Statistics, Fall 2025 - Practice Session 2
Point Estimation

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2/10/2025

Problem 1

Consider a random sample $\{X_i\}_{i=1}^n$ following the Erlang distribution with density

$$f(x;a) = a^k \frac{1}{(k-1)!} x^{k-1} e^{-ax} \mathbb{1}_{[0,\infty)}$$

Let the integer parameter k be known. Find a Method of Moments estimator for a.

Problem 2

Compare the Method of Moments and Maximum Likelihood estimators for the parameter a of the following distribution

$$f(x;a) = a x^{a-1} \mathbb{1}_{(0,1)}(x) \ \ , \ \ a > 0$$

for a random sample $\{X_i\}_{i=1}^n$.

Problem 3

Suppose we toss a four-faced die 30 times and get the following results.

Side	1	2	3	4
No. of obs.	$n_1 = 3$	$n_2 = 13$	$n_3 = 5$	$n_4 = 9$

Based on a point estimation, is the die fair? Use Maximum Likelihood to answer.

Problem 4

Consider an IID sample $\{X_i\}_{i=1}^n$, where X_i follows the Raised Cosine distribution with density

$$f(x; s, a) = \frac{1}{2s}\left[1 + \cos(\frac{x-a}{s}\pi)\right]\mathbb{1}_{[a-s,a+s]}(x)$$

Estimate a and s with the Method of Moments. Why could the Maximum Likelihood estimators be unreliable?

Problem 5

Consider a sequence of IID random variables $\{Z_i\}_{i=1}^n$, distributed according to the logistic distribution with density

$$f(z; a, s) = \frac{\exp\left(-\frac{z-a}{s}\right)}{s\left(1 + \exp\left(-\frac{z-a}{s}\right)\right)^2}$$

Suppose that we know s to be equal to 1. Then, find the Maximum Likelihood estimation of a. If you cannot algebraically solve until the end, it is enough to show which equation we need to find the root of to get the estimate.

Notes

- The Gamma function is directly related to the factorial. It is defined as

$$\Gamma(k) = \int_0^\infty x^{k-1}e^{-x}dx = (k-1)!$$