Advanced Microeconomics, 2025/2026

Master of Science in Economics

Problem Set 1

14/11/2025

Question 1 *Consider the strict preference relation $\succ$ as presented in class. Explain whether you agree with the statement that $\succ$ is transitive but NOT complete.*

Question 2 *Prove the following implication for a rational preference relation: if $x \succ y \succeq z$, then $x \succ z$.*

Question 3 *Consider the budget set $\mathcal{B}(p, w) = \{x \in X : p \cdot x \leq w\}$, with $p >> 0$ and $w > 0$. Show that if X is a convex set, then $B(p, w)$ is also convex.*

Question 4 *For each of the following sets of inequalities, draw the feasible region that they define in $\mathbb{R}^2_+$ and determine if it is closed, non-empty, bounded and convex.*

i) $\{x_2 + 2x_1 \leq 4; x_2 + 4x_1 \leq 6; x_1 \geq 0, x_2 \geq 0\}$.

ii) $\{x_2 + 3x_1 < 6; x_1 \geq 0; x_2 \geq 0\}$.

iii) $\{x_2 + 2x_1 = 4; x_2 + 3x_1 = 7\}$.

Question 5 *Show that a linear function is both concave and convex, but neither strictly concave nor strictly convex.*

Question 6 *Consider the following utility function $u(x_1, x_2) = x_1 - x_2$. Show that it represents a rational preference relation $\succsim$ that is locally non-satiated but not monotone.*

Question 7 *Draw indifference curves relating to the two commodities, Left and Right shoes of the same size, quality and design; and explain how the marginal rate of substitution varies along such indifference curves.*

Question 8 *Suppose preferences are represented by the Cobb-Douglas utility function* $u(x_1, x_2) = Ax_1^{\alpha} x_2^{1-\alpha}$, *for* $A > 0$ *and* $\alpha \in (0, 1)$.

1. *Assuming an interior solution, solve for the Walrasian demand functions.*

2. *Consider the logarithmic transformation* $v(x) = \ln u(x)$ *of the Cobb-Douglas utility function above and verify that the Walrasian demand functions are identical to those derived in the previous item. Explain.*

3. *Show that, for every commodity, the Walrasian demand function is homogeneous of degree one in wealth. Which implications?*

4. *Using the Walrasian demand functions you found before, derive the consumer's indirect utility function.*

5. *Show that the indirect utility function is strictly increasing in* w. *Argue that this is indeed a general property of any indirect utility function* $v(p, w)$.