Exercises 1^{st} Week

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Exercise 1

Mutual funds specializing in small companies gained an average of 8.4% in the first half of 2014. Assume that the distribution of returns in the first half of 2014 for those mutual funds is distributed as a normal random variable with a mean of 8.4% and a standard deviation of 10%.

a) If a sample of 25 mutual funds is selected, find the probability that the sample mean return is greater that 12%

b) the sample mean return is less than 10

Exercise 2

The waiting time at the post office is a uniform distribution between 5 and 45 minutes

a) What is the probability a client will wait less than 30 minutes?

b) You will go to the post office next week every day from Monday to Friday,

   i) what is the probability the maximum time you will be wait is between 30 and 32 minutes?

   ii) what is the probability the minimum time you will be wait is between 10 and 12 minutes?

Exercise 3

The waiting time at the post office is distributed as an exponential with mean 20 minutes.

a) What is the probability a client will wait less than 30 minutes?

b) You will go to the post office next week every day from Monday to Friday: what is the probability the minimum time you will be wait is between 10 and 12 minutes?

Exercise 4

Dartmouth College would like to have 1050 freshmen. This college cannot accommodate more than 1060. Assume that each applicant accepts with probability 0.6 and that the acceptances can be modeled by Bernoulli trials.

a) If the college accepts 1700, what is the probability that it will have too many acceptances?

Exercise 5

The lifetime of an electronic circuit board (in years) is modelled as $X \sim \text{Exp}(\beta)$ with mean $\beta = 2$. A batch of $n = 25$ boards is tested, and $\bar{X}$ denotes the sample mean lifetime.

a) Give $E(\bar{X})$ and $Var(\bar{X})$ exactly.

b) Identify the exact distribution of $S_n = \sum_{i=1}^{n} X_i$ (name it and give its parameters), and use it to compute $P(\bar{X} > 2.5)$ exactly.

c) Approximate the same probability using the CLT, and comment on the agreement with (b), keeping in mind that the population is far from Normal (it is skewed) and n is only moderately large.

Exercise 6

In class it was shown that if X is continuous with cdf F_X, then $Y = F_X(X) \sim \text{Uniform}(0, 1)$.

1. Prove the *converse* direction: in particular: Let $U \sim \text{Uniform}(0, 1)$ and define $X = -\frac{1}{\lambda}\ln(1-U)$. Verify *directly*, by computing $P(X \le x)$ from the definition of X (not by quoting part (b)), that $X \sim \text{Exp}(\lambda)$.