Microeconomics for Business
Practice Session 4 - Solutions

Instructor: Eloisa Campioni
TA: Ugo Zannini*
University of Rome - Tor Vergata

March 13, 2015

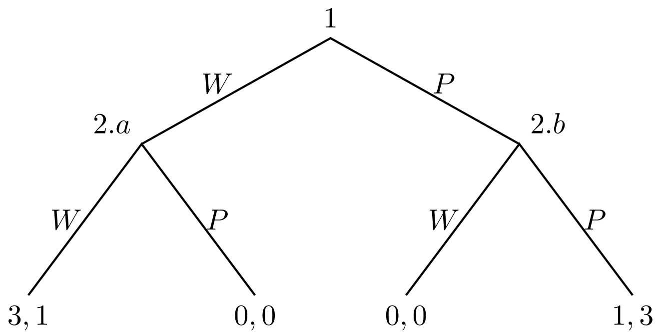
Exercise 1. Paul (1) and Beatrice (2) would like to go on a date. They have two options: a quick dinner at Wendy's (W), or dancing at Pravda (P). Paul first chooses where to go, and knowing where Paul went Beatrice also decide where to go. Paul prefers Wendy's, and Beatrice prefers Pravda. A player gets 3 out of his/her preferred date, 1 out of his/her unpreferred date, and 0 if they end up at different places. All these are common knowledge.

a. Represent the extensive form of the game. Find a subgame-perfect Nash equilibrium.

b. Modify the game a little bit: Beatrice does not automatically know where Paul went, but she can learn without any cost. (That is, now, without knowing where Paul went, Beatrice first chooses between Learn (L) and Not-Learn (N); if she chooses Learn, then she knows where Paul went and then decides where to go; otherwise she chooses where to go without learning where Paul went. The payoffs depend only on where each player goes as before.) Represent the extensive form of the new game. Find all the subgame-perfect equilibria in pure strategies, then represent the new game in normal form and find the Nash equilibria in pure strategies. Comment on the result.

Solution 1. a. The game is represented in figure 1. The game starts at node 1 with player 1 choos-

Figure 1: Exercise 1-a.



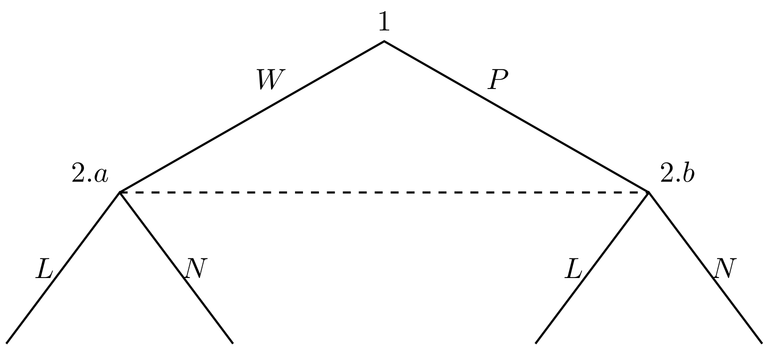
ing a strategy in $S_1 = \{W, P\}$. Then player 2 has perfect information about beeing in node 2.a or

*Email address: ugo.zannini@uniroma2.it

2.b. A strategy of player 2 hosts two actions, one for each node, that is $S_2 = \{WW, WP, PW, PP\}$ where the first letter refers to the node 2.a. The payoffs are easily identifiable.
A subgame-perfect Nash equilibrium (SPNE) can be found using backward induction because information is perfect. Thus, player 2 chooses W in the game starting at node 2.a and P in the one starting in 2.b. Player 1 can associate the corresponding payoffs instead of the subgames, so that he chooses W. The strategy profile (W, WP) is the only SPNE. The outcome is that Paul and Beatrice go to Wendy's, or $(3, 1)$.

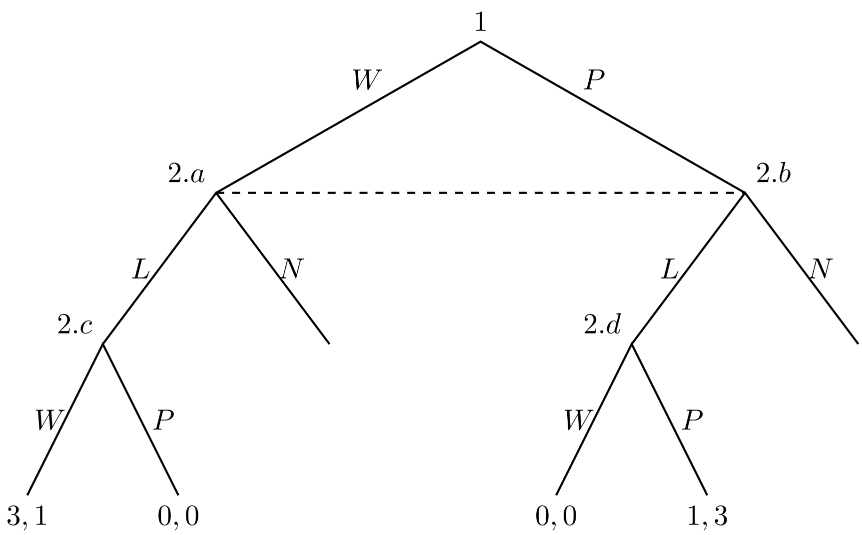
b. In the new setting the first step is to recognize that in the current first stage the players act simultaneously: player 1 chooses between W and P, as before, and *at the same time* player 2 chooses between L and N. This situation can be depicted as in figure 2. Note that the decision

Figure 2: Exercise 1-b, first step.



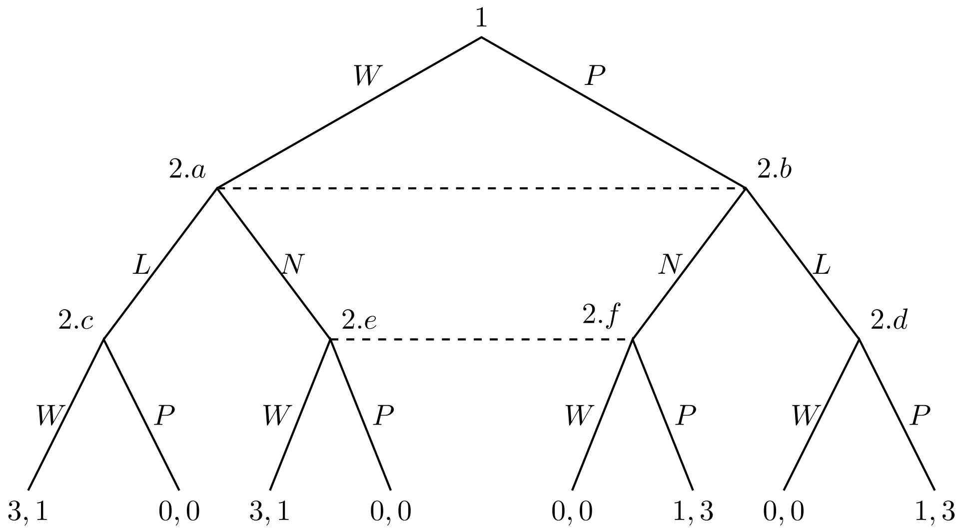
nodes of player 2 are part of the same information set because she cannot distinguish between 2.a and 2.b. Up to now the strategy sets are $S_1 = \{W, P\}$ and $S_2 = \{L, N\}$. In case of L, player 2 is perfectly informed about player 1's choice. Figure 3 depicts the two subgames following L.

Figure 3: Exercise 1-b, second step.



Note that 2.c and 2.d, with their respective terminal nodes, are two singleton information sets. On the contrary, if player 2 chooses N then she is not aware of player 1's choice. Therefore the nodes following N are part of the same information set, i.e. they must linked with a dashed line, which information set is the same of 2.a and 2.b. In order to ease the representation we can shift the branches of node 2.b as in figure 4.

Figure 4: Exercise 1-b, second step.



The player 1's strategy set is unchanged. Player 2, in the first contingency, chooses between L and N. If N, he chooses an action between W and P. Thus a player 2's strategy specifies four actions. In writing the strategy we will follow the above order as convention. If L, he distinguishes between 2.c and 2.d and specifies an action for each node.

In order to find all the subgame-perfect Nash equilibria, we can replace the nodes 2.c and 2.d with the payoffs associated with the Nash equilibria of the subgames, that is W and P respectively, as in figure 5.

Since only one subgame is left (the entire game), we can use the normal form to find the Nash equilibria that, combined with the Nash equilibria of the subgames in 2.c and 2.b, restitute the subgame-perfect equilibria.

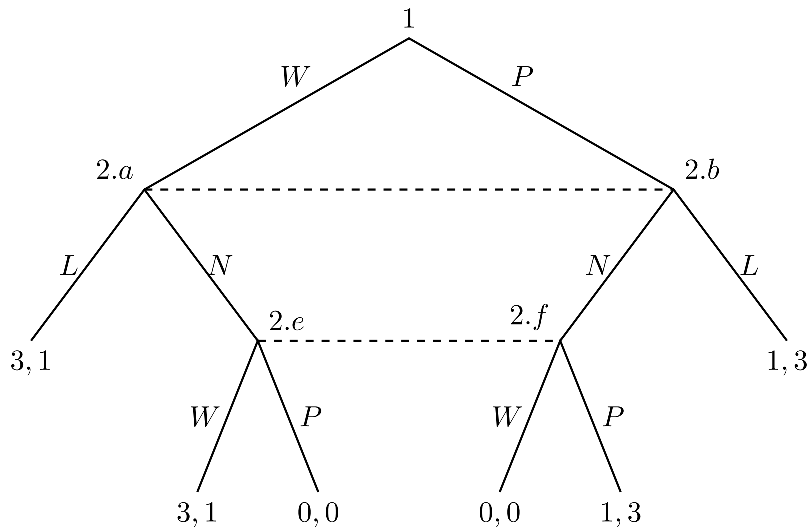
	LW	LP	NW	NP
W	$\underline{3},\underline{1}$	$\underline{3},\underline{1}$	$\underline{3},\underline{1}$	0,0
P	1,$\underline{3}$	1,$\underline{3}$	0,0	$\underline{1},\underline{3}$

Thus

$$SPNE = \{(W,(LWWP)),(W,(LPWP)),(W,(NWWP)),(P,(NPWP)).\}$$

We have four SPNE, the last of which delivers the outcome they go together to Pravda. Note that the last two actions of player 2's equilibrium strategies are W and P because they represent the trees 2.c and 2.d. The first two actions come from the solution of the bi-matrix.

Figure 5: Exercise 1-b, second step.



Solving for the Nash equilibria requires to write the normal form representation of the entire game in figure 4. In this case the strategy set of player 2 has 16 elements (4^2):

$$S_2 = \{LWWW, LWWP, LWPW, LWPP, LPWW, LPWP, LPPW, LPPP, NWWW, NWWP, NWPW, NWPP, NPWW, NPWP, NPPW, NPPP \tag{2}$$

The associated bi-matrix is written with player 2 choosing the row (meaning that the first payoff is for player 2).

	W	P
LWWW	1,3	0,0
LWWP	1,3	0,0
LWPW	0,0	3,1
LWPP	0,0	3,1
LPWW	1,3	0,0
LPWP	1,3	0,0
LPPW	0,0	3,1
LPPP	0,0	3,1
NWWW	1,3	0,0
NWWP	1,3	0,0
NWPW	1,3	0,0
NWPP	1,3	0,0
NPWW	0,0	3,1
NPWP	0,0	3,1
NPPW	0,0	3,1
NPPP	0,0	3,1

Underlining the payoffs of the best replies (straightforward), each strategy profile that involves as outcome that players stay together is a Nash equilibrium. The four of those equilibria involving the creadible choices W and P at $2.c$ and $2.d$ (the third and four actions of player 2's strategy) are also subgame perfect.

Exercise 2. Let the preferences of a given consumer be represented by the following utility function $u(\cdot) = \sqrt{w}$, where w is the initial wealth of such consumer. Assume that the consumer owns 400 Euros and he also possesses a lottery ticket which could let him win a prize of $1,200$ Euros with probability $1/2$ and 0 with the complementary probability $1/2$.

1. What is the expected utility of this consumer?
2. What is the expected value of the lottery?
3. Assume the consumer is offered 500 Euros in exchange for the lottery ticket. Would he be ready to sell that lottery ticket?

Solution 2.

1. Expected utility is

$$\frac{1}{2}u\left(400 + 1,200\right) + \frac{1}{2}u\left(400\right) = \frac{1}{2}\sqrt{1,600} + \frac{1}{2}\sqrt{400} = 30.$$

2. The expected value of the lottery is

$$\frac{1}{2} \times 1,200 + \frac{1}{2} \times 0 = 600.$$

3. If the consumer sells his lottery ticket, his utility is

$$u\left(400 + 500\right) = \sqrt{900} = 30.$$

If he keeps the lottery, his expected utility is 30. Therefore, although the expected value of the lottery is stricly higher than the potential selling price, the consumer is just indifferent between selling the lottery ticket and keeping it.