Notes on the monopolist problem 2015-2016

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May 9, 2016

Overview

Definitions

- A monopoly is a firm that is the sole seller of a product that does not have any close substitute.
- A monopoly arises because of barriers to entry, which may have three may origins:
    - the single firm owns key resources;
    - the government gives to a firm the exclusive right to produce a good (e.g. patents and copyrights),
    - a single firm can supply the entire market at smaller costs than many firms could, i.e. economies of scale originate *natural* monopolies.

Pricing

The monopolist faces a downward-sloping demand curve (the entire market demand), therefore she is price *maker* and chooses both price and quantity (contrary to a competitive firm which is price *taker* as she perceives an horizontal demand curve).

Market demand

In general, the downward-sloping market demand may have any form. Here we assume the *inverse* demand function being a simple linear function $P(Q) = a - bQ$, where P is the market price, Q is the market quantity, $a > 0$ and $b > 0$ are the vertical intercept and the slope, respectively.

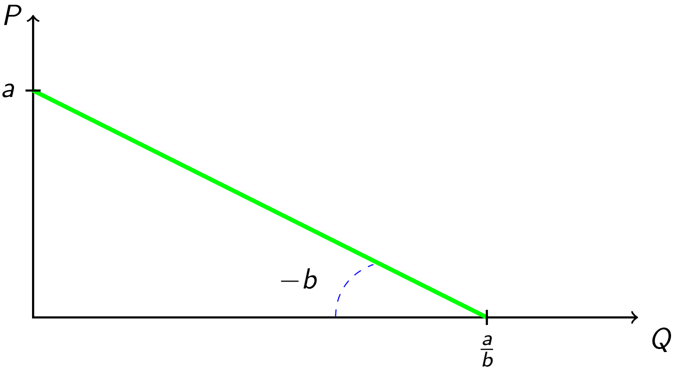

Figure 1: A linear demand curve.

Definitions-1

Profit

The firm's profits, Π, amount to the difference between total revenues, TR, and total costs, TC. It is assumed that the final objective of a firm is the maximization of profits.

Revenues

- Total revenues equal the product of price and quantities, $TR(Q) = PQ$.
- Marginal revenue, MR, is the marginal change of total revenues by selling a unit more of product. Mathematically, it is the derivative of TR w.r.t. Q:

$$MR(Q) = \frac{\partial TR}{\partial Q}. \tag{1}$$

Definitions-2

Costs

- Total costs depend on the cost structure; e.g. $TC(Q) = F + cQ$, with $F > 0$ and $c > 0$, represents a linear cost structure with a fixed part F and a variable part (the cost c for each unit of product).
- The marginal cost of production, MC, is the marginal change in total cost by producing a unit more of good. Mathematically, it is the derivative of TC w.r.t. Q:

$$MC(Q) = \frac{\partial TC}{\partial Q} \tag{2}$$

The maximization problem-1

The problem of the firm can be written as the choice of the quantity maximizing the profit function:

$$\max_{Q} \Pi(Q) = TR(Q) - TC(Q) \tag{3}$$

The optimality condition, or first-order condition (FOC), is

$$\frac{\partial \Pi}{\partial Q} = \frac{\partial TR}{\partial Q} - \frac{\partial TC}{\partial Q} = MR(Q) - MC(Q), \tag{4}$$

or, in general,

$$MR = MC. \tag{5}$$

The above condition states the equality between marginal cost and marginal revenue. The logic is the following: $MR > MC$ can't be optimal because producing a unit more generates more revenues than costs, so increasing profits; $MR < MC$ neither, because reducing production reduce costs more than revenues, so increasing profit.

The maximization problem-2

In case of a competitive firm, as she is price taker, the equation (5) becomes

$$MR = \frac{\partial PQ}{\partial Q} = P = MC, \tag{6}$$

since the price is out of the firm's control. When the firm is monopolistic, she does not take the price as given. Instead, she knows the relation between price and quantity implied by the inverse market demand curve, $P(Q)$ with $\partial P/\partial Q < 0$. Therefore,

$$MR(Q) = \frac{\partial P(Q) * Q}{\partial Q} = \frac{\partial P}{\partial Q} Q + P \frac{\partial Q}{\partial Q} = \frac{\partial P}{\partial Q} Q + P. \tag{7}$$

The monopoly's marginal revenue sums up $P > 0$, the revenue from the marginal quantity (common to the competitve firm), and $\frac{\partial P}{\partial Q} Q < 0$, measuring the loss due to price reduction on the quantities sold before.

The maximization problem-3

Using the linear specification $P(Q) = a - bQ$, the marginal revenues are

$$MR(Q) = \frac{\partial TR(Q)}{\partial Q} = \frac{\partial(aQ - bQ^2)}{\partial Q} = a - 2bQ. \tag{8}$$

or, equivalently,

$$MR(Q) = \frac{\partial P}{\partial Q}Q + P = -bQ + a - bQ = a - 2bQ. \tag{9}$$

Assuming a simple cost function $TC(Q) = cQ$, then the marginal cost curve is flat and reads

$$MC(Q) = c. \tag{10}$$

The maximization problem-3.1

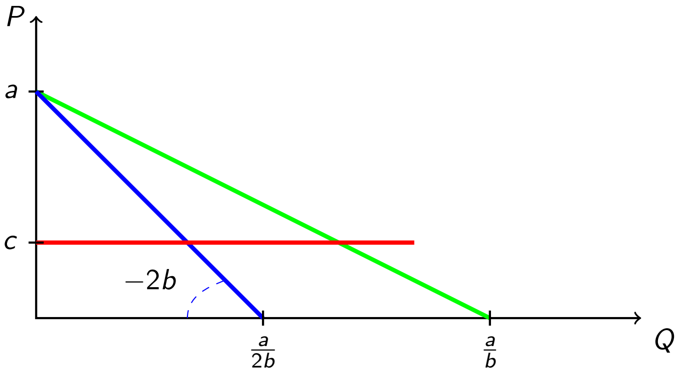

Figure 2: Marginal revenues (blue line) from a linear demand and marginal cost (red) from linear cost function.

The maximization problem-4

The problem (3) can be written as

$$\max_{Q} \Pi(Q) = P(Q)Q - cQ = (a - bQ)Q - cQ, \tag{11}$$

with FOC

$$\frac{\partial \Pi}{\partial Q} = a - 2bQ - c = 0 \Leftrightarrow Q^M = \frac{a-c}{2b}. \tag{12}$$

Q^M defines the optimal monopolistic quantity, labeled with the superscript M. In Figure 2, Q^M is determined by the intersection between MR and MC. The resulting monopolistic price, instead, can be found on the demand curve:

$$P^M = P(Q^M) = a - bQ^M = \frac{a+c}{2} \tag{13}$$

Clearly, if $a < c$ then the firm's optimal choice is $Q^M = 0$ because, otherwise, positive production would lead to losses.

The maximization problem-4.1

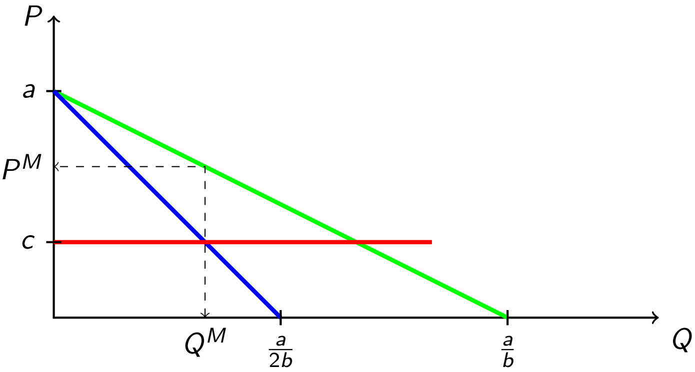

Figure 3: Monopoly Price and Quantity.

Surplus under Monopoly

The green area identifies the consumers surplus, that is the sum of the differences between the price at which consumers are willing to buy each unit (corresponding to the demand function) and the price actually paid for that unit, P^M.

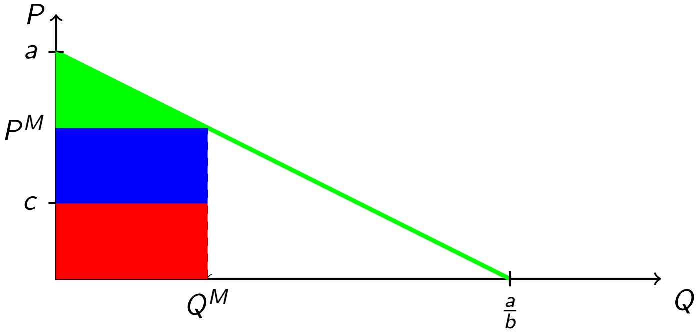

Figure 4: Total cost (red area), total revenue (blue+red), profits (blue), consumers surplus (green), total surplus (blue+green).

Surplus in Perfect Competition

On the same graph we can draw also the market quantity under perfect competition, PC. In this case, from (6), we have $P^{PC} = c$ and

$$P^{PC} = a - bQ^{PC} = c \Leftrightarrow Q^{PC} = \frac{a-c}{b} = 2Q^M. \tag{14}$$

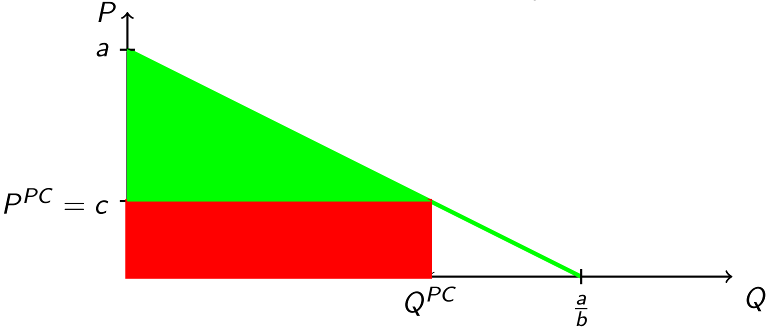

Figure 5: Total cost=total revenues (red area), consumers surplus=total surplus (green).

Monopoly vs PC

- $P = MC$ is an efficient condition for the society as whole because the market quantity reaches the point in which: by producing more, consumer's utility (or the willngness to pay) would be lower than the marginal production cost; by producing less, there would be unexploited gains from trade.
- The total surplus is the consumers surplus, as represented by the green area in Figure 5. Society maximizes surplus but industry's profits are zero.
- Under monopoly, on the contrary, the comparison of figures 4-5 highlights that the society would benefit from increasing production above Q^M where the willingness to pay is higher than the marginal cost. Nevertheless, pricing above the marginal cost allows the firm to extract surplus from consumers and to make positive profits.

Dead-Weight Loss

The social surplus under - this simple version of - monopoly pricing is lower than under perfect competition. This difference is called Deadweight Loss, the yellow area in the above figure.

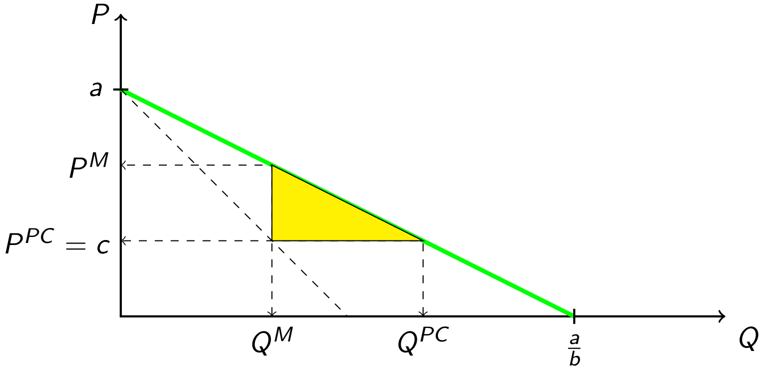

Figure 6: Deadweight loss (yellow) of monopoly w.r.t. perfect competition.

Definition

The price elasticity of demand measures the percentace change in quantity as consequence of a unitary percentage increase in price. It is a pure number, i.e. without unit measure.

Discete measure

$$\epsilon = \frac{\Delta Q/Q}{\Delta P/P} = \frac{\Delta Q}{\Delta P}\frac{P}{Q} < 0 \tag{15}$$

The elasticity is a negative number because the relation between aggregate demand and price is usually negative. When reasoning about elasticity involves positive values, this is because many refer to its absolute value.

Conventions

In general, the value of the elasticity is not fixed over the demand but can vary according the point, the couple (Q, P). A value $0 > \epsilon > -1$ defines a point on the market demand where quantity is sensitive less than proportionally to price. The portion of the demand with $\epsilon \in (0, -1)$ is said to be *inelastic*; $\epsilon < -1$ defines the *elastic* portion; when $\epsilon = -1$ the portion of the demand is unitary elastic.

Continuous measure

$$\epsilon = \frac{\partial Q}{\partial P}\frac{P}{Q} \tag{16}$$

The first fraction is the derivative of the inverse demand function. For example, using the linear demand $P(Q) = a - bQ$, the inverse demand is $Q(P) = \frac{a-P}{b}$ with $\partial Q/\partial P = -1/b < 0$ such that

$$\epsilon = \frac{-P}{a-P} = -\frac{a-bQ}{bQ} < 0 \tag{17}$$

Elasticity and Monololy

It is possible to write the optimality condition of the monoply in terms of price elasticity. From the optimal choice of the monopoly, using expression (7) for the marginal revenue, the general condition (5) can be written as

$$\frac{\partial P}{\partial Q}Q + P = MC \Leftrightarrow \frac{P^M - MC}{P^M} = -\frac{\partial P}{\partial Q}\frac{Q^M}{P^M} > 0. \tag{18}$$

Elasticity and Monololy

In the second equivalence of (18) the RHS is called Lerner Index, L, and measure the price mark-up over the marginal cost. It ranges between 0 ($P = MC$) and 1 ($P > MC = 0$). On the LHS the expression is similar to (16). More precisely,

$$\frac{\partial P}{\partial Q} Q + P = MC \Leftrightarrow L = -\frac{1}{\epsilon}. \tag{19}$$

Since $L \in [0, 1]$, the second equality proves that the monopolistic firm will never set a price where the demand is inelastic, i.e. where consumers' demand is not so sensitive to price. Instead, she maximizes profits choosing a point where $\epsilon \leq -1$. The intuition is that if the firm sets a price where $\epsilon > -1$, then she can profitably increase the price without shrinking the quantity too much, thus increasing total profits.

Elasticity and Monololy

To be sure, consider our linear demand and the costant marginal cost. The elasticity comes from eq. (17). Thus,

$$L = -\frac{1}{\epsilon} \Leftrightarrow \frac{P-c}{P} = -\frac{1}{\frac{-P}{a-P}} \Leftrightarrow P - c = a - P \Leftrightarrow P = \frac{a+c}{2} = P^M, \quad (20)$$

which would not have been otherwise because these expressions are derived by the optimality condition.

The End