1 A Model

the simplified version of Romer (90) proposed by Barro and Sala-i -Martin (1995)

The production function of firm i in the final good sector is given by :

$$Y(i) = AL(i)^{1-\alpha} \sum_{j=0}^{M} x(i,j)^{\alpha} \tag{1}$$

where $Y(i)$ is the amount produced and $L(i)$ is labour used by firm i and $x(i,j)$ is the quantity this firm uses of the intermediate good indexed by j. $0 < \alpha < 1. Total labour supply is constant.$The final good production sector is competitive and we normalize the very large number of final goods producing firms to one (that is we consider a representative firm) and suppress the index i to save pixels (and ink and paper if you print this) The representative final good producing Firm maximizes profits given by

$$Y - wL - \sum_{j=0}^{M} P(j)x(j) \tag{2}$$

where W is the wage. By profit maximization we have:

$$x(j) = L\left(\frac{A\alpha}{P(j)}\right)^{\frac{1}{1-\alpha}} \tag{3}$$

and

$$W = (1-\alpha)\frac{Y}{L} \tag{4}$$

Since the firms in the final good production sector are competitive their profits are zero in equilibrium. In contrast the firms which produce intermediate goods patent the new intermediate good which they invent and then earn monopoly profits for ever. The value of the patent for the jth intermediate good ($v(j,t)$ at time t is the present discounted value of such profits. The value of the jth patent at time t is the present discounted value of the stream of firm j's profits

$$v(j,t) = \int_{t}^{\infty} (p(j) - 1)x(j)e^{-R(s)} \tag{5}$$

where R(t,s) is the integral from t to s of rs), which is the real interest rate at time s (this is the notation used by David Romer)

The inventor of the jth intermediate good chooses $P(j)$ to maximize profits $(P(j)-1)x(j)$ where $x(j)$ is given by 3, so for each j:

$$P(j) = P = \frac{1}{\alpha} \tag{6}$$

and

$$x(j) = x = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} \tag{7}$$

To show there is an equilibrium with a constant real interest rate, we guess that the real interest rate is constant and check the conditions for equilibrium. This is not an assumption. It is a way of solving the model using the guess and check approach.

If the interest rate is constant, we have substituting 6and 7 in **??**:

$$v(j,t) = LA^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\frac{1}{r} \tag{8}$$

The cost of development of new products is η and there is free entry of inventors so by equating the value and the cost of inventions we have:

$$r = \frac{L}{\eta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right) = \frac{L}{\eta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\alpha) \tag{9}$$

Plugging equation 7 in equation 1 gives equation

$$Y = MLA^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}} \tag{10}$$

and plugging 10 in 4 we have:

$$W = M(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}} \tag{11}$$

With time the number of intermediate goods increases. The wage grows proportionally to the number of intermediate goods while the interest rate remains the same. This means that by guessing and checking, we have found an equilibrium with a constant real interest rate.

Although until now we have treated the number of intermediate goods M as an integer, it is the convention to assume that each invention is a tiny amount of progress so the growth in M can be considered continuous and proportional to the flow of research and development spending divided by η.

A silly interpretation of the model is that one unit of final good can be changed (by the monopolist j using intermediate good j production function) into one unit of intermediate good j, and that anyone can change n be change η units of final good into one new invention using the magic research and development function. The more reasonable interpretation is that the production functions for the final good and for each intermediate good are identical and the research and development function (where discoveries are made using labor and intermediate goods) is identical except for the factor η. The silly interpretation makes it clear that total value added (GDP, PIL in Italian) is less than Y given by equation 1 as intermediate goods are used up producing final good.

GDP is divided between consumption and research and development. This decision can be modeled the easy way (Solow's way) assuming that some fixed fraction of GDP is devoted to research and development. Or the Romer 90 production function can be combined with optimal intertemporal consumption choices (that's what Romer did). Either way should be simple by now (yes that's a joke). The point is that from this point on the Romer 90 model acts just like the Romer 86 model with ηM in the place of K.