Exercise 1

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Exercise 2

Let $(X_1, \ldots, X_n)$ be independent identically distributed random variables from the Cauchy distribution, with p.d.f.

$$f(x) = \frac{1}{\pi(x-\theta)^2}$$

1. Find a sufficient statistics for θ

Exercise 2

$$
\begin{aligned}
f(x) &= \frac{1}{\pi(x-\theta)^2} \\
f(x_1,\ldots,x_n) &= \prod_i \frac{1}{\pi(x_i-\theta)^2}
\end{aligned}
$$

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Exercise 3

Let $(X_1, \ldots, X_n)$ be a random sample of i.i.d. random variables distributed as follows:

$$f(x;\theta) = \frac{\theta 2^{\theta}}{x^{\theta+1}} \qquad x > 2$$

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$$
\begin{aligned}
f(x_1, \ldots, x_n; \theta) &= \prod_i \frac{\theta 2^\theta}{x_i^{\theta+1}} \qquad x_i > 2 \\
f(x_1, \ldots, x_n; \theta) &= \frac{\theta^n 2^{n\theta}}{\prod x_i^{\theta+1}} \qquad (x_i > 2\,?) \\
f(x_1, \ldots, x_n; \theta) &= \frac{\theta^n 2^{n\theta}}{(\prod x_i)^{\theta+1}} \qquad (x_i > 2\,?)
\end{aligned}
$$

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Exercise 4

Suppose that X_1 and X_2 are two independent Bernoulli random variables with parameter p; $0 < p < 1$. Show that the statistics $T = X_1 - X_2$ is not a sufficient statistics for p.

Exercise 4

Definition of Sufficient Statistics

A statistic $T(X)$ is a sufficient statistic for θ if the conditional distribution of the sample X given the value of $T(X)$ does not depend on θ.

Distribution of (X_1, X_2), given $T = X_1 - X_2$

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Exercise 4

Distribution of (X_1, X_2), given $X_1 - X_2$

(x_1, x_2)	$x_1 - x_2$	$p(X_1, X_2 \mid X_1 - X_2 = t)$
(0,0)	-1	0
(0,1)	-1	1
(1,0)	-1	0
(1,1)	-1	0
(0,0)	0	$(1-p)^2/(p^2+(1-p)^2)$
(0,1)	0	0
(1,0)	0	0
(1,1)	0	$p^2/(p^2+(1-p)^2)$
(0,0)	1	0
(0,1)	1	0
(1,0)	1	1
(1,1)	1	0

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Exercize 5

Let consider the pdf

$$f(x;\theta) = \frac{1}{\theta}\exp\left(1 - \frac{x}{\theta}\right) \qquad 0 < \theta < x$$

Find a sufficient statistic for parameter θ.

Factorization Theorem

Exercise 5

$$\begin{aligned}
f(x_1, x_2, \ldots, x_n; \theta) &= \prod_{i=1}^{n} \left(\frac{1}{\theta} \exp\left(1 - \frac{x_i}{\theta}\right) I_{(\theta,\infty)}(x_i) \right) \\
&= \frac{1}{\theta^n} \exp\left(\sum_{i=1}^{n} \left(1 - \frac{x_i}{\theta}\right) \right) \prod_{i=1}^{n} I_{(\theta,\infty)}(x_i) \\
&= \frac{1}{\theta^n} \exp\left(n - \frac{1}{\theta} \sum_{i=1}^{n} x_i \right) I_{(\theta,\infty)}(x_{(1)})
\end{aligned}$$

Exercise 5

Therefore we can set: $h(x) = 1$ and

$$g(T_1(x), T_2(x); \theta) = \frac{1}{\theta^n} \exp\left(n - \frac{1}{\theta}\sum_{i=1}^{n} x_i\right) I_{(\theta,\infty)}(x_{(1)})$$

Sufficient Statistics: Sample sum and Sample minimum

$T_1(x) = \sum_{i=1}^{n} x_i$ and $T_2(x) = x_{(1)}$

Two jointly sufficient statistics for the parameter θ

The dimension of a minimal sufficient statistic (two) is greater than the dimension of the parameter space (one).

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