Microeconomics for Business
Practice Session 1 - Solutions

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Exercise 1. In the following normal-form game, what strategies survive iterated elimination of strictly dominated strategies? What are the pure-strategy Nash equilibria?

	L	C	R
T	$2,0$	$1,1$	$4,2$
M	$3,4$	$1,2$	$2,3$
B	$1,3$	$0,2$	$3,0$

Solution 1. The player1's payoffs associated with strategy T is $u(T,\cdot) = (2,1,4)$, where each element corresponds to player2 playing L, C, and R, respectively; with strategy M is $u(M,\cdot) = (3,1,2)$, B:$u(M,\cdot) = (1,0,3)$. For player2: $u(\cdot,L) = (0,4,3)$; and so on. It is immediate to see that each element of B is strictly lower than each element of T, that is B is strictly dominated by T. If we delete B, then the initial game reduces to the following matrix.

	L	C	R
T	$2,0$	$1,1$	$4,2$
M	$3,4$	$1,2$	$2,3$

Iterating what previously done, we can see that C is strictly dominated by R. Removing C we finally obtain the following matrix.

	L	R
T	$2,0$	$\underline{4},\underline{2}$
M	$\underline{3},\underline{4}$	$2,3$

Now there are no strictly dominated strategies. But for each strategy of player2, player1 has a best replay. If L, then player1 prefers to play M; if R, then T. The same works for player2: if T, then R; if M, then L. Thus, in the strategy profile (M,L) nobody has the inventive to change unilaterally its strategy, and also in (T,R). The set of pure-strategy Nash Equilibria is $\{(M,L),(T,R)\}$.

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Exercise 2. Anne and Bob are trying to decide on an evening's entertainment. In particular, they must choose to attend either the opera or a prize fight. Both players would rather spend the evening together than apart, but Bob would rather prefer they be together at the prize fight while Anne would rather prefer they be together at the opera, as represented in the payoff matrix below. Let Anne be the row player, and Bob the column player. What are the Nash equilibria of this game?

	Opera	*Fight*
Opera	$2, 1$	$0, 0$
Fight	$0, 0$	$1, 2$

Solution 2. Best responses are indicated in the following matrix by underlining the corresponding payoff.

	Opera	*Fight*
Opera	$\underline{2}, \underline{1}$	$0, 0$
Fight	$0, 0$	$\underline{1}, \underline{2}$

The set of pure-strategy Nash Equilibria is $\{(Opera, Opera), (Fight, Fight)\}$.

Exercise 3. Consider the following situation: one Euro should be divided between two players (player 1 and player 2). Player 1 makes an offer which specifies how much of the euro he is willing to give to player 2. Player 2, without observing the offer made by player 1, defines the minimum amount she is willing to accept. For both players, the admissible offers are: 0, 0.25, 0.50, 0.75 e 1 Euro. If player 1 offers at least the amount player 2 is willing to accept, then an agreement is achieved and the Euro is divided according to the offer made by player 1. Otherwise no agreement is achieved and both players get nothing.
Represent this game in strategic form. In addition, what are the Nash equilibria of this game?

Solution 3. For each player $i \in \{1, 2\}$, the strategy set is $S_i = \{0, 0.25, 0.50, 0.75, 1\}$.
The normal-form representation of the game is provided in the matrix below. Best responses are indicated by underlining the corresponding payoff.

	0	0.25	0.50	0.75	1
0	$\underline{1}, \underline{0}$	$0, \underline{0}$	$0, \underline{0}$	$0, \underline{0}$	$\underline{0}, \underline{0}$
0.25	$0.75, \underline{0.25}$	$\underline{0.75}, \underline{0.25}$	$0, 0$	$0, 0$	$\underline{0}, 0$
0.50	$0.50, \underline{0.50}$	$0.50, \underline{0.50}$	$\underline{0.50}, \underline{0.50}$	$0, 0$	$\underline{0}, 0$
0.75	$0.25, \underline{0.75}$	$0.25, \underline{0.75}$	$0.25, \underline{0.75}$	$\underline{0.25}, \underline{0.75}$	$\underline{0}, 0$
1	$0, \underline{1}$	$0, \underline{1}$	$0, \underline{1}$	$0, \underline{1}$	$\underline{0}, \underline{1}$

The set of pure-strategy Nash equilibria is $\{(0, 0), (0, 1), (0.25, 0.25), (0.50, 0.50), (0.75, 0.75), (1, 1)\}$.

Exercise 4. In a market there are only two firms (let's denote them as firm 1 and firm 2). For both firms the cost function is $C(q_i) = cq_i$ for $i = 1, 2$ and $c = 10$; while the inverse demand function is:

$$P(Q) = \begin{cases} a - bQ & \text{if } Q \leqslant \frac{a}{b} \\ 0 & \text{if } Q > \alpha \end{cases}$$

where $a = 50$, $b = 2$, and $Q = q_1 + q_2$.
Suppose that the two firms want to maximize their joint profits. Which is the quantity that each firm should produce in order to achieve such objective? Denote with Q^m the level of production that maximizes the joint profits.
Now consider the same economy but with the two firms that compete à la Cournot. Show that in the equilibrium of the Cournot game, the individual profits for each firm are lower than the ones obtained in case each firm produces $\frac{1}{2}Q^m$. Why can't $\left(\frac{1}{2}Q^m, \frac{1}{2}Q^m\right)$ be a Nash equilibrium?

Solution 4. When firms maximize joint profits they behave as a single firm. We have to solve the problem

$$\max_{q_1,q_2} \Pi_1 + \Pi_2 = \max_{q_1,q_2} [50 - 2(q_1 + q_2)](q_1 + q_2) - 10(q_1 + q_2), \tag{1}$$

which is equivalent to

$$\max_{Q} \Pi_{1\&2} = \max_{Q} (50 - 2Q)Q - 10Q = \max_{Q} (50 - 10)\, Q - 2Q^2 = \max_{Q} 40Q - 2Q^2.$$

The first order condition is

$$\frac{\partial \Pi}{\partial Q} = 40 - 4Q = 0.$$

The solution is $Q^m = 10$. Then $P^m = 50 - 2Q^m = 30$.
The joint profits when $Q = Q^m$ are:

$$\Pi^m_{1\&2} = (P^m - c)Q^m = 200. \tag{2}$$

Now suppose that each firm produces 1/2 of the quantity that maximizes the aggregate profits, $q_i^m = \frac{1}{2}Q^m = 5$ for $i = 1, 2$. Then each firm gets profits equal to:

$$\Pi_i^m = (P^m - c)q_i^m = \frac{1}{2}\Pi^m_{1\&2} \text{for} \quad i = 1, 2$$

Note that the price does not change because the aggregate quantity, that determines the price, does not change. Let us now consider the same game but suppose that the two firms are competing à la Cournot. They choose the quantities simultaneously. The payoff function for firm 1 is:

$$\Pi_1(q_1, q_2) = P(Q)q_1 - cq_1 = [50 - 2(q_1 + q_2)]q_1 - 10q_1$$

Its best response can be characterized considering the following problem:

$$\max_{q_1} \Pi_1(q_1, q_2) \Leftrightarrow \max_{q_1} 40q_1 - 2q_1^2 - 2q_1q_2$$

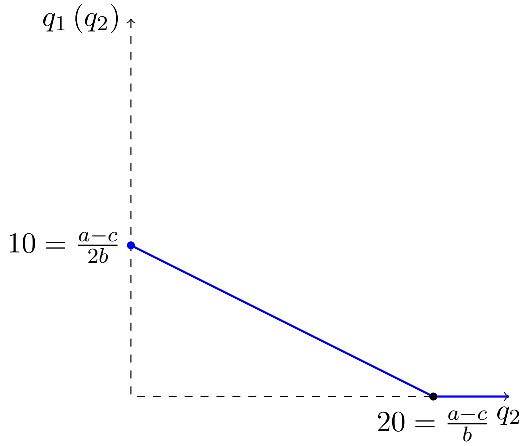
. The first order condition is

$$\frac{\partial \Pi_1}{\partial q_1} = 40 - 4q_1 - 2q_2 = 0.$$

Note that firm 1 takes q_2 as given, like 40. The solutions to this maximization problem will depend on q_2 and are summarized by the best response $q_1(q_2)$, which is

$$q_1(q_2) = \begin{cases} \frac{20-q_2}{2} & \text{if} \quad 20 - q_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Figure 1: Exercise 4 - Best response of firm 1

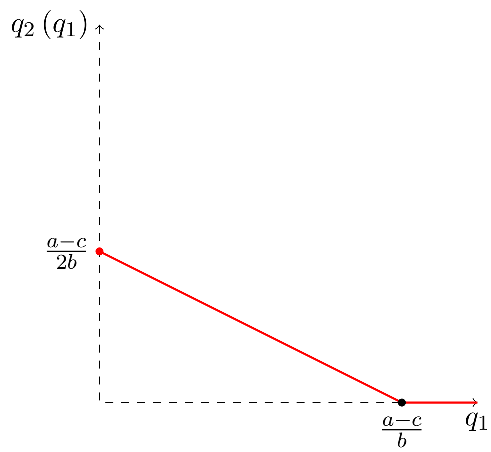


Notice that the optimal level of production for firm 1 depends on the production firm 2 realizes. Moreover, if firm 2 produces zero the optimal reply of firm 1 is to produce 10, that is the monopolistic quantity.

Since the problem for both firms is symmetric, we get symmetric results for firm 2, i.e. its best response is:

$$q_2(q_1) = \begin{cases} \frac{a-c-bq_1}{2b} = \frac{20-q_1}{2} & \text{if} \quad \alpha - c - q_1 \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Figure 2: Exercise 4 - Best response of firm 2



A Nash equilibrium of our game is a pair of output levels, (q_1^*, q_2^*), such that q_1^* is a best response to q_2^* and q_2^* is a best response to q_1^*. Therefore:

$$q_1^* = q_1(q_2^*) \quad \text{and} \quad q_2^* = q_2(q_1^*), \tag{3}$$

that is:

$$q_1^* = \frac{20 - q_2^*}{2} \quad \text{and} \quad q_2^* = \frac{20 - q_1^*}{2}. \tag{4}$$

We can either substitute the expression for q_2^* in the that for q_1^*, leading to

$$q_1^* = \frac{20 - \frac{20-q_1^*}{2}}{2} = \frac{20+q_1^*}{4} \Leftrightarrow q_1^* = \frac{20}{3} \tag{5}$$

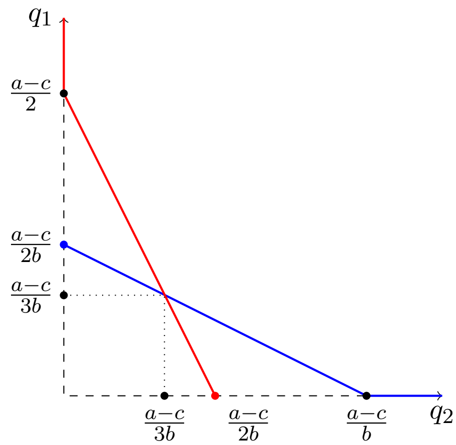
and then substitute $q_1^* = \frac{20}{3}$ in the expression for q_2^*, or we can guess that $q_1^* = q_2^*$ because *in this case* the problem is symmetric and write

$$q_1^* = \frac{20 - q_1^*}{2} \Leftrightarrow q_1^* = \frac{20}{3}. \tag{6}$$

In the end, we get

$$q_1^* = q_2^* = \frac{20}{3} = \frac{a-c}{3b}. \tag{7}$$

Figure 3: Exercise 4 - Nash equilibrium and best responses



In equilibrium the total output is $Q^* = q_1^* + q_2^* = \frac{2}{3}\left(\frac{a-c}{b}\right) = 2\frac{20}{3} = \frac{40}{3}$ and the equilibrium price is: $P\left(\frac{40}{3}\right) = 50 - 2\frac{40}{3} = \frac{70}{3} = \frac{a+2c}{3}$. The equilibrium aggregate profits are:

$$\Pi_{1\&2}^* = \Pi_1^*(q_1^*, q_2^*) + \Pi_2^*(q_1^*, q_2^*) = [P(q_1^* + q_2^*) - c](q_1^* + q_2^*) = \frac{40}{3} * \frac{40}{3} = \frac{1600}{9} = \frac{2}{9}\frac{(a-c)^2}{b}. \tag{8}$$

Aggregate profits in Cournot equilibrium are lower than in the case in which firms choose the production level that maximizes joint profits, $\Pi_{1\&2}^m = 200 > \Pi_{1\&2}^* = \frac{1600}{9}$.
The profile of production levels $\left(\frac{1}{2}Q^m, \frac{1}{2}Q^m\right)$ can't be an equilibrium since if a firm i sets its production level to $\frac{1}{2}Q^m$ then the other firm j has a profitable deviation. Let's consider the best response of firm j when i produces $\frac{1}{2}Q^m$:

$$q_j\left(\frac{1}{2}Q^m\right) = \frac{20 - \frac{Q^m}{2}}{2} = \frac{20-5}{2} = \frac{15}{2} = \frac{3}{8}\frac{a-c}{b} > 5 = \frac{1}{2}Q^m = \frac{1}{4}\frac{a-c}{b}. \tag{9}$$

In words, the strategy profile $\left(\frac{1}{2}Q^m, \frac{1}{2}Q^m\right)$ is not a Nash Equilibrium because each firm has an incentive to deviate, i.e. to produce more and to increase profits, according to her best response.