Practice session 3

Game Theory - MSc EEBL

Guillaume Pommey
guillaume.pommey@uniroma2.eu

Simone Senesi
simone.senesi@students.uniroma2.eu

October 5, 2023

Exercise 1. *Headphones war*

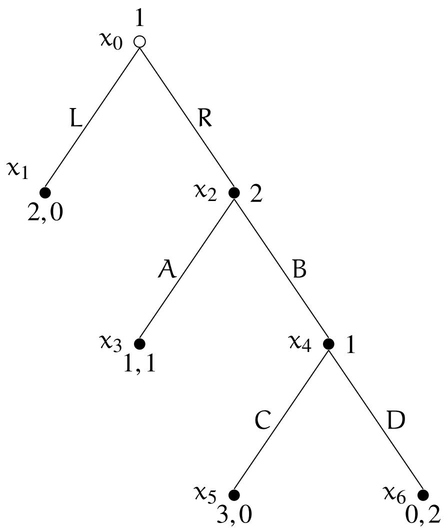
Two mobile phone producers, 1 and 2, use different standards for plugging headphones. Firm 1 is dominant and plays first. Firm 1 can either choose "no common standard" (N), a "basic common standard" (B) or a "sophisticated common standard" (S). If Firm A chooses N, the game ends. If Firm 1 chooses B or S then Firm B has to choose whether to "accept" (A) or "reject" (R) the standard. After Stage 2, the game ends. Payoffs are as follows:

- $(2,4)$ if Firm 1 chooses N;
- $(3,2)$ if Firm 1 proposes B and Firm 2 accepts (A);
- $(1,1)$ if Firm 1 proposes B and Firm 2 rejects (R);
- $(1,3)$ if Firm 1 proposes S and Firm 2 accepts (A);
- $(1,2)$ if Firm 1 proposes S and Firm 2 rejects (R).

1. Write the game as an *extensive-form* game and the game tree.
2. Carefully write the strategy space of each player.
3. Find the *pure-strategy Nash equilibria* of the game.
4. Find the *subgame-perfect Nash equilibrium* of the game.
5. For each Nash equilibria which is not a SPNE, indicate what is the strategy that is not *sequentially rational* (noncredible threats).

Exercise 2. ***A three-stage game***

Consider the following game.



1. Write the extensive form of this game.
2. Find all pure-strategy Nash equilibria.
3. Find the subgame-perfect Nash equilibrium.
4. Comment.

Exercise 3. ***Stackelberg Duopoly***

Two firms compete in a market by setting the quantities of a (homogeneous) good to produce (q_i, q_j). Each firm faces a *constant marginal cost* $c_k \in \mathbb{R}_+$. The two firms face the *inverse demand function* $P(Q) = a - bQ$, where Q is the aggregate quantity produced and $(a, b) \in \mathbb{R}^2_+$ are demand parameters. Payoffs are given by each firm's profits.

The difference with the Cournot setting is that we are not going to assume that firms choose their quantity simultaneously. Instead, we are going to assume that one firm, say i, is the *leader* while firm j is the *follower*. The leader, firm i, sets its quantity q_i first. Then the follower, firm j, observes q_i and decides its own quantity q_j. This is what we call a Stackelberg Duopoly.

The game is a dynamic game in which firm i plays first a $q_i \in \mathbb{R}_+$ and second firm j plays a $q_j \in \mathbb{R}_+$. Then, the two quantities determine the market price through the inverse demand function and firms receives their profits.

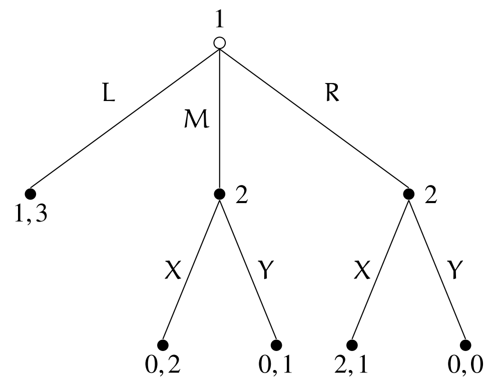
1. Write the profit of each firm at the end of the game, for any given pair of quantities (q_i, q_j).

2. Starting at the second period, assume that firm i's choice of quantity in the first period is q_i. Find the best-response function of firm j to this quantity q_i. Call this best-response function $q_j(q_i)$.

3. Starting now at the first period, find firm i's optimal choice of q_i given that it anticipates firm j to best respond according to $q_j(q_i)$. Call the SPNE of this game (q_i^*, q_j^*).

Exercise 4. ***Additional exercise***

Consider the following dynamic game between player 1 and player 2.



1. Write this game as a payoff matrix. Carefully explain why you write player 2's strategies as a couple of actions.

2. Find all *pure-strategy Nash equilibria* of this game.

3. Find the unique *subgame-perfect Nash equilibrium.*

4. Explain why (L, XY) is a Nash equilibrium of the dynamic game but not a subgame-perfect Nash equilibrium.