Exercises 3^{rd} Week

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Exercise 1

Let $X_1, \ldots, X_n$ be iid r.v. distributed as continuous uniform distribution on $[0, \theta]$. The probability distribution function of X_i for each i is:

$$f(x|\theta) = \begin{cases} \theta^{-1}, & 0 \leq x \leq \theta \\ 0, & otherwise \end{cases}$$

Consider $T = max(X_1, X_2, \ldots, X_n)$, discuss whether is an unbiased estimator for θ.

Exercise 2

Let $(X_1, \ldots, X_n)$ be independent identically distributed random variables with p.d.f.

$$f(x) = \theta^2 x \exp(-\theta x) \qquad x > 0$$

Is $T(X_1, \ldots, X_n) = 1/X_1$ an unbiased estimator of θ?

Exercise 3

Let $(X_1, \ldots, X_n)$ be independent identically distributed random variables with $E(X) = \mu$, $Var(X) = \sigma^2$, Are the following estimators unbiased estimator for σ^2?

$$\begin{aligned} T_1(X_1, \ldots, X_n) &= \frac{(X_1 - X_2)^2}{2} \\ T_2(X_1, \ldots, X_n) &= \frac{(X_1 + X_2)^2}{2} - X_1 X_2 \end{aligned}$$

Exercise 4

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables with expected value μ and variance σ^2. Consider the following estimator of μ:

$$T_n(a) = a \times X_n + (1-a) \times \bar{X}_{n-1}$$

where X_n is the $n-th$ observed random variable and $\bar{X}_{n-1}$ is the sample mean based on $n-1$ observations.

1. Find value of a such that $T_n(a)$ is an unbiased estimator for μ

2. Find value of $a\star$ such that $T_n(a\star)$ is the most efficient estimator for μ within the class $T_n(a)$?