LECTURE 1 : THE INFINITE HORIZON REPRESENTATIVE AGENT MODEL

In the IS-LM model consumption is assumed to be a static function of current income. It is assumed that consumption is greater than income at low income levels, which implies that if someone always has low income they will spend more than their lifetime income in their lifetime. No one would be allowed to borrow to do this. More generally, the Keynesian consumption function is unappealing to most neoclassical economists because the consumption/savings choice is not based on utility maximization.

In this lecture I will discuss how a rational utility maximizer would choose how much to consume, draw testable implications and discuss statistical tests of the hypothesis that aggregate consumption behaves as it would if there were a single rational representative agent who lived forever. The lecture summarizes the work reported by Hall in (Hall 198?) on the reading list.

First consider a consumer who lives for two periods. For simplicity I will first assume that he knows that his income will be w_1 in the first period and w_2 in the second. Later I will discuss the case in which he does not know exactly what his income will be in future. Although it is clearly important I will not discuss the case of agents who can affect their income by choosing how much to work.

The consumer can save or borrow at the interest rate r, for each lira he saves in the first period he gets an additional lira in the second. For simplicity there is only one kind of consumption good. This means that the only choice the consumer has to make is how much of the good to consume in the first period and how much to consume in the second.

The consumer's lifetime consumption is equal to his lifetime income. No one will lend more than he can repay and he can't take his wealth with him when he dies. The intertemporal utility maximization hypothesis of consumption is called the life-cycle permanent income hypothesis because it assumes that consumption is determined by preferences and the lifetime budget constraint. For a two period lifetime the budget constraint

1) $c_1 + c_2/(1+r) <= w_1 + w_2/(1+r)$

where c_i is consumption in period i and w_i is income in period i. Since the consumer wants to consume, 1 holds with equality. The right side of equation 1 is called permanent income.

The consumer chooses c_1 to maximize the intertemporal utility function

2) $\underset{c_1,c_2}{\text{maximize}}\ U = u(c_1) + u(c_2)/(1+d)$ subject to 1

d is the subjective rate of discounting of future happiness. This very simple problem can be solved in many ways. The simplest is to use equation 1 to solve

for c_2 as a function of c_1
and plug the result into equation 2.

3) $c_2 = w_1(1+r) + w_2 - c_1(1+r)$

so

4) $U = u(c_1) + [u(w1(1+r) + w_2 - c_1(1+r))]/(1+d)$

Equation 4 gives the first order condition equation 5

5) $0 = u'(c_1) - u'(c_2)(1+r)/(1+d)$

Equation 5 states that $u'(c_1) = u'(c_2)$, it is conventional to write $u'(c_1) = u'(c_2)/(1+r) =$ lambda, and call lambda a Lagrange multiplier. Lagrange generalized the trivial derivation above, but basically he did the same thing. Now that lambda is defined it is easy to check that maximizing 2 subject to the constraint 1 is equivalent to unconstrained maximization of equation 6 with respect to c_1 and c_2

6) $\max_{c_1,c_2} u(c_1) + u(c_2)/(1+d) - \text{Lambda}[c_1 + c_2/(1+r) -w_1 - w_2/(1+r)]$

The advantage of Lagrange multipliers is that no matter how many constraints must be satisfied, it is possible to write an equation analogous to equation 6 with a different Lambda for each constraint. It can be very tedious to solve so many equations, but the validity of the Langrange multiplier approach depends on the fact that it can be done.

It is fairly easy to generalize the two period case to the infinite horizon case. Now assume that the consumer lives forever, consumes c_i in period i and earns w_i in period i. The consumer chooses c_i in each period to maximize the intertemporal utility function

$$7)\quad U = \sum_{i=1}^{\infty} u(c_i)/(1+d)^i$$

subject to the budget constraint

$$8)\quad \sum_{i=1}^{\infty} (c_i - w_i)/(1+r)^i <= 0$$

By noting the analogy with equation 6 and appealing to Lagrange you might convince yourself that maximizing 7 subject to 8 is equivalent to unconstrained maximization of equation 9

$$9)\quad \max \sum_{i=1}^{\infty} [\, u(c_i)/(1+d)^i] - (\text{Lambda})\{\sum_{i=1}^{\infty} [(c_i-w_i)/(1+r)^i\}$$

So long as w_i is known for all i, Lambda is constant. 9 gives an infinite number of first order conditions, one for
each time period all described by equation 10 for each i

$$10)\quad u'(c_i) = (\text{Lambda})(1+d)^i/(1+r)^i$$

Since Lambda is unknown equation 10 is not very useful by itself but Lambda can be eliminated by comparing equation 10 for i = j and for i = j+1 giving equation 11 for each j

11) $u'(c_{j+1}) = u'(c_j)[(1+d)/(1+r)]$

The important point is that if d and r are constant, the derivative of the utility function is multiplied by the same amount each period. Since I have assumed that w is known with certainty each period I make a clearly false prediction. If I Assume that u'(c) is a decreasing function of c, I predict that consumption either decreases always or increases always. One way to explain up and down fluctuations in consumption in the intertemporal utility maximizing framework is that consumers do not know their future income and adjust their consumption as they learn about it.

Fortunately it is not too difficult to adjust to an uncertain world if we assume that consumers maximize the expected value of the intertemporal utility function and know the probability of having any income in the future. The second assumption is the rational expectations hypothesis as formulated by Muth and Lucas. As consumers learn more about their income stream, they adjust their consumption, so future consumption is not known exactly. Also consumption can decrease and increase as it does.

Assume that at time i consumers don't know w_{i+1}, w_{i+2}

etcetera but do know their expected value this means that at each time t the consumer maximizes the expected value of future consumption taking expectations conditional on all information available at time t

$$12)\quad \underset{c_i,\, i>=t}{\text{MAXIMIZE}}\; E_t\{ \sum_{i=t}^{\infty} [u(c_i)/(1+d)^{i-t}]\}$$

subject to the constraint

$$13)\quad \sum_{i=t}^{\infty} (c_i-w_i)/(1+r)^{i-t} \le S_t$$

Where S_t is financial wealth at time t (which can be negative).

The analysis above can be repeated with expected values and gives equation 14

If c_t is optimal and the plan which gives c_i i>t as a function of new information is optimal then there is no other c'_i and new plan c'_i i > t which improves expected utility. In particular there is no change in which $c'_i = c_i$ for all i > t+1 which increases expected utility. Given the budget constraint this implies that there is no δ such that expected utility is increased if $c'_t = c_t + \delta$ and $c'_{t+1} = c_{t+1} - (1+r)\delta$. Note c_{t+1} is not known at t it depends on things the consumer learns after t, but the rational consumer plans c_{t+1} as a function of this new information and can imagine changing the plan by consuming $(1+r)\delta$ less in every case. When considering modified consumption plans of this type it is clear that the FOC is that the derivative of expected utility with respect to δ is zero at $\delta = 0$ or

14) $E_i[u'(c_{i+1})/u'(c_i)] = (1+d)/(1+r)$

Equation 14 is still not testable, since without knowing or assuming anything about u, u'(c) can't be measured. Needless to say few economists have been stymied by an unwillingness to assume a particular shape for a utility function. With the exception of Hall, most economists eager to test equation 14 have assumed constant elasticity of substitution utility functions of the form

$u(c) = [c^{(6-1)/6}]6/(6-1)$

which implies that $u'(c) = c^{-1/6}$ and turns equation 14 into the highly useful equation 15

15) $E_i(c_{i+1}/c_i)^{-1/6} = (1+d)/(1+r)$

Which making a bold approximation for logarithms implies

16) $E_i[\log(c_{i+1}) - \log(c_i)] = 6(r - d)$

which can be written in the usable form

17) $\log(c_{i+1}) = \log(c_i) + 6(r-d) + e_i$

where e_i is a disturbance term which is according to the rational expectations hypothesis uncorrelated with any lagged information.

Equation 17 is testable, it implies that if log consumption is regressed on lagged log consumption and on lagged variables, the coefficient on log consumption will be one and the other coefficients will be zero.

For example if log consumption is regressed on lagged log consumption and twice lagged log consumption the coefficient on twice lagged log consumption should be zero.

Hall's contribution was to point out that it is not necessary to specify a consumption function in order to derive equation 17. Previously economists studying consumption had specified consumption as a function of income etc. Hall showed this was not necessary in order to test the intertemporal utility maximization model of consumption.

Put briefly most of the tests have rejected the restrictions implied by equation 17. lagged information often helps predict changes in consumption. This implies that one of the many assumptions made in deriving equation 17 must be false.

Many assumptions have to be made in order to derive equation 17. First it is assumed that consumers are expected intertemporal utility maximizers. Second it is assumed that they have rational expectations. Third it is assumed that consumers are free to borrow and to borrow at the same interest rate they earn when they save. That is it is assumed that they are not liquidity constrained. These assumptions are critical and rejection of the restrictions imposed by equation 17 might imply that any (or all) of them are false.

A form of the utility function is assumed. It is easy to check that results do not depend on the particular form of the one period utility function u by using different assumptions.

More importantly it was assumed that the intertemporal utility function is time separable, that is that it is the discounted sum of functions of consumption at each time. This implies that consumption now does not affect the marginal utility of consumption in the future. This is clearly false if consumption includes the purchase of durable goods. Few people are eager to buy another house the day after buying one (few not none). For this reason, consumption of non-durables and services is used instead of total consumption. Nonetheless, the assumption may still be false and more general utility functions are often proposed.

Equation 17 was derived for a single consumer. It is usually tested with aggregate data. An additional assumption is made -- that aggregate consumption behaves as if there were a single representative consumer. Nonetheless the model has also been tested and rejected with data on individual consumption.

In this lecture I have assumed that the real interest rate r is known and constant. This assumption can be relaxed by testing whether lagged information helps predict changes in consumption only because it helps predict real interest rates. The modified hypothesis is also rejected by the data.

This leaves the following possible explanations for the rejection of the predictions of the life-cycle permanent income hypothesis -- that consumers are not utility maximizers, that they do not have rational expectations, that the utility function is not additively separable and that they are not free to borrow.

Appendix 1: The best part again. This is another effort to tell the story needed to get to equation 14.

Recall we had gotten to the most interesting part, maximizing utility over an infinite horizon. I had just described the budget constraints equations 14 and 15 then I jumped to the first order condition and result (also called 14 sorry again). This can be derived in a manner strictly analogous to the finite horizon case. consider an alleged solution c^*_1, c^*_2, c^{*3} &c This consumption path implies wealth at each period S^*_1, S^*_2, S^{*3} &c. If this is optimal it is impossible to find an improvement and in particular it is impossible to find an improvement with the additional restriction that S_t is equal to S^*_t for every t not equal to i+1. This means that the consumer chooses c_i to maximise 13

13) $U= \sum_{j=i}^{\infty} u(c_j)/d^{j-i}$

given the constraint that $S_{i+2} = S^*_{i+2}$ &c

This gives

new 1) $c_{i+1} = (1+r)(S^*_i + w_i - c_i) + w_{i+1} - S^*_{i+2}/(1+r)$
and $c_j = c^*_j$ for j greater than i+2.

this gives

new 2) $dU/dc_i = u'(c_i) - u'(c_{i+1})(1+r)/(1+d)$

If the alleged solution is really an optimum this must equal zero as asserted. The problem of finding optimal consumption for each period can be quite tedious. The problem of checking that the stated first order condition holds is as you have seen trivial. The only trick (and this is very common) is the trick of arguing that if their is no improvement which satisfies the original budget constraint, then there must be no improvement that satisfies it and additional restrictions.

In this additional bit I have solved the problem under uncertainty. The exact same technique for looking for an improvement works under uncertainty. The only difference is that If I specify S^{*i+2} I must imagine specifying it as a function of new information such as w_{i+1}.

I could also use the restriction

new 3) $c_{i+1} = c^*_{i+1} - (1+r)(c_i - c^{*i})$

this leaves wealth and consumption the same for all periods i+2 and after so the first order condition new 2 holds under certainty and the first order condition 14

14) ${}_tE[u'(c_{i+1}/c_i) = (1+d)/(1+r)$

holds even if wages are uncertain.

Appendix II An implicit assumption which I used above

Recall equation 13 read

$$13) \quad \sum_{i=t}^{\infty} (c_i - w_i)/(1+r)^{i-t} \le S_t$$

Where S_t is financial wealth at time t and can be negative and the argument about increasing c_t to $c_t + \delta$ and cutting c_{t+1} to $c_{t+1} - (1+r)\delta$.

I assume that the budget constraint must be satisfied with certainty. Disappointingly low w must be balanced by low c. This may not always be possible if the required c is negative. Then the consumer goes bankrupt. Creditors would not loan at the safe rate r to a consumer who might go bankrupt. For creditors to be willing to loan any amount the consumer wants to borrow at the same interest rate consumers receive on savings it is necessary that consumers do not want to risk bankruptcy -- that they choose to borrow only so much that there consumption is certainly strictly positive in each period. Consumers will choose to do this if the consequences of zero or extremely low consumption are sufficiently horrible, that is, if the slope of the u(c) goes to infinity sufficiently quickly as c goes to 0. Returning to the argument behind equation 14, for the FOC to hold it is necessary that the consumer not choose to be at a corner. I argue that the derivative of expected utility with respect to δ must be zero at $\delta = 0$. Otherwise it would be possible to increase expected utility for δ slightly positive or slightly negative. For this argument to be valid, slightly positive and slightly negative δ must be feasible. In other words c_t must be positive making it possible to reduce c_t by a small amount and c_{t+1} must be a random variable bounded away from zero (certainly greater than or equal to some positive amount) making it certain that c_{t+1} can be reduced by $(1+r)\delta$ for some positive δ. If this is not always true with certainty for every t, equation 14 is not valid.

The assumption that consumers are free to borrow any amount at the same real interest rate, and the assumption that lenders have rational expectations together require and imply that consumers will never choose to risk bankruptcy which should imply equation 14. If consumers are willing to risk bankruptcy (as we certainly are) creditors will charge different interest rates depending on the risk of bankruptcy or refuse to lend at all (as they certainly do). The possibility that consumers might choose to risk bankruptcy not only implies that we sometimes violate our budget constraint, but also implies that rational creditors are not willing to lend us any amount that we wish to borrow at the same interest rate. This might explain why equation 14 does not hold in practice and is as noted the most popular proposed explanation.