Surname .... First name ....
Matricola Number .... PhD/Master ....
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Mathematics – Teacher: Paolo Gibilisco

(Simulation 4 of) Written Examination

Rules: you cannot use any textbook, lecture notes, neither any electronic device (very welcome a traditional watch). Write your solutions on the enclosed sheets. You will receive additional sheets for the preliminary drafts. The solutions of the exercises must be detailed. One of the exercises requires the proof of a theorem. For the quizzes you have simply to indicate your choice in the True-False alternative. You have **two hours and half** to finish your work. An **identity card** or a passport is needed to participate.

Quiz 1. *Correct answer: points 2. Wrong answer: points -1. No answer: points 0.*

$$\cos(5) \leq |e^{5i}|$$

TRUE FALSE

Quiz 2.
If λ is an eigenvalue of the real matrix A then λ^2 is an eigenvalue of the matrix A^2.

TRUE FALSE

Quiz 3.
If A, B are real matrices and $AB = BA$ then A and B are 1×1 matrices (namely they are numbers).

TRUE FALSE

Quiz 4.
Suppose that $f \in \mathcal{C}^1(\mathbb{R})$ (this means that the derivative of f exists and is continuous on $\mathbb{R}$) and that f is not constant. Then the number of stationary points of f is always finite.

TRUE FALSE

Quiz 5.

If the complex number $1+i$ is an eigenvalue of the real matrix A then there exists a $\mathcal{C}^2$ function f and a point P_0 such that for the Hessian matrix one has $H_f(P_0) = A$.

TRUE FALSE

Quiz 6.

If A^c, B are independent events then also A, B^c are independent events.

TRUE FALSE

Quiz 7.

The correlation coefficient of two random variables is scale invariant.

TRUE FALSE

Exercise 1. *6 points*

Solve the Cauchy problem

$$\begin{cases} y' = \sqrt{x}y & x \geq 0 \\ y(0) = 2 \end{cases}$$

Exercise 2. *6 points*

Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 8 & 8 \\ 3 & 8 & 19 \end{pmatrix}, \qquad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}.$$

i) Find the Cholesky decomposition $A = LL^t$.
ii) For L given by the point *i)* solve the system $LX = b$.

Exercise 3. *6 points*

Find the stationary points of the following function and discuss the behavior of the function in those points.

$$f(x, y, z) = y^2 + z^4 + z^2 + x^3 - 2xy$$

Exercise 4. *6 points*

The function $g : \mathbb{R} \to \mathbb{R}$ is defined by

$$g(x) = b \cdot \cos x \cdot 1_{[-\frac{\pi}{2}, \frac{\pi}{2}]}(x)$$

a) Fix b so that g is a density.
b) Let X be a random variable such that g is its density: calculate the c.d.f $F_X(\cdot)$
b) Solve the equation $F_X(t) = \frac{1}{2}$.

Exercise 5. *6 points*

Prove the (weak) Law of Large Numbers.