some exercizes

1) Consider the model of investment with no financial market imperfections presented by Romer. The real interest rate is assumed by all economic agents to be a known constant.

a)Write down the equations for Kdot and Qdot -- the time derivatives of K and Q (you are not obliged to re-derive them)
b) draw a graph of Q on K showing the Qdot = 0 curve and the Kdot=0 curve (that is draw the phase diagram)
c) K* is the steady state level of capital. Write down the equation for K* (that is find the point where Kdot=0 and Qdot= 0)
d) Now imagine that initial k = K* but that the state suddenly imposes a tax τ on profits so $\pi(K)$ will be replaced by $(1-\tau)\pi(k)$ Agents assume that the tax will remain constant. Look at the equations for Qdot and Kdot. Which one is changed by the tax ?
e) Illustrate what happens after the unexpected introduction of the tax using the phase diagram.

2) 1: Consider a Ramsey Cass Koopman growth model with no population growth but with technological progress at rate g = 0.02..

1) $Y=0.05K^{0.5} L^{0.5}$

a) Ramsey Cass Koopmans Find the steady state capital labour ratio if consumers act to maximize the presented discounted value of the square root of consumption with a discount rate of 0.04 (so theta = 0.5 and rho = 0.04)

b) Draw a phase diagram of c and k illustrating the convergence to the steady state you found in part c.

3a) consider a consumer who lives for one period and earns and consumes w (so initial wealth = 0). c=w= 2 with probability 0.5 and c=w = 2/3 with probability 0.5. The consumer gets utility ln(c). What is the expected value of the consumer's marginal utility of consumption u'(c) ?

b) Now consider another consumer who lives for two periods earning w1 in period 1 and w2 in period 2. The consumer consumes c1 in period 1 and c2 in period to to maximize
1) ln(c1) + ln(c2)
Subject to the budget constraint
2) c1+c2 = w1+w2

So, using my horrible notation, d = 0.0, r = 0.0 and initial wealth K = 0.

a) assume w1 = 4/3 and w2 is stochastic with
W2 = 5/3 with probability 0. and 1/3 with probability 0.5
Find optimal c1. Also find optimal c2 as a function of w2
b) now assume w2 = 1 with probability 1 (so the expected value of w2 is the same as in part a)
Find optimal c1 and c2.