Microeconomics for Business
Practice Session 5 - Solutions

Instructor: Eloisa Campioni
TA: Ugo Zannini*
University of Rome - Tor Vergata

May 11, 2016

Exercise 1. Consider the following game with two players, 1 (he) and 2 (she). Player 1 chooses either **L** or **R**. After observing 1's move, player 2 chooses either **l** or **r**. Afterwards, if at the beginning **R** was played by 1, then the game continues and player 1 chooses either **a** or **b**. If instead **L** was chosen at the beginning, then the game ends with player 2's move.
The payoffs of the game are the following. If 1 chooses **L** and 2 chooses **l**, then the payoff vector is $(2, 3)$; if 2 plays **r** then the payoff vector is $(1, 1)$. If player 1 chooses **R** and player 2 plays **l**, then the payoff vector is $(3, 1)$ if 1 plays **a** and $(4, 3)$ if he chooses **b** in the second round. If instead player 1 chooses **R** and 2 plays **r**, then the payoff vector is $(2, 0)$ if 1 chooses **a** and $(0, 2)$ if he plays **b** in the second round.

1. Find the strategy set for each player and construct the normal-form representation of this game.
2. Find all pure-strategy Nash equilibria.
3. Draw the extensive-form representation of this game. How many subgames does this game have?
4. Find the pure-strategy subgame perfect Nash equilibria of the game.

Solution 1. 1. The strategy set for Player 1 is $S_1 = \{Laa, Lab, Lba, Lbb, Raa, Rab, Rba, Rbb\}$.
The strategy set for Player 2 is $S_2 = \{ll, lr, rl, rr\}$.
The normal form representation of the game is provided in the following payoff matrix.

	ll	lr	rl	rr
Laa	$2,3$	$2,3$	$1,1$	$1,1$
Lab	$2,3$	$2,3$	$1,1$	$1,1$
Lba	$2,3$	$2,3$	$1,1$	$1,1$
Lbb	$2,3$	$2,3$	$1,1$	$1,1$
Raa	$3,1$	$2,0$	$3,1$	$2,0$
Rab	$3,1$	$0,2$	$3,1$	$0,2$
Rba	$4,3$	$2,0$	$4,3$	$2,0$
Rbb	$4,3$	$0,2$	$4,3$	$0,2$

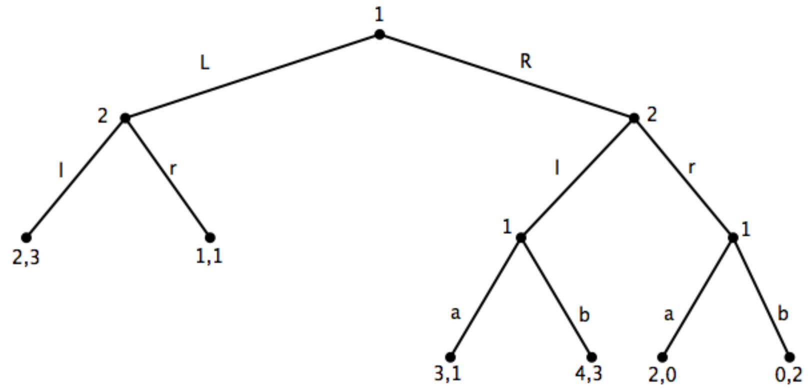
2. The set of pure strategy Nash Equilibria is

$$NE = \{(Laa, lr), (Lab, lr), (Lba, lr), (Lbb, lr), (Rba, ll), (Rbb, ll), (Rba, rl), (Rbb, rl)\}.$$

*Email address: ugo.zannini@uniroma2.it

3. The game has 5 subgames (including the entire game). The extensive form representation is provided in the game tree below.

Figure 1: Exercise 1 - Game tree



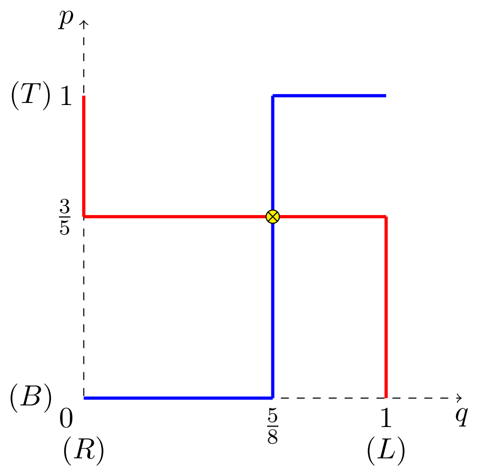
4. The unique Subgame Perfect Equilibrium is (Rba, ll).

Exercise 2. Find all the Nash equilibria of the following normal-form game, both in pure strategies and in mixed strategies.
Then assume that Player 2, before playing simultaneously with Player 1, must chose between action A, leading her to a payoff of 3, and action B, leading her to the simultaneous game. Which action will she take?

	L	R
T	$6, 2$	$0, 6$
B	$0, 6$	$10, 0$

Solution 2. It is immediate to see that the game has no Nash Equilibria in pure strategies.
To find Nash Equilibria in mixed strategies, assume that Player 1 chooses T with probability p (and B with probability $1 - p$), while Player 2 chooses L with probability q.
Player 1's expected payoff from T is $6q$ and from B is $10 - 10q$. For Player 2, expected payoff from L is $6 - 4p$ and from R is $6p$.
Therefore, Player 1 wants to play T if and only if $6q \geq 10 - 10q$, which is equivalent to $q \geq \frac{5}{8}$. In addition, Player 2 is willing to play L if and only if $6 - 4p \geq 6p$, which is equivalent to $p \leq \frac{3}{5}$.
Best responses are represented in the following graph, where the blue line is for Player 1.

Figure 2: Exercise 2 - Best responses



The mixed-strategy Nash Equilibrium is such that Player 1 randomizes over his actions with probability $p^* = \frac{3}{5}$ and Player 2 randomizes according to $q^* = \frac{5}{8}$.
In order to evaluete the choice of Player 2 between A and B, it must be computed the payoff of the mixed-strategy equilibrium, which corresponds to the payoff from playing B and going through the simultaneous game. The mixed-strategy equilibrium restitutes for Player 1 and 2, respectively, the following payoffs:

$$P1: 6 \times p^* q^* + 0 \times p^*(1-q^*) + 0 \times (1-p^*)q^* + 10 \times (1-p^*)(1-q^*) = \frac{15}{4}$$
$$P2: 2 \times p^* q^* + 6 \times p^*(1-q^*) + 6 \times (1-p^*)q^* + 0 \times (1-p^*)(1-q^*) = \frac{18}{5}$$

The equilibrium payoff is $\left(\frac{15}{4}, \frac{18}{5}\right)$. Player 2 will choose B since it restitutes a higher expected payoff than A, i.e. $\frac{18}{5} > 3$. Note that in this case Player 1's payoff is not involved in the choice and it could have been not computed at all.

Exercise 3. Consider the following demand curve:

$$q(p) = 100 - 50p$$

- Define the price-elasticity of the demand.
- Draw the demand curve corresponding to the above parameters, and explain what is the elastic and what the inelastic portion of that curve.

Solution 3.

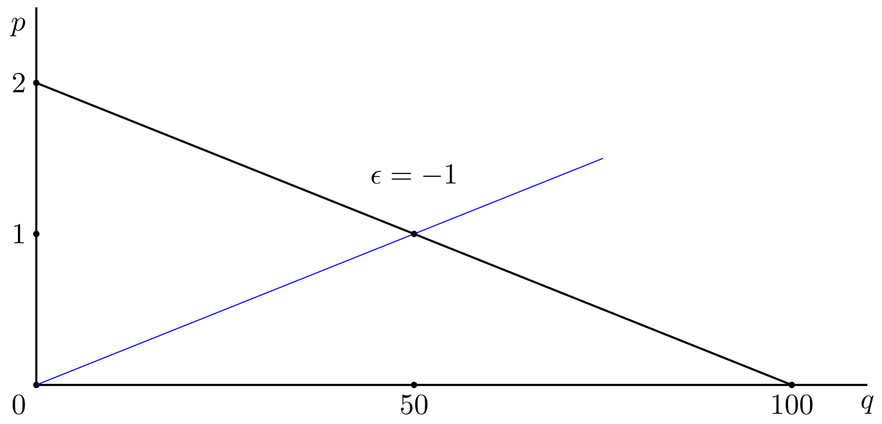
- The price-elasticity of the demand is the percentage change of output for a unit percentage change of price. Formally,

$$\epsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{\Delta q}{\Delta p}\frac{p}{q},$$

  where $\frac{\Delta q}{\Delta p} = \frac{dq}{dp}$ is the slope of the (direct) continuous demand curve. In this case, $\frac{dq}{dp} = -50$, and $\epsilon = -50\frac{p}{q} < 0$.

- Using the *inverse* demand function $p = 2 - \frac{1}{50}q$, the curve corresponding to the above parameters in the space (q, p) is represented in Figure 3.

Figure 3: Exercise 3 - Unit price-elasticity



In order to find the elasticities of the (q, p) combinations on the demand curve, we can use the observation that

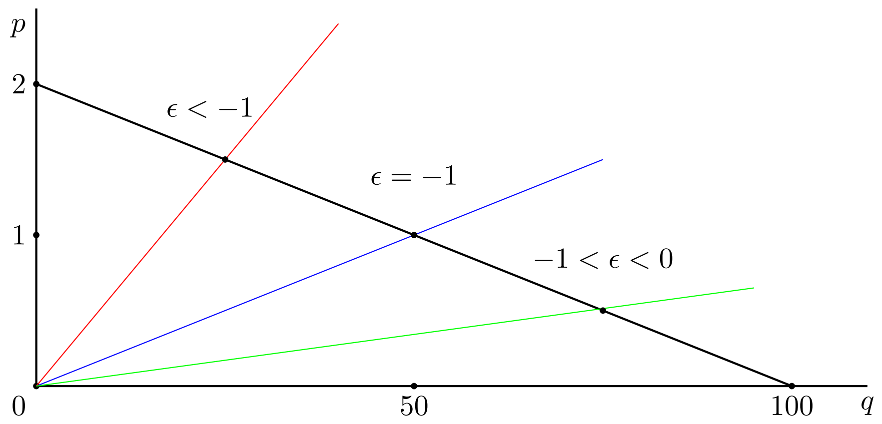
$$\epsilon = -50 * \frac{p}{q} \Leftrightarrow \frac{p}{q} = -\frac{1}{50} * \epsilon.$$

In the point associated with $\epsilon = -1$, this relation restitutes $\frac{p}{q} = \frac{1}{50}$. Therefore, the point (q, p) with unit elasticity belongs to the line $p = \frac{1}{50}q$, which is represented in the figure and intersects the demand function in $(50, 1)$.

The elastic part of the demand has $|\epsilon| > 1$, or equivalently $\epsilon < -1$. In this case, $\frac{p}{q} > \frac{1}{50}$, which means that the line from the origin has a slope greater than $\frac{1}{50}$. The red line in Figure 5 shows this case.

On the other hand, the inelastic part of the demand has $0 < |\epsilon| < 1$, or equivalently $-1 < \epsilon < 0$. In this case, $\frac{p}{q} < \frac{1}{50}$, meaning that the line from the origin has a slope smaller than $\frac{1}{50}$, as the green line in Figure 4 shows.

Figure 4: Exercise 3 - Elastic part of the demand



In general, the marginal revenues of a monopolist can be written as

$$MR = p + q\frac{dp}{dq} = p\left(1 + \frac{q}{p}\frac{dp}{dq}\right) = p\left(1 + \frac{1}{\epsilon}\right).$$

The unit elasticity is associated to $MR = p\left(1 + \frac{1}{-1}\right) = 0$. Because the profit maximization condition requires $MR = MC$, the monopolist faces two cases. If the marginal costs are zero, then the monopolist produces where $|\epsilon| = 1$, i.e. $q = 50$. If the marginal costs are positive, the optimality condition requires $MR > 0$, which means $|\epsilon| > 1$. Hence, the monopolist's choice will be on the elastic part of the demand, with $q < 50$.

Exercise 4. Let the demand curve of a market in which there exists a single firm, be given by $P = 360 - 10Q$. Let the total cost function of such firm be $TC = 10Q^2$.

1. Compute the optimal quantity and price offered by the monopolist in the market and compute its profits.

2. Provide a graphical representation of the marginal cost curve (MC), marginal revenue curve (MR), and demand curve, then show the optimal price and quantity calculated in 1.

3. Using the same graph, represent the consumer surplus, the producer surplus, and the deadweight loss that emerges with a monopolistic market.

4. Compute the value of the consumer surplus and compare it with its value in a competitive version of the same economy. Compute the value of the deadweight loss.

5. Suppose now that the monopolist can use perfect price discrimination. Recalculate the monopolist profits and the consumer surplus in this case, and compare them with the results obtained in 4. Explain your results.

Solution 4.

1. The monopolist maximizes profits $\pi = TR - TC$, corresponding to

$$\max_q \pi = P(Q) * Q - 10Q^2 = (360 - 10Q)Q - 10Q^2 = 360Q - 20Q^2$$

The optimal quantity Q_M is determined by the first order condition equated to zero, i.e.

$$\frac{\partial \pi}{\partial q} : 360 - 40Q_M = 0 \Leftrightarrow Q_M = \frac{360}{40} = 9.$$

The price is $P_M = 360 - 10Q_M = 270$, and profits amount to $\pi_M = 1620$.

2. In Figure 5, the marginal cost curve is $MC \equiv \frac{\partial TC}{\partial Q} = 20Q$, the marginal revenue curve is $MR \equiv \frac{\partial TR}{\partial Q} = \frac{\partial(360Q - 10Q^2)}{\partial Q} = 360 - 20Q$[1].

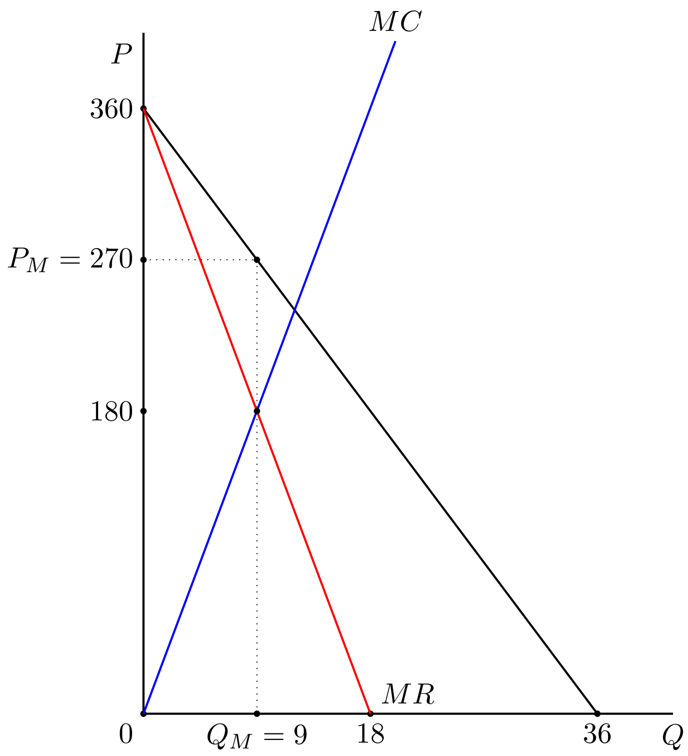
3. In Figure 6, the consumer surplus (CS_M), the producer surplus (PS_M), and the deadweight loss (DWL) are represented respectively by the green, yellow, and red area. The CS_M is the sum, for each consumed quantity, of the difference between the consumer's willingness to pay (i.e. the demand curve) and the price actually paid. The producer surplus is the sum, for each quantity sold, of the the difference between the price and the marginal cost to produce it, i.e. the profits. The DWL is the potential surplus from the non-exploited trade. Indeed, for those quantities, consumer's willingness to pay is higher than the marginal cost to produce them.

[1]More generally,

$$MR = \frac{\partial[Q * P(Q)]}{\partial Q} = \frac{\partial Q}{\partial Q} P + Q\frac{\partial P}{\partial Q} = P + Q\frac{\partial P}{\partial Q}$$

. Check that the last expression restitutes the same result in the text.

Figure 5: Exercise 4 - Marginal revenue and cost curves



4. The consumer surplus under monopoly is

$$CS_M = \frac{Q_M * (P_{max} - P_M)}{2} = \frac{9 * (360 - 270)}{2} = 405.$$

In order to compute the consumer surplus under perfect competition (CS_P), the market price and quantity are determined by the intersection of the demand and the marginal cost curves, which corresponds to solving the optimality condition characterizing perfect competition:

$$P = MC \Leftrightarrow 360 - 10Q_P = 20Q_P \Leftrightarrow Q_P = 12.$$

The market price under perfect competiotion is therefore $P_P = 240$, and

$$CS_P = \frac{Q_P * (360 - P_P)}{2} = \frac{12 * (360 - 240)}{2} = 720 > CS_M.$$

The consumers are better off under perfect competition.
The producer surplus is

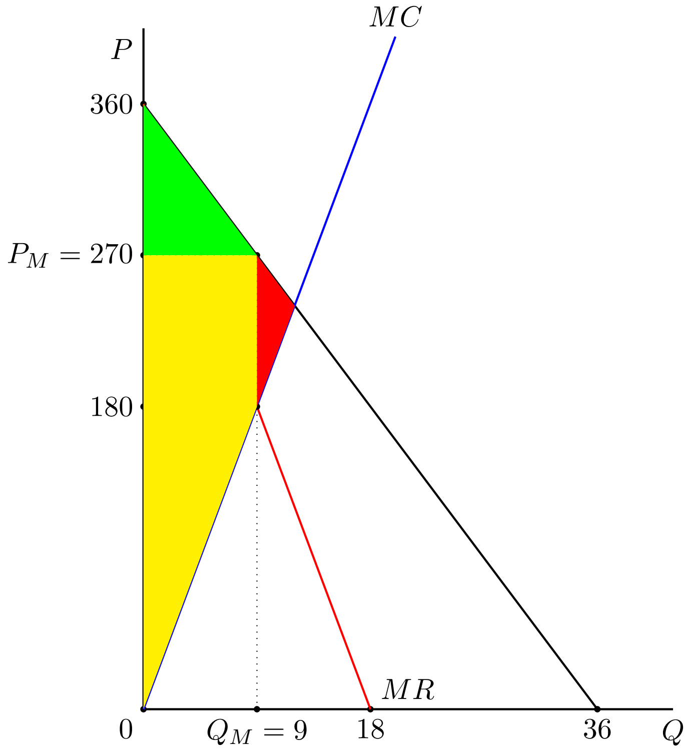
$$PS_P = P_P Q_P - 10Q_P^2 = 1440 < PS_M = 1620,$$

such that the producer is worse off[2].
Finally, the DWL can be calculated as the difference between the total surplus (TS) under perfect competition and under monopoly:

$$DWL = TS_P - TS_M = (CS_P + PS_P) - (CS_M + PS_M) = 2160 - 2025 = 135.$$

[2]Note that producer surplus and profit coincides because fixed costs, F, are absent –otherwise $PS = \pi + F$ –and and the pricing function is linear –i.e. $t = pq$ instead of $t = pq + f$, otherwise $PS = \pi - f$.

Figure 6: Exercise 4 - Consumer and producer surplus, and deadweight loss



In this case the society is better off with perfect competiotion because it is able to realize the maximum attainable surplus.

5. If the monopolist has perfect information on the consumer's willingness to pay, a *Two-part tariff* can be used to maximize the total surplus and, contemporaneously, to collect all of it.
The tariff T has a variable part, $P * Q$, and a fixed part, A. By pricing at the perfect competition price, the variable part of the tariff maximizes the consumer surplus, because he will buy 12 units of the product, with a corresponding surplus of $CS_P = 720$. This implies that the monopolist can ask to the buyer a fixed amount $A \leq 720$, and the consumer is still willing to buy. Clearly, the optimal strategy is to set $A = 720$ in order to extract all the consumer surplus. With $T = A + P_P Q = 720 + 240Q$, the representative consumer buys $Q_P = 12$, with total profits

$$\pi = T(A, Q) - TC(Q) = 720 + 240Q - 10Q^2 = 2160.$$

With perfect price discrimination the consumer surplus plus the producer surplus equal the social surplus under perfect competition. From a social point of view, without redistributive concerns, the society is indifferent between a perfect competition regime and a monopoly with perfect price discrimination, since the monopolist is eventually maximizing the social surplus.

Exercise 5. Assume that a businessman wants to contract a worker, but there are aspects of the worker that are unknown to the businessman. He does know that the worker receives a positive utility form the wage w, but with respect to disutility of effort, the worker could be either of two types. His disutility is either e^2 or $2e^2$. That is, the second type (θ_b) suffers greater disutility to effort than the first type (θ_g). Therefore, the worker's utility function is either $U^g(w, e) = w - e^2$ or $U^b(w, e) = w - 2e^2$ depending on his type. The probability that a worker is type G is q. Both worker's types have reservation utility equal to $\bar{U} = 0$. The businessman values the workers' effort ke with

k large enough for the businessman to be interested in contracting the worker, independently of her type. Hence, for each unit of effort that the worker exerts, the businessman gets k units of revenues. In addition he pays the wage w to the worker.

1. Describe the indifference curves of every type of worker. Write down the marginal rate of substitution (MRS), is it increasing? Provide a graphical representation of these curves in the space (w, e).
2. Write down the iso-profit curves for the businessman. Represent them on the same graph (w, e).
3. Formulate and solve the problem of the businessman, who wants to maximize his profit and has perfect information on the worker's type. What effort levels are demanded, and what wages are paid?
4. Are these contracts (effort-wage pairs) still optimal if the businessman cannot observe the worker's type? Explain.

Solution 5.

1. The indifference curves corresponding to the arbitrary utility level $\tilde{U}$ have equations

$$\tilde{U^g} = w - e^2$$
$$\tilde{U^b} = w - 2e^2$$

for type θ_g and θ_b, respectively. The Marginal Rate of Substitution (MRS) between wage and effort (how much wage for a marginal increase in effort the worker must receive in order to stay on the same indifference curve) for θ_g and θ_b are, respectively,

$$MRS_g = -\frac{MU_e}{MU_w} = 2e$$
$$MRS_b = -\frac{MU_e}{MU_w} = 4e,$$

where MU_e is the marginal utility of effort e and MU_w it the marginal utility of wage w. The above expressions come from partially differentiating an indifference curve, that is:

$$\frac{\partial U}{\partial w}dw + \frac{\partial U}{\partial e}de = 0 \Leftrightarrow \frac{dw}{de} = -\frac{\partial U/\partial e}{\partial U/\partial w} = -\frac{MU_e}{MU_w}.$$

2. The isoprofit curves describe the combinations of wage and effot that keep the businessman at the same profit level, $\tilde{\Pi}$. If θ_g and θ_b are hired, then isoprofit curves are, respectively,

$$\tilde{\Pi}_g = ke_g - w_g$$
$$\tilde{\Pi}_b = ke_b - w_b.$$

3. If the businessman wants to hire θ_g, he will offer the contract $\left(w_g^*, e_g^*\right)$ which solves

$$\max_{\{w_g, e_g\}} \quad \Pi_g = ke_g - w_g$$

$$s.t. \quad U^g(w, e) = w_g - e_g^2 \geq 0.$$

Profit maximization requires that the worker gets $U^g(w, e)$, that is the worker's participation constraint must be binding. This implies that

$$w_g = e_g^2.$$

Substituting the latter into the objective function, the maximization problem reduces to

$$\max_{\{e_g\}} \quad \Pi_g = ke_g - e_g^2.$$

The First Order Condition to this problem is

$$\frac{\partial \Pi_g}{\partial e_g} = 0 \Longleftrightarrow k - 2e_g = 0 \Longleftrightarrow e_g = \frac{k}{2}.$$

Therefore, $e_g^* = \frac{k}{2}$ and $w_g^* = (e_g^*)^2 = \left(\frac{k}{2}\right)^2 = \frac{k^2}{4}$.
If the businessman wants to hire θ_b, he will offer the contract (w_b^*, e_b^*) which solves

$$\max_{\{w_b, e_b\}} \quad \Pi_b = ke_b - w_b$$

$$s.t. \quad U^b(w,e) = w_b - 2e_b^2 \geq 0.$$

Even in this case one can easily show that the worker's participation constraint must be binding. This implies that $w_b = 2e_b^2$. Substituting the latter into the objective function, the maximization problem reduces to

$$\max_{\{e_b\}} \quad \Pi_b = ke_b - 2e_b^2.$$

The First Order Condition to this problem is

$$\frac{\partial \Pi_b}{\partial e_b} = 0 \Longleftrightarrow k - 4e_b = 0 \Longleftrightarrow e_b = \frac{k}{4}.$$

Therefore, $e_b^* = \frac{k}{4}$ and $w_b^* = 2\left(\frac{k}{4}\right)^2 = \frac{k^2}{8}$.

Concluding, the good worker is paid more and works more than the bad worker. The businessman makes more profits with the good worker since

$$\Pi_g(w_g^*, e_g^*) = k\frac{k}{2} - \frac{k^2}{4} = \frac{k^2}{4} > \Pi_b(w_b^*, e_b^*) = k\frac{k}{4} - \frac{k^2}{8} = \frac{k^2}{8}. \tag{1}$$

4. Under asymmetric information, contracts $\left(w_g^*, e_g^*\right)$ and (w_b^*, e_b^*) are no longer optimal. In particular, if both contacts were offered, type θ_g would select (w_b^*, e_b^*) rather than $\left(w_g^*, e_g^*\right)$. To see this, notice that θ_g's utility from (w_b^*, e_b^*) is

$$U^g\left(w_b^*, e_b^*\right) = w_b^* - (e_b^*)^2 = \frac{k^2}{8} - \frac{k^2}{16} = \frac{k^2}{16} > U^g\left(w_g^*, e_g^*\right) = 0.$$