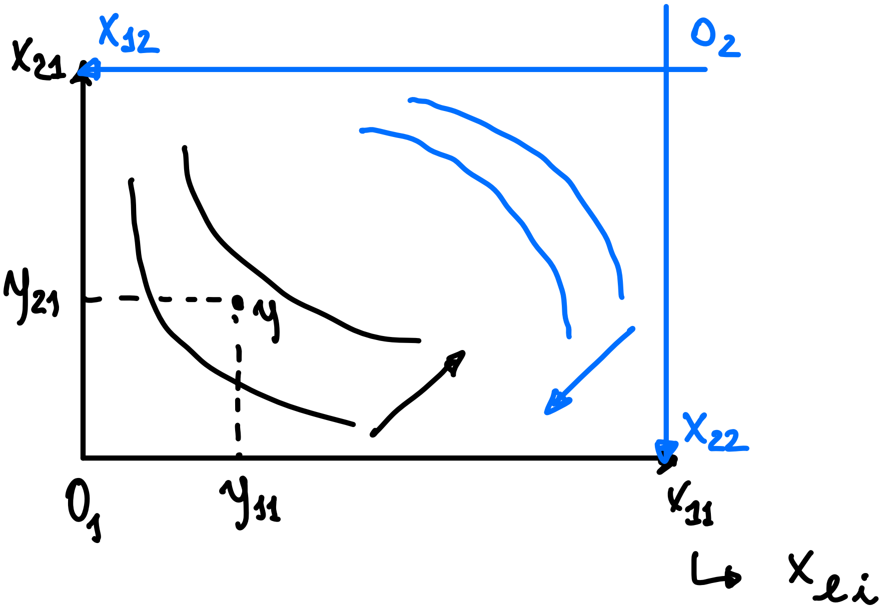
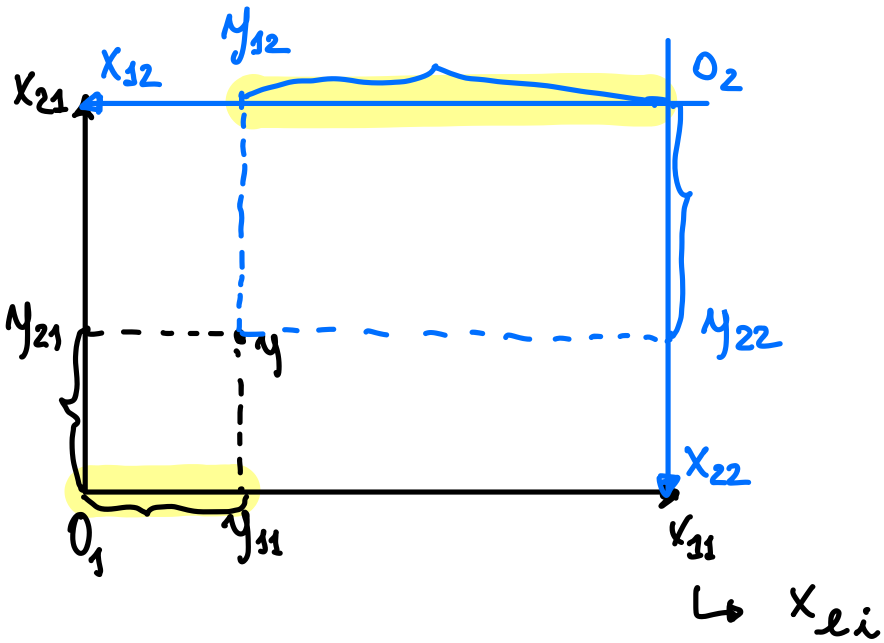
5/12/2024

GE SETTING WITH I = L = 2

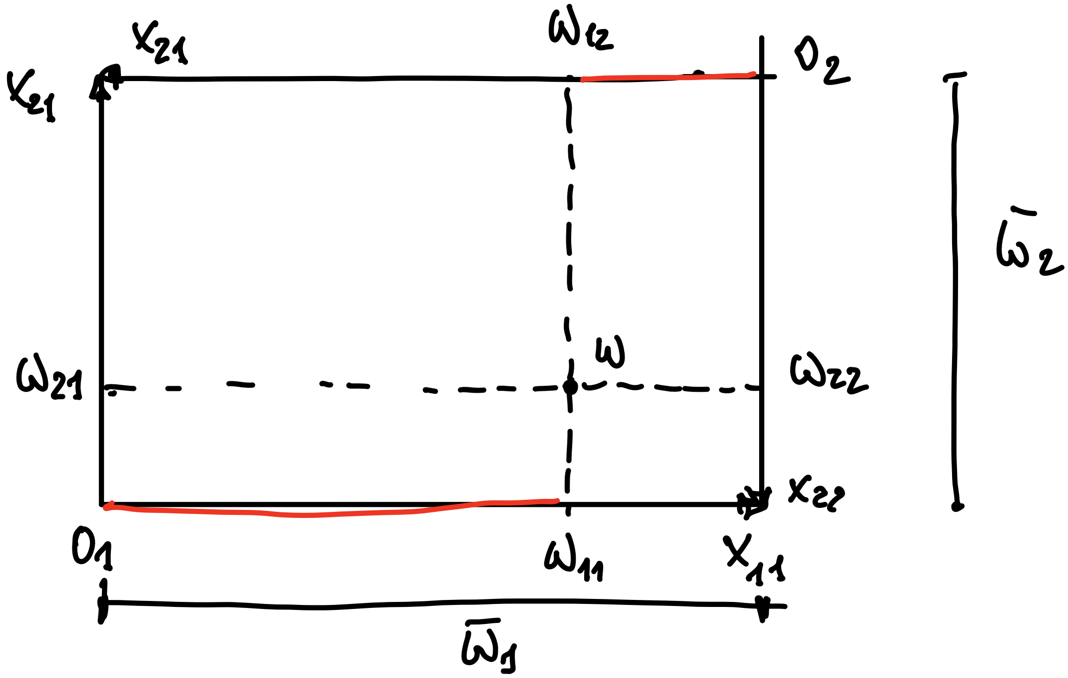




$y = (y_1, y_2) = ((y_{11}, y_{21}), (y_{12}, y_{22}))$ is an allocation

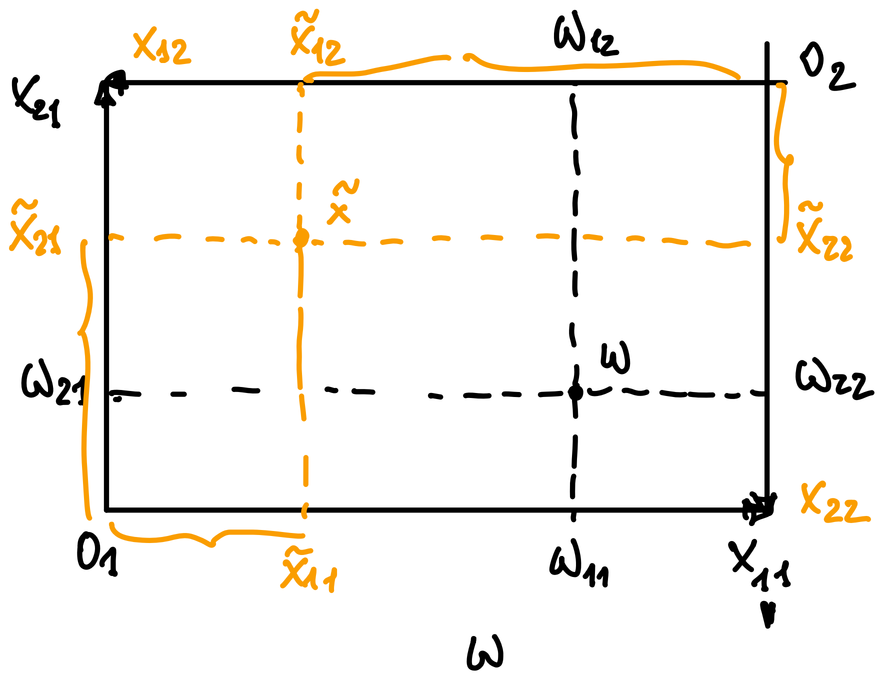
$\bar{\omega}_1 = \omega_{11} + \omega_{12}$ aggregate endowments of commodity 1 and 2

$\bar{\omega}_2 = \omega_{21} + \omega_{22}$ $\omega_{\ell i}$



EDGEWORTH BOX is a tool to analyse exchange economies $\mathcal{E} = \{u_i, \omega_i\}_{i=1}^{2}$ with $I = L = 2$

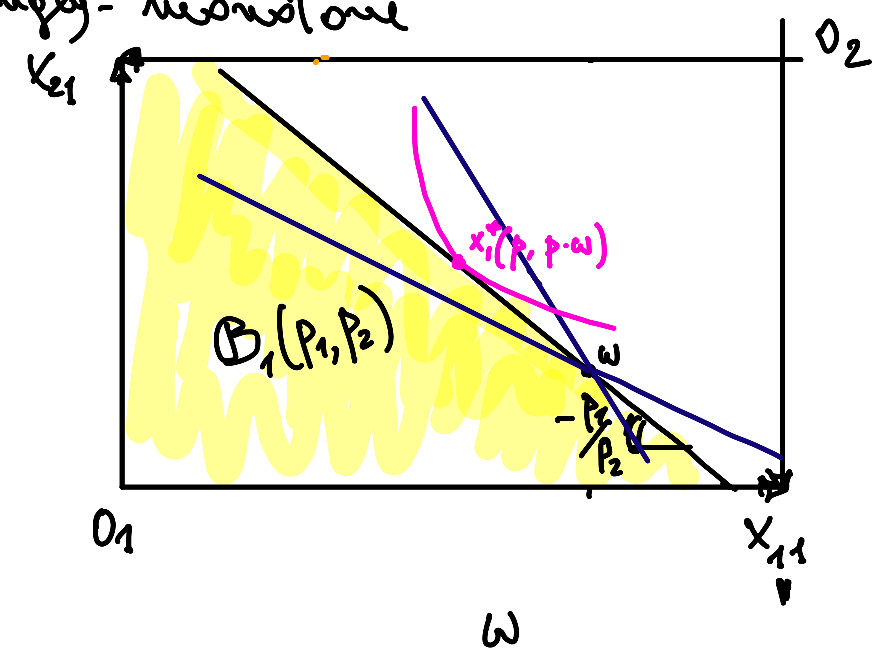
No waste property:
$\forall \ell \quad x_{\ell 1} + x_{\ell 2} = \bar{\omega}_\ell$



$$\tilde{x} = (\tilde{x}_1, \tilde{x}_2) = ((\tilde{x}_{11}, \tilde{x}_{21}), (\tilde{x}_{12}, \tilde{x}_{22}))$$

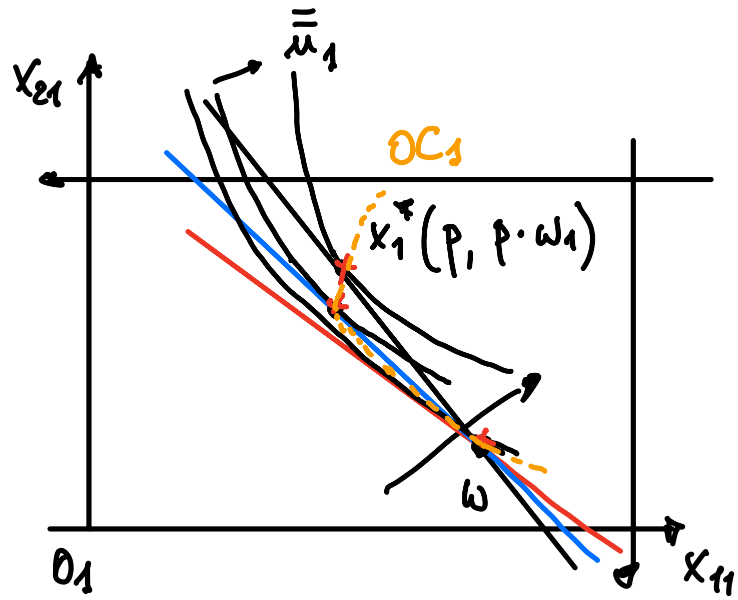
$$\forall \ell, \forall i: \quad \tilde{x}_{12} = \bar{\omega}_1 - \tilde{x}_{11}$$

Let $\succsim_i$ be continuous, strictly convex and strongly-monotone

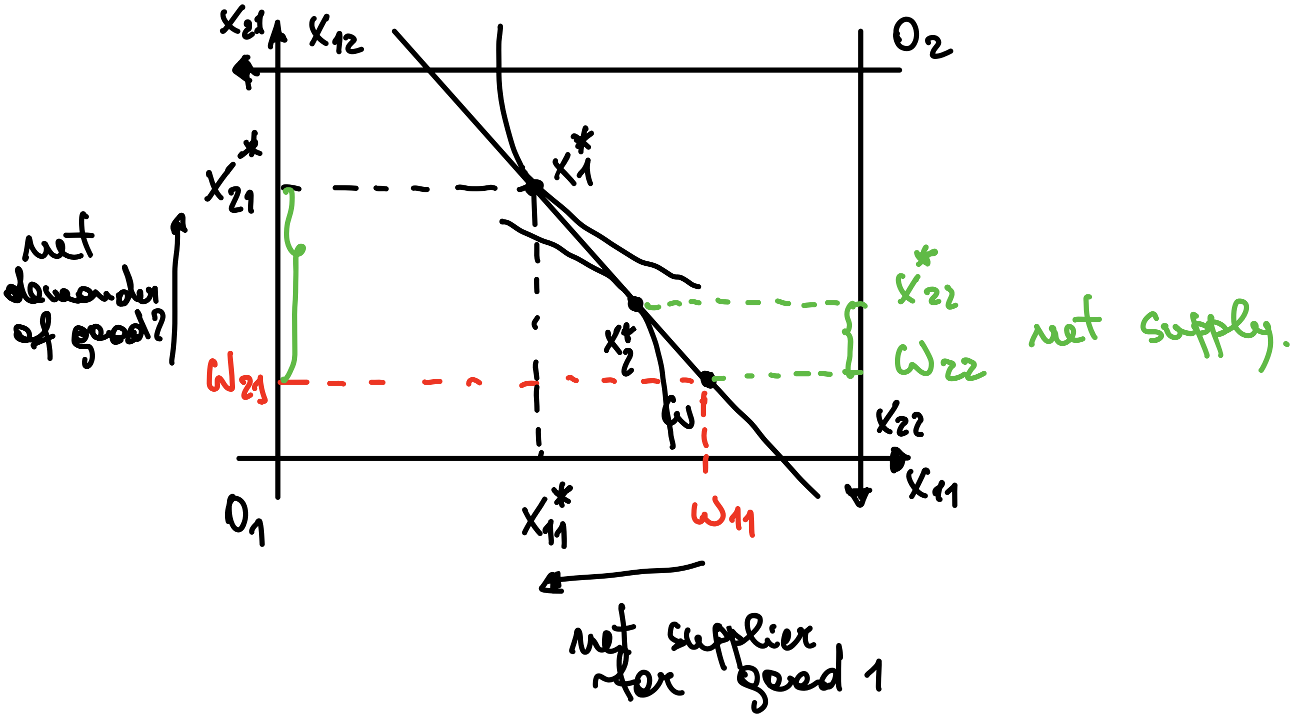


$$B_i(p_1, p_2) = \{ x_i \in \mathbb{R}^2_+ : p \cdot x_i \leq p \cdot \omega_i \}$$

$$p_1 x_{11} + p_2 x_{21} \leq p_1 \omega_{11} + p_2 \omega_{21}$$



Consider how $x_1^*(p, p\cdot\omega_1)$ changes as p change $\Rightarrow$ the collection/set of $x_1^*(p, p\cdot\omega_1)$ $\forall p \gg 0$ is the Offer curve (OC_1) of consumer 1. Since consumer 1 can always consume her endowment, the OC_1 is in the upper-contour set of ω_1, and it is tangent to the indifference curve that passes through the endowment ω_1.



A Walrasian equilibrium in the Edgeworth is a profile of prices p^* and an allocation $x^* = (x_1^*, x_2^*)$ st. $\forall\, i = 1,2$

$$x_i^* \succsim x_i' \quad \forall \quad x_i' \in B_i(p^*)$$

$$\text{At } x_i^* : \quad p^* \cdot x_i^* = p^* \cdot \omega_i \qquad \forall\, i = 1,2$$

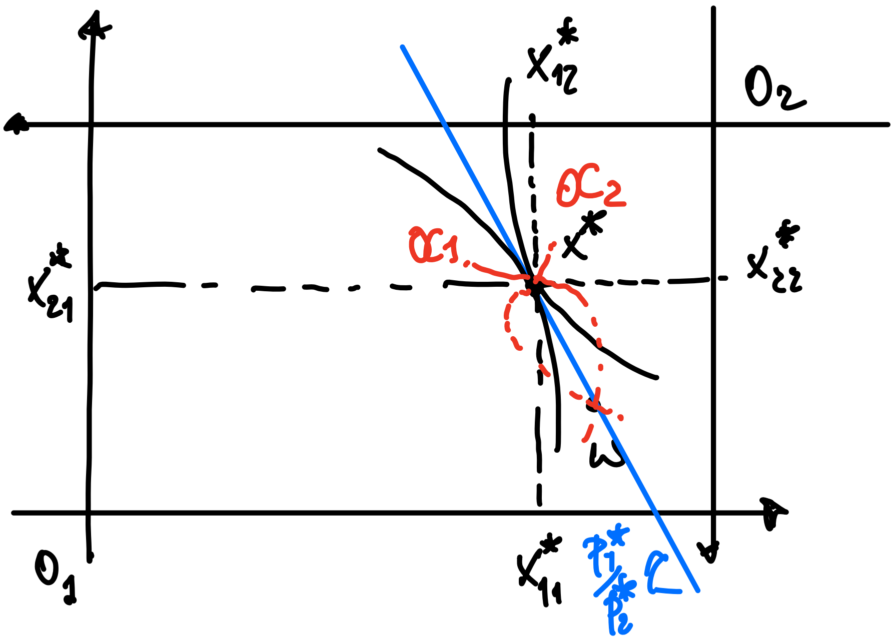
$$\begin{cases} p_1^* x_{11}^* + p_2^* x_{21}^* = p_1^* \omega_{11} + p_2^* \omega_{21} \\ p_1^* x_{12}^* + p_2^* x_{22}^* = p_1^* \omega_{12} + p_2^* \omega_{22} \end{cases}$$

Walras' law

$\bar{\omega}_1$

$$p_1^*(x_{11}^* + x_{12}^*) + p_2^*(x_{21}^* + x_{22}^*) = p_1^*(\omega_{11} + \omega_{12}) + p_2^*\underbrace{(\omega_{21} + \omega_{22})}_{\bar{\omega}_2}$$

$$p_1^*\underbrace{(x_{11}^* + x_{12}^* - \bar{\omega}_1)}_{\text{no excess demand}} + p_2^*(x_{21}^* + x_{22}^* - \omega_2^*) = 0$$



If (x^*, p^*) is a Walrasian eq. then $(x^*, \alpha p^*)$ with $\alpha > 0$ is also a Walrasian eq.

$\hookrightarrow$ relative prices matter

Ex. $I = L = 2$

$$u_i(x_{1i}, x_{2i}) = x_{1i}^{\alpha} \, x_{2i}^{1-\alpha} \qquad \alpha \in (0,1)$$

$$\omega_1 = (1, 2)$$

$$\omega_2 = (2, 1)$$

$m_i = p \cdot \omega_i$

for $l = 1$ $\quad x_{1i}(p, m_i) = \dfrac{\alpha m_i}{p_1} \qquad \forall i = 1,2$

$$m_1 = p_1 \cdot 1 + p_2 \cdot 2 = p_1 + 2p_2$$

$$m_2 = p_1 \cdot 2 + p_2 \cdot 1 = 2p_1 + p_2$$

$$x_{11}^*(p, p \cdot \omega_1) = \frac{\alpha(p_1 + 2p_2)}{p_1}$$

$$x_{21}^*(p, p \cdot \omega_1) = \frac{(1-\alpha)(p_1 + 2p_2)}{p_2}$$

$$x_{12}^*(p, p \cdot \omega_2) = \frac{\alpha(2p_1 + p_2)}{p_1}$$

$$x_{22}^*(p, p \cdot \omega_2) = \frac{(1-\alpha)(2p_1 + p_2)}{p_2}$$

$$x_{11}^*(p, p\cdot\omega_1) + x_{12}^*(p, p\cdot\omega_2) = \bar{\omega}_1$$

$$\frac{\alpha(p_1 + 2p_2)}{p_1} + \frac{\alpha(2p_1 + p_2)}{p_1} = 3$$

$$\alpha + 2\alpha\frac{p_2}{p_1} + \alpha 2 + \alpha\frac{p_2}{p_1} = 3$$

$$\cancel{3}\alpha + \cancel{3}\alpha\frac{p_2}{p_1} = \cancel{3}^{\,1}$$

$$\frac{p_2^*}{p_1^*} = \frac{1-\alpha}{\alpha}$$

check that the market for good 2 also clears at $\frac{p_2^*}{p_1^*} = \frac{1-\alpha}{\alpha}$.

What is a Pareto efficient allocation in the Edgeworth box?

An allocation x is Pareto efficient in the Edgeworth box if there exists no other allocation x' (in Edgeworth box) such that

$$x_i' \succsim x_i \quad \forall\ i = 1,2$$

and

$$x_i' \succ x_i \quad \text{for some } i$$