

Consumption and Investment Exam January 23 2014
Please do one exercise (not both)

1) Consider a consumer who chooses consumption in period t (C_t) for $t = 1, 2, 3, \dots$ to maximize the Sum from $t = 1$ to infinity of $(C_t)^{0.5}/(1.10)^t$ subject to the budget constraint that the present discounted value of consumption is equal to initial wealth (K_1) plus the present discounted value of wages (w_t) where wages and consumption are discounted at a constant rate r
so $K_1 = \text{the sum from } 1 \text{ to infinity of } (C_t - w_t)/(1+r)^{t-1}$

- a) If initial wealth $K_1 = 100$ and wages are zero what is C_1 as a function of r ?
- b) If Initial Wealth $K_1 = 0$ and $w_t = 10$ for every t , what is C_1 if $r = 0.10$. ?

2) Consider the model of investment with no financial market imperfections presented by Romer.

- a) what is the slope of the $\dot{K}=0$ curve ?
- b) draw a graph of \dot{Q} on K showing the $\dot{Q}=0$ curve and the $\dot{K}=0$ curve (that is draw the phase diagram)
- c) K^* is the steady state level of capital. Write down the equation for K^* (that is find the point where $\dot{K}=0$ and $\dot{Q}=0$)
- d) suppose that initial K is half of K^* . Illustrate what would happen using the phase diagram you just drew.
- e) Now imagine that initial $K = K^*$ but that, in a surprise move, the state introduces a tax τ on profits so from the point of view of the firm $\pi(K)$ is replaced by $(1 - \tau)\pi(K)$. Look at the equations for \dot{Q} and \dot{K} . Which one is changed by the tax ?
- f) Illustrate what happens after the unexpected introduction of the tax using the phase diagram