Mathematics – Teacher: P. Gibilisco – Probability Test

1. The birthday problem. Suppose that in each year you have 365 days. Consider a group of n randomly chosen people. Define:

$$A_n = \text{in the group there exists a pair with the same birthday.}$$

Trivially $P(A_1) = 0$ and by the pigeon-hole principle $P(A_{366}) = 1$. According to you the smallest n such that $P(A_n) > \frac{1}{2}$ is ...

$a)183 \qquad b)23 \qquad c)100$

2. The False positive problem. You want to check if you have a certain disease (say an infection). In the population of your country the 1% is infected. Moreover the doctors say to you that: a) the probability that the test is positive if you really have the disease is 79% (true positive); the probability that the test is positive if you do not have the disease is 10% (false positive). Your test is positive. Which is the probability that you really are infected?

$a)79\% \qquad b) > 50\% \qquad c) < 8\%$

3. The Monty Hall problem. Suppose you are on a game show, and you are given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?". Is it to your advantage to switch your choice?

a) It's better to switch b) It's better not to switch c) It doesn't matter.

4. The queue problem. You are in battalion (n soldiers) in a war zone. The commander needs a volunteer for a suicide mission but nobody is willing to propose himself. So, to choose the soldier for the mission, the commander prepares $n-1$ sticks of the same length and one longer stick (he holds the sticks in his hand so that nobody can see the length). The soldier must form a queue to choose the stick. Which position is the safest in the queue?

a) At the beginning. b) At the end. c) It doesn't matter.