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Calculus and Optimization
Problem Set 5 - Solutions

↪**Topics**
Derivative of power series, stationary points (min, max, saddle points)

Exercise 1
Compute the first and second derivatives of the following power series.

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

Solution
We know that the derivative of the sum is equal to the sum of the derivatives. This property holds also for the infinite sums in some special cases (among which the power series). We rewrite the series as

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x} = (1-x)^{-1}$$

Terefore, the derivative of the left hand side will be

$$0 + 1 + 2x + 3x^2 + \cdots$$

The derivative of the right hand side is

$$-1(1-x)^{-2}(-1)$$

We can conclude that

$$\sum_{n=1}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}$$

Second derivative:
Right hand side

$$\sum_{n=2}^{\infty} n(n-1)x^{n-2} =$$

Left hand side

$$-2(1-x)^{-3}(-1)$$

We can conclude that

$$\sum_{n=2}^{\infty} n(n-1)x^{n-2} = \frac{2}{(1-x)^3}$$

Exercise 2

Find the stationary points of the following functions e discuss the behavior of the functions in those points.

$$g(x,y) = x^3 - 3xy^2 + y^4$$

Local min in $(\frac{3}{2}, \frac{3}{2})$, $(\frac{3}{2}, -\frac{3}{2})$ (eigenvalues all positive $\frac{9}{2}(3 \pm \sqrt{5})$). Saddle point in $(0,0)$ (use $g(x,0), g(0,y)$).

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$$h(x,y) = x^4 + x^2y^2 - 2x^2 + 2y^2 - 8$$

Local min in $(1,0), (-1,0)$ (eigenvalues 6,8). Saddle point in $(0,0)$ (eigenvalues ± 4).

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$$f(x,y) = 2x^3 + y^3 - 3x^2 - 3y + 5$$

Local Max in $(0,-1)$ (eigenvalues -6); local min in $(1,1)$ (eigenvalues 6). Saddle points in $(0,1), (1,-1)$ (eigenvalues ± 6).

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