

Neyman Pearson test - Exponential

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as $Exponential(\lambda)$ such that $E(X) = \frac{1}{\lambda}$ known

$$H_0 : \lambda = \lambda_0 \text{ vs } H_1 : \lambda = \lambda_1 \quad \lambda_0 > \lambda_1$$

Reject when

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$$\frac{f(x_1, x_2, \dots, x_n | \lambda_0)}{f(x_1, x_2, \dots, x_n | \lambda_1)} < k$$

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$$\frac{f(x_1, x_2, \dots, x_n | \lambda_0)}{f(x_1, x_2, \dots, x_n | \lambda_1)} < k$$
$$\frac{\prod_i \lambda_0 \exp(-\lambda_0 x_i)}{\prod_i \lambda_1 \exp(-\lambda_1 x_i)} < k$$
$$\frac{\lambda_0^n \exp(-\lambda_0 \sum x_i)}{\lambda_1^n \exp(-\lambda_1 \sum x_i)} < k$$

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- Reject when

$$\begin{aligned} \left(\frac{\lambda_0}{\lambda_1}\right)^n \exp\left(-(\lambda_0 - \lambda_1) \sum_i x_i\right) &< k \\ \exp\left(-(\lambda_0 - \lambda_1) \sum_i x_i\right) &< k_2 \\ -(\lambda_0 - \lambda_1) \sum_i x_i &< k_3 \\ \sum_i x_i &> k \end{aligned}$$

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$$R = \{(x_1, x_2, \dots, x_n) : \sum_i x_i > k\}$$

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Which value for k ? USE α

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$$R = \{(x_1, x_2, \dots, x_n) : \sum_i x_i > k\}$$

$$P((x_1, x_2, \dots, x_n) \in R | H_0 \text{ true}) = \alpha$$

$$P\left(\sum_i x_i > k | \lambda = \lambda_0\right) = \alpha$$

$$\sum_i X_i \sim \text{Gamma}(\lambda_0, n)$$

$$R = \{\underline{x} : \sum_i x_i > \text{InvGamma}(1 - \alpha, \lambda_0, n)\}$$

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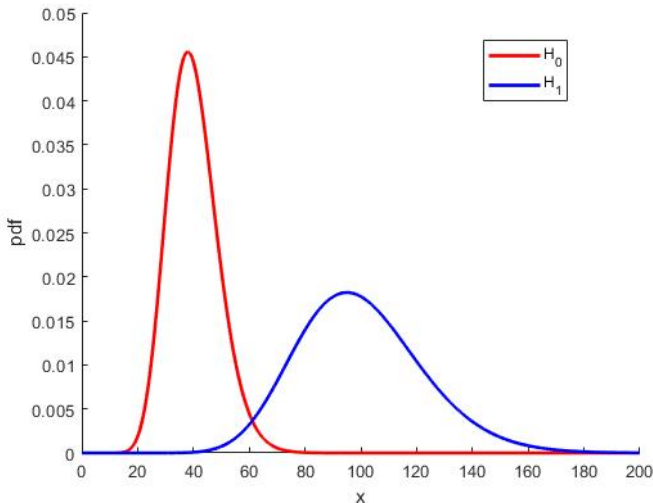
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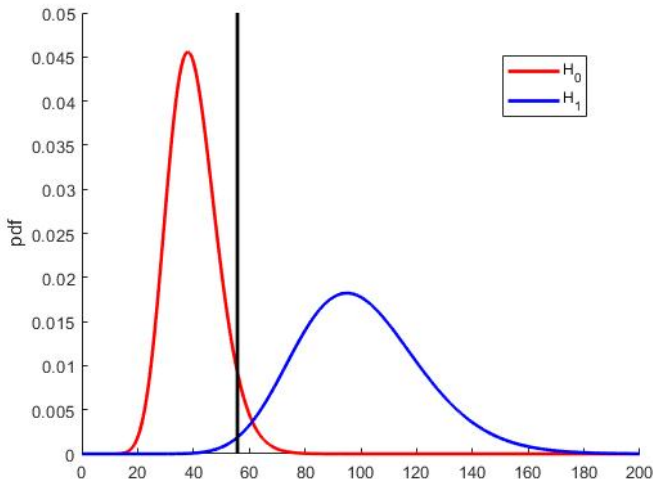
Neyman Pearson - Exponential, $\lambda_0 = 0.5$ $\lambda_1 = 0.2$ $n = 20$

Figure: Distribution of $\sum X_i$ under null and alternative hypothesis



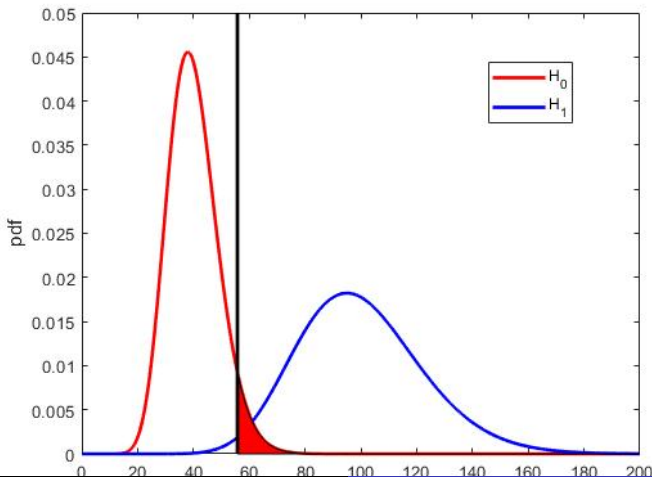
Neyman Pearson - Exponential, $\lambda_0 = 0.5$ $\lambda_1 = 0.2$ $n = 20$

Figure: Distribution of $\sum X_i$ under null and alternative hypothesis and Rejection Region



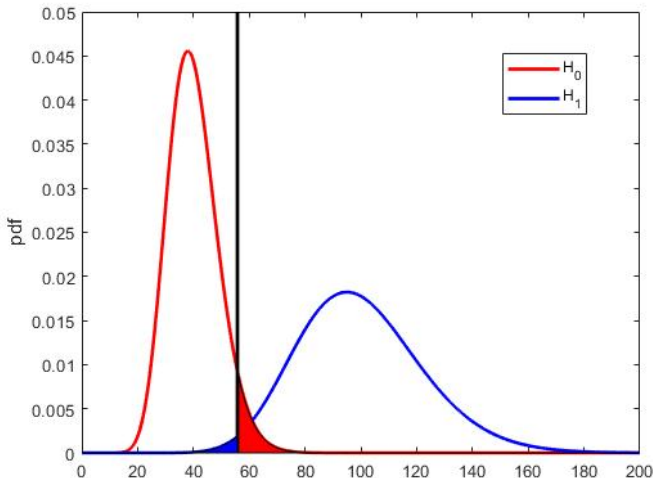
Neyman Pearson - Exponential, $\lambda_0 = 0.5$ $\lambda_1 = 0.2$ $n = 20$, α

Figure: Distribution of $\sum X_i$ under null and alternative hypothesis



Neyman Pearson - Exponential, $\lambda_0 = 0.5$ $\lambda_1 = 0.2$ $n = 20$

Figure: Distribution of $\sum X_i$ under null and alternative hypothesis, α and β



Neyman Pearson test - Gaussian

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 known,

$$H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu = \mu_1 \quad \mu_0 > \mu_1$$

Reject when

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Reject when

$$\frac{f(x_1, x_2, \dots, x_n | \mu_0)}{f(x_1, x_2, \dots, x_n | \mu_1)} < k$$
$$\frac{\prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}} \right) \exp \left(-\frac{(x_i - \mu_0)^2}{2\sigma^2} \right)}{\prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}} \right) \exp \left(-\frac{(x_i - \mu_1)^2}{2\sigma^2} \right)} < k$$

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Neyman Pearson test - Gaussian

$$\begin{aligned}\frac{f(x_1, x_2, \dots, x_n | \mu_0)}{f(x_1, x_2, \dots, x_n | \mu_1)} &< k \\ \frac{\prod_i \left(\frac{1}{\sqrt{2\pi}\sigma^2} \right) \exp \left(-\frac{(x_i - \mu_0)^2}{2\sigma^2} \right)}{\prod_i \left(\frac{1}{\sqrt{2\pi}\sigma^2} \right) \exp \left(-\frac{(x_i - \mu_1)^2}{2\sigma^2} \right)} &< k \\ \frac{\left(\frac{1}{\sqrt{2\pi}\sigma^2} \right)^n \exp \left(-\sum_i \frac{(x_i - \mu_0)^2}{2\sigma^2} \right)}{\left(\frac{1}{\sqrt{2\pi}\sigma^2} \right)^n \exp \left(-\sum_i \frac{(x_i - \mu_1)^2}{2\sigma^2} \right)} &< k\end{aligned}$$

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$$\left(-\sum_i \frac{(x_i - \mu_0)^2}{2\sigma^2} \right) - \left(-\sum_i \frac{(x_i - \mu_1)^2}{2\sigma^2} \right) < k_1$$

$$\left(-\sum_i (x_i - \mu_0)^2 \right) - \left(-\sum_i (x_i - \mu_1)^2 \right) < k_2$$

$$-\mu_0^2 + \mu_1^2 + 2 \sum_i x_i (\mu_0 - \mu_1) < k_3$$

$$\sum_i x_i < k$$

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Reject when

$$\lambda(\mathbf{x}) = \frac{\sup_{\mu_0} L(\mu|\mathbf{x})}{\sup_{\mu} L(\mu|\mathbf{x})} \leq k$$

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Reject when

$$\frac{\prod_i \left(\frac{1}{\sqrt{2\pi\hat{\sigma}_0^2}} \right) \exp \left(-\frac{(x_i - \mu_0)^2}{2\hat{\sigma}_0^2} \right)}{\prod_i \left(\frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \right) \exp \left(-\frac{(x_i - \hat{\mu})^2}{2\hat{\sigma}^2} \right)} < k$$
$$\hat{\sigma}_0^2 = \frac{\sum_i (x_i - \mu_0)^2}{n}$$
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$$\frac{\left(\frac{1}{\sqrt{2\pi}\hat{\sigma}_0^2}\right)^n \exp\left(-\frac{n}{2}\right)}{\left(\frac{1}{\sqrt{2\pi}\hat{\sigma}^2}\right)^n \exp\left(-\frac{n}{2}\right)} < k$$
$$\frac{\hat{\sigma}^2}{\hat{\sigma}_0^2} < k$$

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Reject when

$$\begin{aligned} \frac{\sum_i (x_i - \bar{x})^2}{\sum_i (x_i - \mu_0)^2} &< k \\ \frac{\sum_i (x_i - \bar{x})^2}{\sum_i (x_i - \bar{x})^2 + n(\bar{x} - \mu_0)^2} &< k \\ \frac{1}{1 + n \frac{(\bar{x} - \mu_0)^2}{\sum_i (x_i - \bar{x})^2}} &< k \end{aligned}$$

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Reject when

$$n \frac{(\bar{x} - \mu_0)^2}{\sum_i (x_i - \bar{x})^2} \geq k$$

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Which value for k ?

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Reject when

$$R = \left\{ \left| \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \right| \geq t_{1-\alpha/2}^{n-1} \right\}$$

Likelihood Ratio Test - Gaussian

Let $(X_1, \dots, X_n)$ be a random sample of i.i.d. random variables distributed as $Gaussian(\mu, \sigma^2)$, σ^2 unknown,

$$H_0 : \mu = \mu_0 \text{ vs } H_1 : \mu \neq \mu_0$$

Reject when

$$R = \left\{ \left| \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \right| \geq t_{1-\alpha/2}^{n-1} \right\}$$

Likelihood ratio test: Example

Suppose that the distribution of lifetimes of TV tubes can be adequately modelled by an exponential distribution with mean λ so

$$f(x|\lambda) = \frac{1}{\lambda} \exp\left(-\frac{x}{\lambda}\right)$$

for $x \geq 0$. Under usual production conditions, the mean lifetime is 50 hours but if a fault occurs in the process, the mean lifetime drops to 40 hours. A random sample of 20 tube lifetimes is taken in order to test the hypotheses

$$H_0 : \lambda = 50 \quad H_1 : \lambda = 30.$$

And the following statistics is observed $\sum_i x_i = 680$

Use the Neyman-Pearson lemma to find the most powerful test with significance level $\alpha = 0.05$.

Likelihood ratio test: Example

$$R = \left\{ x : \sum_i x_i \leq K \right\}$$

$$R = \left\{ x : \sum_i x_i \leq 662.73 \right\}$$

$$R = \{ x : \bar{x} \leq 33.14 \}$$

ACCEPT NULL HYPOTHESIS