is some $x \in \mathbb{R}^L_+$ satisfying $u(x) \geq u$.

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Question 5 Consider preferences represented by the constant elasticity of substitution (CES) utility function:

$$u(x_1, x_2) = (x_1^{\rho} + x_2^{\rho})^{1/\rho}$$

- Compute the Hicksian demands for goods 1 and 2 and the expenditure function.